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Rational Determination of Dynamic Characteristics of Fluid-Flow Reactors

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Abstract

A new model for water treatment and waste disposal reactors was developed. The startpoint to this model is a "triangular" approximation in the shape of the real flow-through curve. This approximation is defined by some characteristic points of time, which are strongly related to the characteristic velocities of waste water. Combination of this simplified function with the kinetic equation of the reaction of the 1st order yields the sequence of the governing equations. Three practical examples show that the proposed "rational" model gives much better results than the traditional plug-flow model.

Key words: fluid-flow reactors, waste treatment, dispersion

Notation

The following symbols are used in this paper:

B – width of reactor,
c – concentration,
c_m – modal concentration,
D_e – effective coefficient of mass transport (diffusive or dispersive),
f_z – extrinsic distribution of detention time,
H – reactor depth,
k – reaction rate constant,
L – reactor length,
L_w – distance between inlet and outlet of reactor,
M_0 – initial mass of pollutant,
M_r – mass of pollutant removed in reactor,
n – direction normal to the outlet,

Q – discharge of fluid,
r – degree of pollutant removal (reactor efficiency),
r_{ef} – total efficiency of reactor,
r_n – expected (necessary) degree of removal,
t – time,
t_d – real detention time,
t_m – modal detention time,
t_p – frontal time,
t_{ps} – mean detention time,
t_{rn} – time of necessary pollutant removal,
v – mean velocity of fluid,
v_d – mean velocity of inflowing stream,
V – reactor volume,
Z – source function.

Additional subscripts:

a – value determined for approximated (“triangular”) distribution,
E – experimental value,
i – index of the considered substance,
j – identifier of mass element,
k – terminal value,
L – organic matter,
m – value determined for ideal-mixing model,
N – nitrogen compounds,
o – initial value,
R – value determined for rational model,
S – suspension,
T – value determined for plug-flow model.

1. Introduction

The present condition of the human environment, is a factor which places high demands on sanitary engineers’ efforts. Seriously polluted natural resources of drinking water deliver increasingly worse raw material to the water treatment plants. At the same time, increasingly severe ecological legal requirements force the continuous improvement of the waste disposal methods.

In consequence, one can state that the discussion and improving of the existing design tools is a dictate of both theory and practice. The more so, that the existing methods are very far from being perfect. There are two main kinds of such methods – “algebraic” and “differential”.

The design methods of the first kind, presented in the bibliography and applied by engineers, are almost entirely founded upon the plug-flow model (Chapra 1998). The essence of this concept is the statement that each mass element passes through a reactor during the time, equal to the mean detention time:

$$t_{ps} = V/Q, \tag{1}$$

and that this value must be identical with the time of necessary removal of pollutants:

$$t_{ps} = t_{rn}. \tag{2}$$

The time t_{rn} can be evaluated experimentally (in a laboratory or using a semi-technical installation) or determined on the basis of practical recommendations (e.g. *Design* . . . 1992).

Another model of this kind is based on the assumption that the content of the reactor is ideally (or fully) mixed. In other words, the concentrations of the waste-water components are the same for the whole reactor and depend on time only.

The second analysed category includes those models, which are based on the equation of the dissolved mass conservation. In the general case this equation has the following form:

$$\frac{Dc}{Dt} = \frac{\partial c}{\partial t} + \left(\mathbf{u}\,\mathrm{grad}\right) c = \mathrm{div}\left(D_e\ \mathrm{grad}\, c\right) + Z. \tag{3}$$

In contradistinction to the “algebraic” methods, which were mentioned above, there exists a significant number of particular “differential” models (Sawicki 2002a, 2003). However, each model of this kind, when applied to the flow through a reactor, is charged by two important sources of faults.

The Eq. (3) in the considered class of problems (i.e. flows through reactors) can be supplemented by the Dirichlet boundary condition for the inlet ($x = 0$), but it would not be possible for the outlet ($x = L$). Such conditions, which can be written as follows:

$$c\,(0,t) = c_0\,(t)\,,\ \ c\,(L,t) = c_k\,(t)\,, \tag{4}$$

would be very attractive from the mathematical point of view, but in the outlet it is just the information in demand. So, the simplified Neumann condition:

$$\frac{\partial c}{\partial n} = 0, \tag{5}$$

is most commonly used in practice. In consequence, the solution to the governing differential equations, which fulfils the relation (5), introduces a considerable error to the final result. The absolute value of this error can reach even 10% (Sawicki and Zima 1995, Zima 2000).

The second source of the error brings the choice of the turbulence model. One can match such a model with quite a high accuracy for almost each individual case. However, the error created by this model during the technological forecasting for new states of the analysed system, cannot be determined exactly.

Finally, the third factor – which must be mentioned – is an economical aspect. The calculations with the use of differential models are rather expensive and time-consuming. Especially when a more complex, three-dimensional model is applied (Olsen and Skoglund 1994). For this reason, some authors try to apply some simplified differential relations (e.g. Mąkinia and Wells 2000). Such relations can be easily solved, but the errors, discussed above, are especially high in these cases.

Concluding, one can state that the models based on the CFD methods are surely a very powerful and important tool for engineers. They should be applied especially when complex and difficult tasks are considered. However, this kind of mathematical description of the problem cannot be accepted as a typical, basic tool in every-day engineering practice. So, it would be very reasonable to look for some compromising method – simple in form (algebraic if possible), but reach in content (at least, giving better results, than the traditional plug-flow model). Such a simplified method is presented in this paper.

2. Functional Characteristics of the Reactor

2.1. Kinetics of the Process

One of the basic parameters is a function describing the temporal course of removal of each substance. For the sanitary engineering problems one can usually assume, that the considered process can be described by the model of the reaction of the first order, when the following equation is valid:

$$\frac{dc_i}{dt} = -kc_i, \;\; c_i \left(t = 0\right) = c_{i0}, \tag{6}$$

whence the degree of pollutant removal equals (Sawicki 2002a):

$$r\left(t\right) = 1 - \frac{c_i\left(t\right)}{c_{i0}} = 1 - \exp\left(-kt\right). \tag{7}$$

An example of the function $r\left(t\right)$ is shown in Fig. 1a.

For simple cases, this relation can be determined theoretically (the reaction velocity constant k can be taken from the bibliography). However, for special cases the function $r(t)$ must be evaluated experimentally.

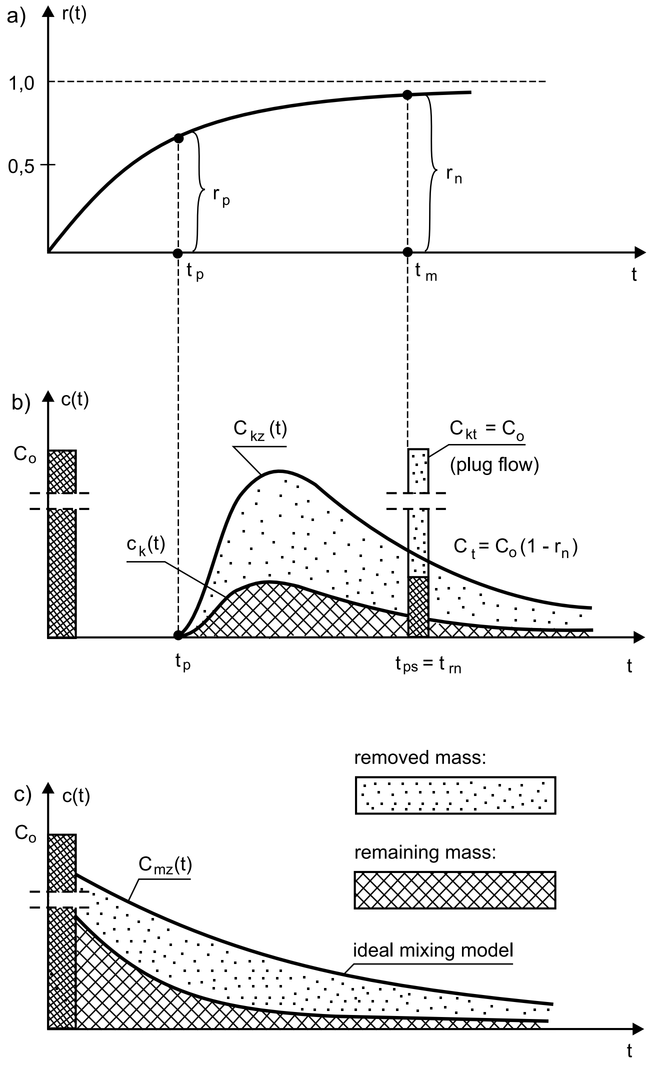

Fig. 1. Qualitative characteristics of the reactor (a – kinetics of the reaction, b – transformation of mass in the reactor, c – ideal-mixing reactor)

2.2. Flow Through the Reactor

The main physical quantity, which expresses the functional characteristics of each reactor, is the real detention time t_{dj}. This parameter is equal to the time in which a mass element number “j” traverses this reactor.

Because of the unavoidable complexity of the velocity field inside each reactor, this time differs for the different parts of the mass “slice”, which enters the reactor at the same initial moment of time. Assuming that the concentration of a pollutant in this initial “slice” equals c_0, the temporal distribution of the mass in the outflowing stream is given by the function $c_{kz}(t)$, when a conservative compound is considered, or by the function $c_k(t)$ for the degradable matter (Fig. 1b). In practice, the function $c_{kz}(t)$ is very often replaced by so-called extrinsic distribution of detention time (Sawicki et al. 2003):

$$f_z(t) = c_{kz}(t)/M, \tag{8}$$

where the auxiliary parameter:

$$M = \int_0^\infty c_{kz}(t)\ dt. \tag{9}$$

For the plug-flow model, mentioned above, the detention time is identical for each mass element (of the fluid and each component dispersed in this fluid, especially – for the tracer, used for the experimental investigations of the reactor) and equal to t_{ps} (Eq. 2, Fig. 1b).

In the theory of reactors another simplified model of flow can be met, viz. the reactor of the ideal mixing (and some combinations of these simple concepts), for which the line $c_{kz}(t) = c_{mz}(t)$ has a specific shape (Fig. 1c).

2.3. Efficiency of the Reactor

Dispersed substance (one or more) can be subject to some transformations inside the reactor. If the kinetics of each reaction can be described by the function $r(t)$, (Fig. 1a), the terminal concentration of the considered component is equal to:

$$c_k(t) = c_{kz}(t)[1 - r(t)]. \tag{10}$$

The removed mass of this component can be calculated from the relation:

$$M_r = \int_0^\infty c_{kz}(t)\ r(t)\ Q\ dt, \tag{11}$$

whereas the total efficiency of the reactor can be expressed as follows:

$$r_{ef} = M_r/M_0, \tag{12}$$

where M_0, defined by the integral:

$$M_0 = \int_0^\infty c_{kz}(t)\ Q\,dt, \tag{13}$$

can also be determined as the initial mass of the tracer, introduced into the reactor. Introducing the extrinsic distribution of detention time (Eq. 8) we can rewrite Eq. 12 as below (for Q = const):

$$r_{ef} = \int_0^\infty f_z(t)\, r(t)\, dt. \tag{14}$$

These relations become more simple for the plug-flow model. The terminal concentration is equal to:

$$c_t = c_0 (1 - r_n) = \text{const} \tag{15}$$

and the total efficiency equals:

$$r_{eft} = r_n. \tag{16}$$

3. Practical Problems

As already mentioned, in technical practice, the plug-flow model is almost always applied. The main cause of this situation is the high level of the formal simplicity of this model. However, what results from the above discussion, real behaviour of each reactor is much more complex.

The acceptability of each model of a reactor is determined by the level of conformity between two functions – real and calculated by the model distributions of the terminal concentration of dispersed matter. Considering the plug-flow model one has to state that these two functions differ very much (Fig. 1b). In consequence, the real efficiency of the reactor (Eq. 14) apparently differs from the value calculated for the plug-flow model.

Two examples of the real cases are shown in Figs. 2 and 3. The first one is related to the technical scale and shows the characteristics of the activated-sludge chamber (waste water discharge Q = 0.524 m^3/s, effective chamber volume V = 4500 m^3, length and width of the chamber L = 84.0 m, B = 15.5 m, mean detention time t_{ps} = 143 min) in Hillsboro (Oregon, USA, Mąkinia and Wells 2000). The second case (Fig. 3) illustrates an experiment which was carried out in a Hydraulic Laboratory of the Gdańsk University of Technology (Sawicki 2002b). The goal of these measurements was to determine the transformation of the conservative tracer (k = 0) through the reservoir. The shape of the investigated object is shown in Fig. 4 and the results of 8 performed tests are presented in Table 1. The symbol t_{20}, used there, denotes the time after which the terminal concentration of the tracer (dissolved NaCl) reaches the value $c_k = 0.20\ c_m$.

The analysis of lines, which are shown in Figs. 2 and 3, confirms the statement, that the quality of the plug-flow model is very low.

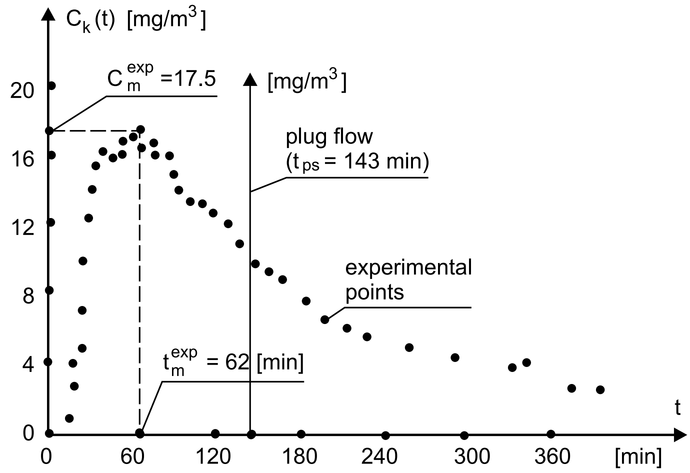

Fig. 2. Real distribution of the tracer concentration (Mąkinia and Wells 2000)

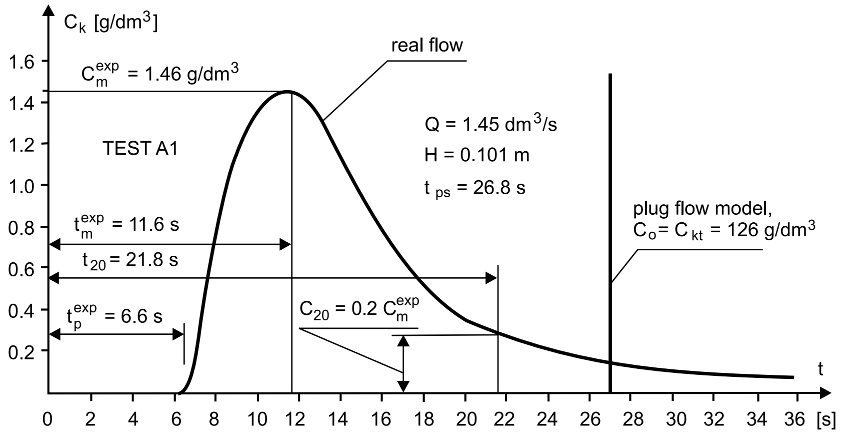

Fig. 3. Laboratory distribution of the tracer concentration (Sawicki 2002b)

Table 1. Parameters of the laboratory experiment

Test	h	H	Q	V	t_p^{exp}	t_m^{exp}	t_{20}	t_{ps}
	[m]	[m]	[dm^3/s]	[dm^3]	[s]	[s]	[s]	[s]
A1	0.101	0.101	1.45	38.89	6.6	11.6	21.8	26.8
B1	0.041	0.106	1.45	40.81	5.8	10.9	19.6	28.1
A2	0.133	0.133	1.45	51.21	8.1	11.3	27.4	35.3
B2	0.088	0.153	1.45	58.91	7.6	11.2	24.4	40.6
A3	0.127	0.127	6.22	48.90	2.7	4.4	8.6	7.9
B3	0.086	0.151	6.22	58.14	2.4	5.0	9.7	9.3
A4	0.155	0.155	6.22	59.68	3.6	5.2	11.3	9.6
B4	0.099	0.164	6.22	63.14	3.0	5.1	12.3	10.2

But on the other hand, as stated above, application of the real characteristics of the reactor would be unrealistic, due to the mathematical difficulties. Even the formulation of the problem is a very complex task. In consequence, everybody must admit that an intermediate method – situated between the over-simplified algebraic plug-flow concept and specialized differential equations – would be welcome.

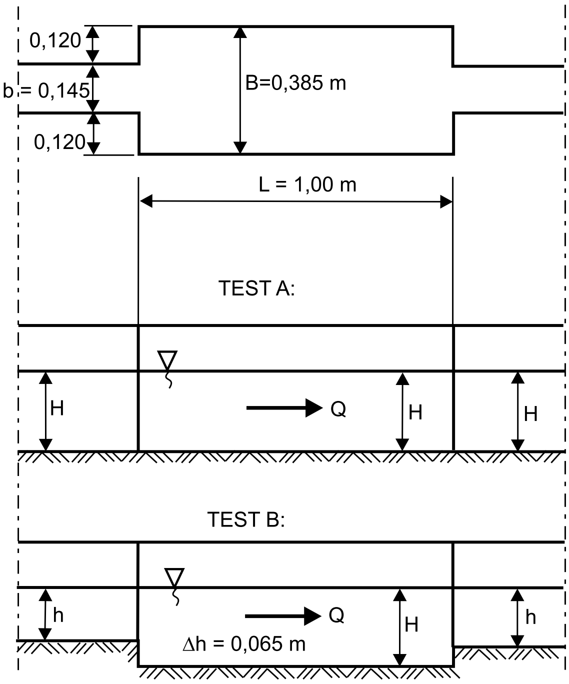

Fig. 4. Laboratory model

4. The Rational Model

4.1. Approximation of the Terminal Concentration Distribution

Our previous considerations can be reduced to the statement that the essential question is a reasonable approximation of the real curve $c_{kz}(t)$. An important difficulty is caused by the specific “tail”, which characterizes this curve and is a consequence of the detention time elongation in the boundary layer and “dead zones” of the reactor. However, taking into account the fact that this “tail” is placed in the region of high intensity of removal of the dissolved matter as the function $r \to 1$ (Fig. 1a), we can attach less importance to the existence of this segment of the line $c_{kz}(t)$.

In effect, one can propose (arbitrarily and intuitively) the approximation of the real diagram $c_{kz}(t)$ by a triangle (Fig. 5). Without any doubt it is a much better solution than the classical case, given by the plug-flow model.

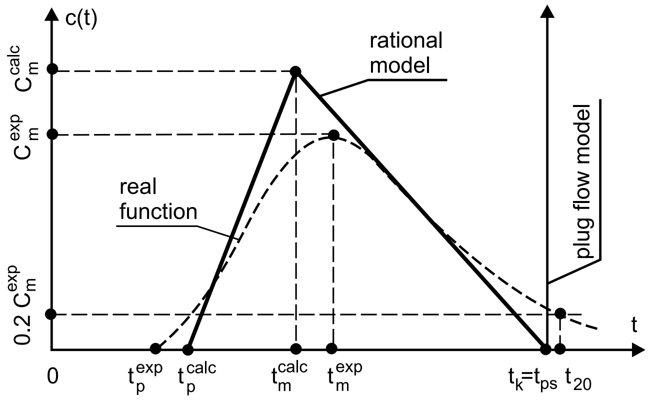

Fig. 5. General concept of the “triangular” approximation

In order to determine the position of this triangle, three characteristic points of time must be found: initial t_p, modal t_m (i.e. the time in which the outflowing concentration has the maximum value) and terminal t_k. The maximal ordinate of this triangle c_m (modal concentration) can be calculated using the condition of the dissolved matter conservation, as the total mass of each component is expressed by Eq. 13.

According to the requirement of the formal simplicity (which is a very important premise of this paper) these three characteristic points of time should be related to other important parametres of the reactor. The initial time t_p can be connected with the maximal characteristic velocity. The most convincing value of this parameter is probably the mean velocity of the inflowing stream v_d (channel or pipe), as when this value is too large – one can observe “a hydraulic short-circuit” in a reactor. Dividing the distance L_w by the velocity v_d one obtains:

$$t_p = L_w / v_d, \tag{17}$$

(very often the distance between the inlet and outlet of the reactor L_w is identical with L). After that period of time the fluid begins to leave the reservoir.

In order to evaluate the modal time t_m one should define some intermediate characteristic velocity of the fluid. It cannot be identified with the mean velocity in the reactor:

$$v = Q/(BH), \tag{18}$$

as in this case Eq. 1 would be obtained, which gives too high a value of the mean time of flow (as was already discussed). However, analysing the results of measurements presented above (Figs. 2 and 3, Table 2) one can note that the time

t_{ps} is approximately twice as long as the modal value t_m. This statement can, to some degree, be related to the fact, that the maximal value of the real velocity profile is considerably greater than the averaged value of this profile (for the laminar flow – two times). Thus it is logical to accept the following evaluation:

$$t_m = t_{ps}/2 = V/2Q = L/2v. \tag{19}$$

Table 2. Comparison of calculated and measured parameters of flow

Test	v_d	t_p^{calc}	t_p^{exp}	v	t_m^{calc}	t_m^{exp}	t_k^{calc}	t_{20}
	[m/s]	[s]	[s]	[m/s]	[s]	[s]	[s]	[s]
A1	0.10	10.0	6.6	0.04	13.4	11.6	26.8	21.8
B1	0.24	4.2	5.8	0.04	14.1	10.9	28.1	19.6
A2	0.08	12.5	8.1	0.03	17.6	11.3	35.3	27.4
B2	0.11	9.1	7.6	0.02	20.3	11.2	40.6	24.4
A3	0.34	2.9	2.7	0.13	4.0	4.4	7.9	8.6
B3	0.50	2.0	2.4	0.11	4.7	5.0	9.3	9.7
A4	0.28	3.6	3.6	0.10	4.8	5.2	9.6	11.3
B4	0.43	2.3	3.0	0.10	5.1	5.1	10.2	12.3

Finally, we have to determine the third characteristic point of time, namely t_k. The repeated analysis of the experimental data leads to the conclusion that the most convenient solution would be the following:

$$t_k = t_{ps} = V/Q. \tag{20}$$

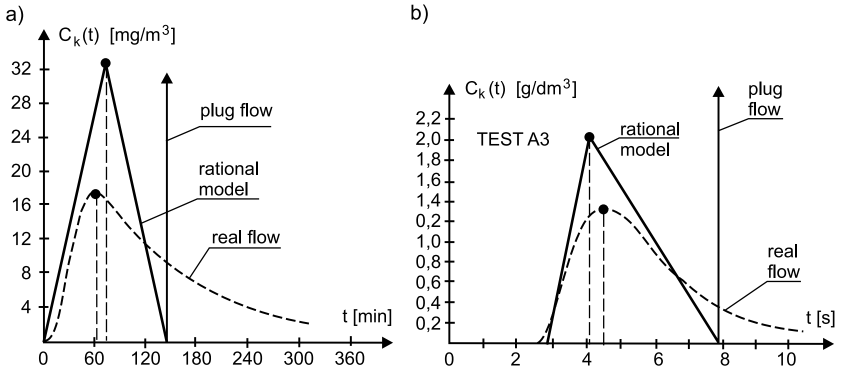

Fig. 6. Examples of the "triangular" approximation (a – real reactor, b – laboratory model)

The method of practical application of the function c_{kz} by a triangular distribution, introduced above, is certainly not delightful, but surely shows much better conformity with the real situation than the plug-flow model (Fig. 6, Table 2), both for the laboratory model and for the technical object (width and depth of the inlet channel of the bioreactor at the Rock Creek WWTP were equal to $B = 2.00$ m and $H = 0.50$ m respectively, which for $Q = 0.525$ m^3/s and $L = 84.00$ m gives $t_p = 160$ s).

4.2. Efficiency of the Reactor

Now we can calculate the total efficiency of the reactor. Let us assume that the intensity of removal of the considered dissolved component is described by Eq. 7. Substituting $c_a\,(t)$ according to the "triangular scheme" in place of the real function $c_{kz}\,(t)$ in Eq. 11 and making use of Eq. 7 one can analytically calculate the proper integral, separately for the "ascending" (mass M_{ra}) and "descending" (mass M_{rd}) parts of this triangle. According to definition (11) we have:

$$M_{ra} = \int_{t_p}^{t_m} c_a\,(t)\; r\,(t)\;\; Q\,dt, \tag{21}$$

$$M_{rd} = \int_{t_m}^{t_k} c_d\,(t)\; r\,(t)\;\; Q\,dt. \tag{22}$$

Making use of the evident geometrical relations (Fig. 5), one can write:

$$c_a\,(t) = \frac{c_m}{\left(t_m - t_p\right)}\left(t - t_p\right), \tag{23}$$

$$c_d\,(t) = \frac{c_m}{\left(t_k - t_m\right)}\left(t_k - t\right). \tag{24}$$

Substituting these formulae into Eqs. 21 and 22, and expressing $r\,(t)$ according to Eq. 7, one obtains:

$$M_{ra} = c_m Q\left[\frac{t_m - t_p}{2} + \frac{\exp\left(-kt_m\right)}{k} + \frac{\exp\left(-kt_m\right)}{k^2\left(t_m - t_p\right)} - \frac{\exp\left(-kt_p\right)}{k^2\left(t_m - t_p\right)}\right], \tag{25}$$

$$M_{rd} = c_m Q\left[\frac{t_k - t_m}{2} - \frac{\exp\left(-kt_m\right)}{k} - \frac{\exp\left(-kt_k\right)}{k^2\left(t_k - t_m\right)} - \frac{\exp\left(-kt_m\right)}{k^2\left(t_k - t_m\right)}\right]. \tag{26}$$

Efficiency of the reactor (12) in this case can be expressed as follows:

$$r_{ef} = \frac{M_{ra} + M_{rd}}{M_0}. \tag{27}$$

The initial mass M_0 for the triangular distribution is equal to:

$$M_0 = 0.5Qc_m\left(t_k - t_p\right). \tag{28}$$

After some rearranging, we finally obtain:

$$\begin{aligned} r_{ef} &\approx r_a = \\ &= 1 - \frac{2}{k^2}\left[\frac{\exp\left(-kt_p\right)}{\left(t_m - t_p\right)\left(t_k - t_p\right)} + \frac{\exp\left(-kt_k\right)}{\left(t_k - t_m\right)\left(t_k - t_p\right)} - \frac{\exp\left(-kt_m\right)}{\left(t_k - t_m\right)\left(t_m - t_p\right)}\right], \end{aligned} \tag{29}$$

4.3. Ideal-Mixing Model

It is evident (see Fig. 1c) that the ideal-mixing model can be considered as a special case of the rational model proposed above, when:

$$t_p = t_m = 0, \tag{30}$$

and t_k is expressed by Eq. 20. Such an attitude affords the possibility of estimating the assumption concerning ideal mixing. On the one hand Eqs. 30 are very simple in form (advantage), but the results given by this model differ very much from the real terminal distribution of the dissolved mass concentration (defect).

An analysis of this case leads also to the conclusion, that the evaluation of the frontal time (Eq. 17) can be generalized in order to include the case, when the reactor is provided with some mixing installations (e.g. aerators). Existence of such mechanical devices practically destroys the individuality of the inlet stream. In consequence we have to replace the velocity v_d in Eq. 17 by another value v_m, characteristic of the local flow, induced by this mixing device, which yields:

$$t_p = t_m = L_w / v_m. \tag{31}$$

This question should be analysed separately. In the extreme case we can have $v_m \to \infty$, which leads to the traditional version of the ideal-mixing model (Eq. 30).

5. Dispersive Model

Also, the dispersive model can be compared with the rational one. In general this first belongs to the higher class, as it is based on the analysis of the intrinsic structure of the fluid velocity (although to a limited degree, as such a model does not contain the full 3D velocity field, but only its mean value, averaged with respect to the stream cross-section). Applying the dispersive model one can obtain the function $c_{kz}(t)$ for a conservative substance, or the function $c_k(t)$ for degradable compounds (Fig. 1b).

However, the dispersive model contains two important faults. First, such a model can be used only when the length of the stream exceeds some critical value (Fisher et al. 1979). In the case of reactors, especially when the flow direction changes in several places of the chamber, this condition can be not fulfilled. Secondly, the accuracy of this model depends greatly on the proper determination of the coefficient of dispersion, which is determined by the individual features of the considered system.

Thus, the dispersive model (e.g. Mąkinia and Wells 2000) is a separate possibility, different from the suggestion presented above.

6. Practical Verification of the Rational Model

6.1. General Remarks

The essence of the proposed method is expressed by Eq. 29, which describes the total efficiency of the reactor. First of all, this method should be applied by engineers, designing new or improving the existing objects. The calculation procedure depends on which parameters and variables are known or given, and which must be found.

The presented version of the rational method refers to those transformations and reactions, which can be described by the 1st order model (Eq. 6). One can have some reservations concerning the acceptability of this model. However, in practice, this attitude is very popular (e.g. Chapra 1998) and moreover – if necessary, Eq. 6 can easily be replaced by a more complex relation, expressing a reaction of the higher order.

In order to verify the rational model, an analysis of the reaction-velocity constant was performed. For three real cases this parameter was determined experimentally and theoretically, by means of the traditional Eq. 7 and using the rational method (Eq. 29).

6.2. Sedimentation

Analysing the experimental data (e.g. Imhoff and Imhoff 1979) one can state that the sedimentation of particles suspended in waste water can be quantitatively expressed in compact form by an equation of the 1st order reaction. The process rate constant equals:

$$k_{SE} = 11.8 \text{ d}^{-1}. \tag{32}$$

As an example let us consider the rectangular secondary settling tank at the Tczew WWTP (Poland), for which we have:

Q = 3600 m^3/d, V = 2590 m^3, $L = L_w$ = 46.0 m, B = 13.4 m, H = 4.2 m, v = 64.0 m/d, v_d = 895.5 m/d, t_{ps} = 0.72 d, t_p = 0.05 d, t_m = 0.36 d, c_{So} = 619 g/m^3, c_{Sk} = 21 g/m^3, r_S = 96.6%.

Making use of Eq. 7 for $t = t_{ps}$ we obtain:

$$k_{ST} = 4.7 \text{ d}^{-1} < k_{SE}, \tag{33}$$

whereas according to the rational method (Eq. 29) the sedimentation rate is equal to:

$$k_{SR} = 12.3 \text{ d}^{-1} \sim k_{SE}. \tag{34}$$

6.3. Organic Matter Biodegradation

The biodegradation rate for the organic substances contained in waste water strongly depends on the technological characteristics of each individual process of purification. Let us, for instance, analyse the situation at the Gdynia WWTP (Poland), where:
Q = 14150 m^3/d, V = 12000 m^3, $L = L_w$ = 192.0 m, B = 12.5 m, H = 5.0 m, v = 226.4 m/d, v_d = 28165 m/d, t_{ps} = 0.848 d, t_p = 0.007 d, t_m = 0.424 d, initial and terminal BOD_5- c_{L0} = 330.0 g/m^3, c_{Lk} = 3.45 g/m^3, r_L = 98.96%.

A laboratory test (Geneja 1992) for such a kind of process gave the following experimental value:

$$k_{LE} = 19.7 \text{ d}^{-1}. \tag{35}$$

For the plug-flow model (Eq. 7) we have:

$$k_{LT} = 5.38 \text{ d}^{-1} < k_{LE}, \tag{36}$$

while Eq. 29 yields:

$$k_{LR} = 18.3 \text{ d}^{-1} \approx k_{LE}. \tag{37}$$

6.4. Denitrification of Waste Water

Elimination of the nitrogen compounds from the waste water is a complex combination of several unit processes. However, even in this case, it is possible to apply the 1st order reaction model (at least as a first approximation – see Chapra 1998). For the Gdynia WWTP (which has been described above) the empirical effective denitrification rate equals (*Mathematical*… 1978):

$$k_{NE} = 3.34 \text{ d}^{-1}. \tag{38}$$

initial total nitrogen content – $c_{No} = 80.67$ g/m^3;
terminal total nitrogen content – $c_{Nk} = 17.22$ g/m^3;
reduction of nitrogen compounds – $r_N = 78.65\%$.

At present the denitrification process at the Gdynia WWTP is characterized by the following parameters:

According to our procedure:

$$k_{NT} = 1.82 \text{ d}^{-1} < k_{NE}, \tag{39}$$

and:

$$k_{NR} = 3.74 \text{ d}^{-1} \approx k_{NE}. \tag{40}$$

7. Conclusions

The method proposed in this paper is based on an approximated "triangular" distribution of the terminal concentration of a substance (pollutant or admixture) dissolved in waste water flowing through a reactor. Combination of this simplified function with the equation of the kinetics of the reaction of the 1st order yields a sequence of mathematical equations which express the interrelations among the characteristic parameters of this reactor. Especially important is Eq. 29, describing the total effectivity of the system under consideration.

The rational model surely proves three important advantages:

1) the "triangular" approximation of the terminal concentration is more logical and correct from the mathematical point of view than the oversimplified distribution, resulting from the plug-flow model;
2) the governing mathematical relations of this new model (although not as simple as traditional) still have an algebraic form and can easily be applied in practice;
3) the reaction-velocity constants, determined for practical situations (sedimentation, biodegradation and denitrification) at the Gdynia WWTP and Tczew WWTP by means of the rational model (Eqs. 34, 37 and 40) are much closer to the proper experimental values (Eqs. 32, 35 and 38) than the respective parameters, obtained by means of the plug-flow model (Eqs. 33, 36 and 39).

The final verification of the rational model requires more practical data. However, even now, one can state that this model can be an attractive offer for sanitary and environmental engineers.

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