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Horizontal Motion of a Rigid Block Resting on Accelerating Subsoil

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Abstract

The horizontal motion of the system: rigid block-accelerating subsoil is analysed, using the most simple approach. First, the method of analysis is described and illustrated for constant coefficient of friction between the block and subsoil. Then, the changing coefficient of friction is taken into account, and its influence on the motion shown. In the next step of analysis, the influence of horizontal force on permanent relative displacement of the block with respect to subsoil is illustrated for constant and changing coefficients of friction. The method presented in this paper form a basis for critical discussion of the Newmark approach, that is a kind of standard in earthquake geotechnics.

Key words: vibrations, earthquake, gravity structure

1. Introduction

The aim of this paper is to analyse the horizontal vibrations of a rigid block, resting on accelerating subsoil. This is the most simple mechanical system, that can help in understanding much more complex behaviour of gravity structures during earthquake excitations, just to mention quay-walls in harbours, see Fig. 1.

It seems that the most extensive studies on the quay-walls behaviour during an earthquake were taken up after the 1995 Hyogoken-Nambu Earthquake which devastated the area around Kobe, a major port city in central Japan. This earthquake had a magnitude of 7.2 with the greatest horizontal acceleration of 0.54 g. Most of the caisson walls in this harbour moved towards the sea about 5 m maximum and about 3 m average (Inagaki et al. 1996). According to Kamon et al. (1996), some of the caisson quay-walls in Kakogawa moved even 16 m seaward. Some earthquake-induced displacements of port structures were also observed in such other places as, for example, the Port of Derince after the 1999 Kocaeli Earthquake (Turkey), see Sumer et al. (2002), Yuksel et al. (2003), or in the

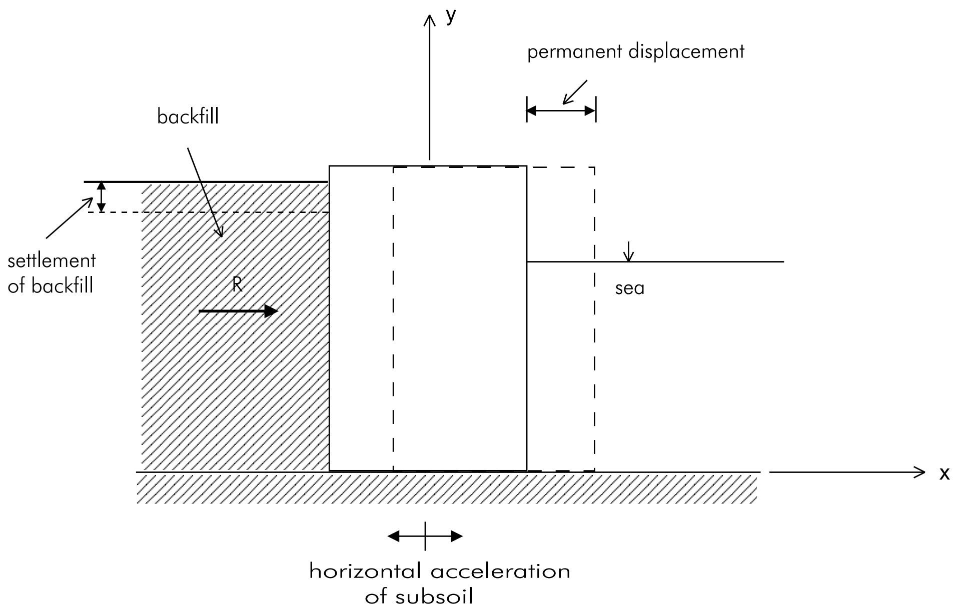

Fig. 1. The complex system: structure-subsoil-backfill-water. Permanent seaward displacement of the structure due to horizontal subsoil shaking. *R* denotes the resultant of forces exerted by backfill and water

Greek Harbour of Kalamata, Pitilakis & Moutsakis (1989). These displacements were coupled with settlements of backfills behind quay-walls.

The dynamic behaviour of the system: gravity retaining structure – subsoil – backfill – water is extremely complex, mainly due to complicated couplings between particular elements of this system. Therefore, various analyses attempting to grasp the problem were restricted only to some chosen features of this system, just to mention papers of Newmark (1965), Sarma (1975), Richard & Elms (1979), Nadim & Whitman (1983), Kamon et al. (1996), Hamada & Wakamatsu (1998), Iai et al. (1998), Ghalandarzadeh et al. (1998), Wood & Kalasin (2004), etc.

The present paper deals with the most simple model of the system shown in Fig. 1, where the uni-axial movement, along the x-axis, of a gravity block, caused by the horizontal cyclic acceleration of subsoil, is considered. The influence of friction between the block and the subsoil on periodic movement of the system is analysed first. The analysis is performed for constant coefficient of friction, as well as for the case when the coefficient of friction suddenly drops after the initiation of relative motion of a block with respect to subsoil. Then, the influence of horizontal force, acting on a block as resultant of forces exerted on the structure by surrounding media, on permanent displacements of this block with respect to subsoil, is analysed. The results presented in this paper form a theoretical basis for possible experiments which can be performed on a "shaking table".

2. Constant Coefficient of Friction

Consider the most simple case of a periodic motion of the gravity block resting on moving subsoil for the case $R = 0$, where R – resultant of forces exerted by backfill and water, cf. Fig. 1. Assume a particular form of this motion, given by the following equations:

$$a_s = -a_0 \sin \omega t \,, \tag{1}$$

$$v_s = \frac{a_0}{\omega} \cos \omega t, \tag{2}$$

$$x_s = \frac{a_0}{\omega^2} \sin \omega t, \tag{3}$$

where: a_s – subsoil's acceleration; v_s – subsoil's velocity; x_s – subsoil's displacement; a_0 – amplitude of acceleration; ω – angular frequency, t – time. Obviously: $v = \dot{x}$ and $a = \dot{v} = \ddot{x}$, where the dot represents differentiation with respect to time.

The above equations were chosen for the sake of simplicity. For example, the net displacement of subsoil, after a single cycle, is equal to zero. The system of co–ordinates x, y is fixed in space, as shown in Fig. 2.

Mass of the block is $m = Q/g$, where Q – own weight and g = gravity acceleration. The coefficient of friction μ between the block and subsoil is assumed constant, i.e. in the case considered, we do not distinguish the static and dynamic friction. Movement of the block is caused by the resultant friction force F, see Fig. 2b. During a single cycle of the subsoil's motion, one can generally distinguish two types of dynamic behaviour of the system considered, depending on the magnitude of ground acceleration. From the equilibrium of horizontal forces it follows that:

$$F = ma. \tag{4}$$

Note that the resultant friction force cannot exceed its maximum value:

$$F_{\max} = \mu Q, \tag{5}$$

therefore, in the case $F < F_{\max}$, which is equivalent to the following inequality:

$$|a_s| < \mu g, \tag{6}$$

no relative movement of the block with respect to subsoil takes place, i.e. the motion of the block is exactly the same as that of the subsoil. It is the first type of dynamic behaviour, rather trivial.

The relative motion of the block with respect to subsoil will take place when

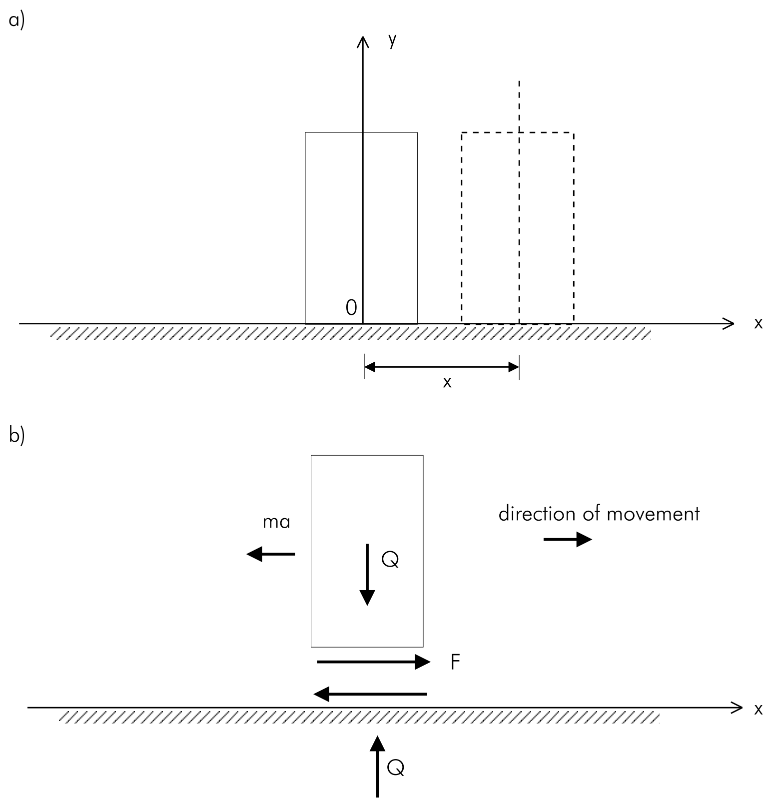

Fig. 2. (a) Absolute movement of a block with respect to the fixed system of co–ordinates x, y; (b) Forces acting on a block

$$|a_s| \geq \mu g, \tag{7}$$

which is the second type of behaviour of the system considered. Denote by t^* the time corresponding to the beginning of relative motion, that corresponds to acceleration $|a_s| = |a^*| = \mu g$. From this moment, the block moves according to the following equation:

$$\ddot{x}_b = -\mu g. \tag{8}$$

Subsequent integrations lead to the well known equations:

$$\dot{x}_b = v_b = -\mu g t + C_1, \tag{9}$$

$$x_b = -\frac{1}{2}\mu g t^2 + C_1 t + C_2, \tag{10}$$

where the subscript "b" distinguishes the motion characteristics related to the block. The integration constants C_1 and C_2 can be determined from the initial condition at $t = t^*$, when the initial displacement and velocity of the block are known, i.e.

$$x_b\left(t = t^*\right) = x_s\left(t = t^*\right) \text{ and } v_b\left(t = t^*\right) = v_s\left(t = t^*\right). \tag{11}$$

The relative motion of the block with respect to subsoil ends, when the velocities of both elements of the dynamic system become equal:

$$v_b\left(t = t^{**}\right) = v_s\left(t = t^{**}\right), \tag{12}$$

where time t^{**} should be determined from Eqs. (2), (9) and (12). Then, the block moves together with the subsoil, until respective condition for initiation of the relative motion is satisfied again.

Example 1

This example illustrates the motion of the system: block-subsoil for Eqs. (1)–(3) and the following data: $a_0 = 9$ m/s^2; $\mu = 0.6$; $\omega = 2\pi$ s^{-1} (period $T = 1$ s).

Within a single harmonic cycle of subsoil motion, one can distinguish four characteristic intervals, obviously for the data assumed, as depicted in Fig. 3. During the interval OA, the block moves together with the subsoil, having the same acceleration, velocity and displacement. At point A, corresponding to $t^* =$ 0.1125 s, the sliding of the block with respect to subsoil begins. Such a relative motion ends at point B ($t^{**} = 0.535$ s) where both velocities are equal, which means that the block sticks to the subsoil again. The common motion is going on in the interval BC. At point C, the sliding of the block begins again, and lasts to the end of the cycle. Respective equations describing the motion of the block are presented in Table 1.

Table 1. Equations describing the motion of the block for data assumed in Example 1

	OA $0 < t < 0.1125$	AB $0.113 < t < 0.535$	BC $0.535 < t < 0.614$	CD $0.614 < t < 1.035$
Acceleration m/s^2	9 sin(6.283 t)	0.5886	9 sin(6.283 t)	0.589
Velocity m/s	–0.143 cos(6.283 t)	–1.7514+5.885 t	–0.1432 cos(6.283 t)	4.69439–5.886 t
Displacement m	–0.228 sin(6.283 t)	0.012–1.751 t + 2.943 t^2	–0.133 –0.228 sin(6.283 t)	–1.756+4.69 t –2.943 t^2

Fig. 4 illustrates the relative velocity and displacement of the block with respect to subsoil, extracted from Fig. 3, for the first two cycles, and Fig. 5 shows these motions in the phase space x, v for the first cycle of ground shaking. Anyone can perform a simple experiment that illustrates qualitatively the behaviour described above. Take a hard-backed book with a smooth surface and put some object on it, as for example a box of matches. Then shake the book horizontally and observe the motion of this object.

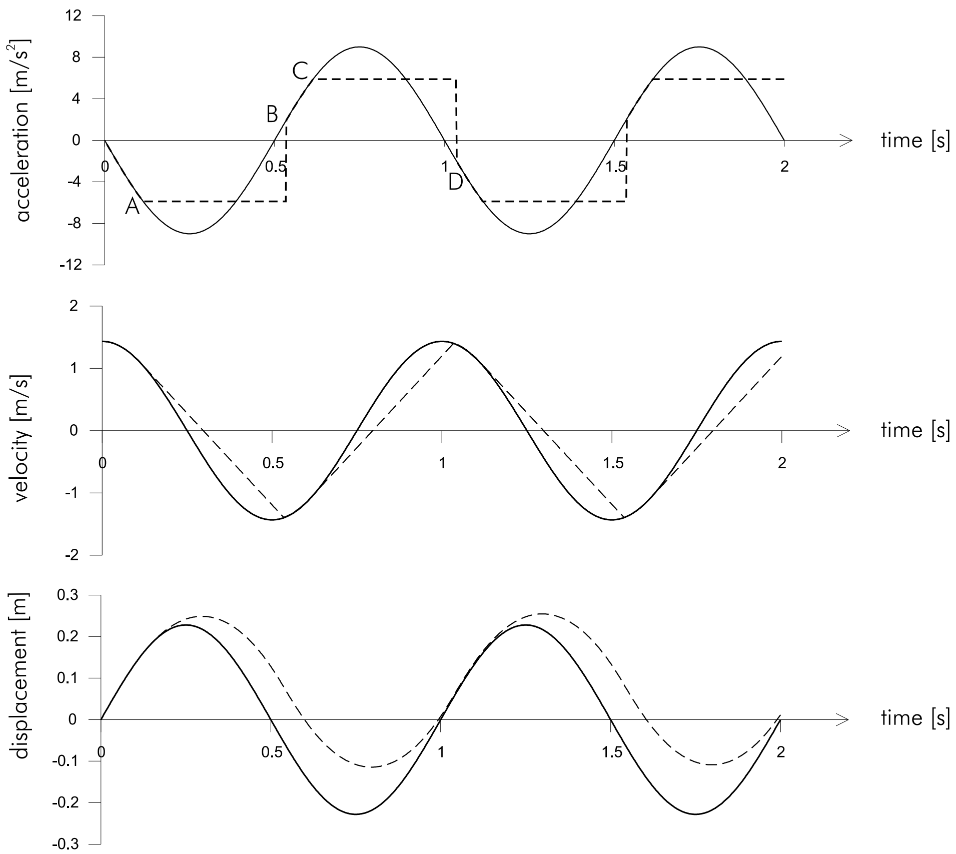

Fig. 3. Accelerations, velocities and displacements of the subsoil (solid line) and the block (broken line) for constant coefficient of friction

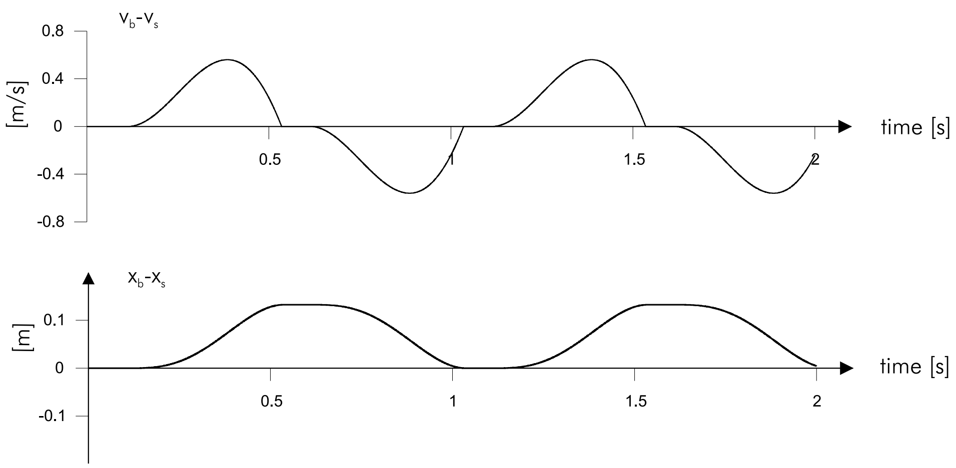

Fig. 4. Relative velocity (a) and displacement (b) of the block with respect to subsoil

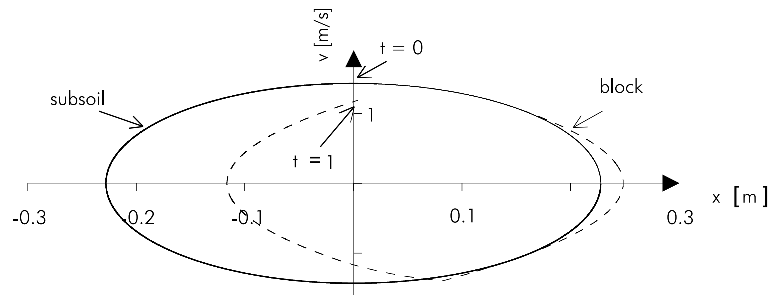

Fig. 5. Motions of the subsoil and the block illustrated in phase space, for constant coefficient of friction

3. Influence of the Coefficient of Dynamic Friction

It is commonly known from theoretical mechanics that the coefficient of friction is reduced after the initiation of relative motion. In order to take into account this phenomenon in the description of the system considered in this paper, let us assume that the coefficient of friction suddenly drops after initiation of relative motion of the block and subsoil, as shown in Fig. 6. Previously introduced coefficient μ will be designated as the coefficient of static friction. The coefficient of dynamic (or kinetic) friction will be denoted as f.

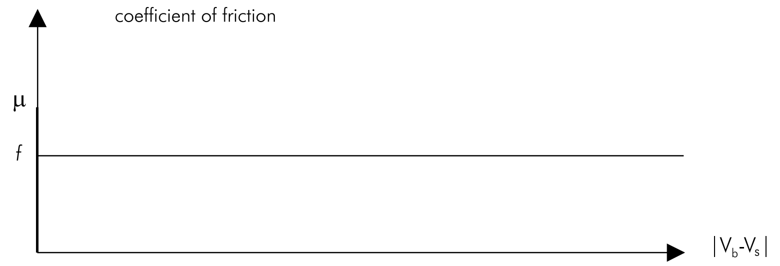

Fig. 6. Coefficients of static and dynamic friction

The method of analysis is similar to that described in the previous section, with the exception that after the initiation of relative motion, the total friction force at the block-subsoil interface equals *mfg*.

Example 2

This example illustrates the motion of the system: block-accelerating subsoil for the data from the previous example, and for $f = 0.3$ (cf. Fig. 6). Fig. 7 shows the accelerations, velocities and displacements of the block and subsoil respectively, and Fig. 8 illustrates the motions in phase space. Note that the behaviour of the system considered is different from that analysed previously, cf. Figs. 3 and 5. In the second case considered, there is practically no common motion of the block and subsoil. There is an initial permanent displacement, caused by assumed initial conditions, and after a few cycles of subsoil shaking, the motion of the block becomes stable.

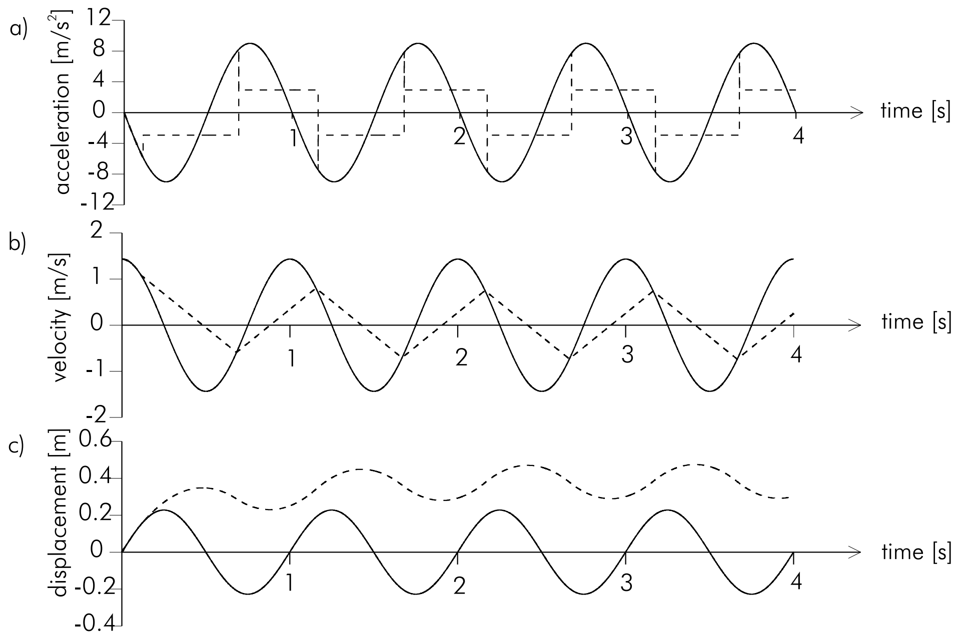

Fig. 7. Accelerations, velocities and displacements of the subsoil (solid line) and the block (broken line) for changing coefficient of friction, cf. Fig. 3

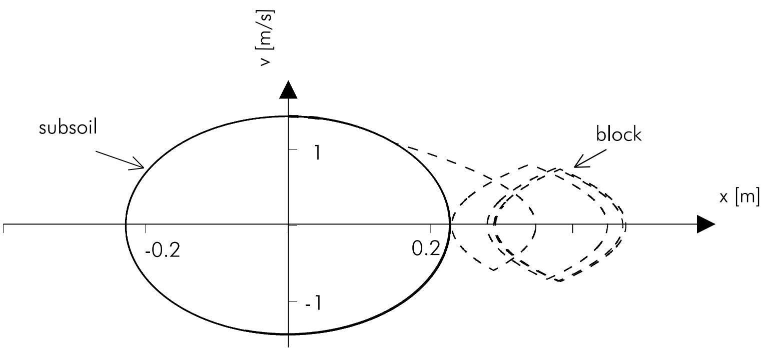

Fig. 8. Motions of the subsoil and the block in phase space for changing coefficient of friction, cf. Fig. 5

4. Influence of Constant Horizontal Force

So far, we have considered the periodic motion of rigid block resting on accelerating subsoil, in the absence of horizontal forces. In order to study permanent displacements of the block with respect to subsoil, caused by reactions of surrounding media, we have to introduce the resultant of some reactions R, see Fig. 1. At the present stage of analysis, we shall neglect the problem of exact determination of this force and its variation in time. In the case of constant coefficient of friction, the block moves according to the following equation:

$$\ddot{x}_b = \left(-\mu + \frac{R}{Q}\right) g, \tag{13}$$

where $Q = mg =$ own weight if the block.

Note that the ratio of R/Q apparently reduces the coefficient of friction when the relative displacement of block with respect to subsoil is positive. In the opposite case, the apparent coefficient of friction increases. The movement of the block is analysed in a way similar to that described previously, with the help of numerical algorithm.

Example 3

Figs. 9 and 10 illustrate the relative motion of the block with respect to subsoil for the following data: $a_0 = 7$ m/s^2; $R/Q = 0.3$; $\mu = 0.6$ and $f = 0.6$ (Fig. 9) or $f = 0.4$ (Fig. 10).

In both cases, one can notice progressive permanent displacements of the block with respect to the subsoil. The magnitude of these displacements depends on such factors as: the history and magnitude of ground acceleration; coefficient of friction; the ratio R/Q. Using the method described in this paper, one can analyse the influence of a particular factor on the motion of the system considered.

5. Discussion of Newmark's Method

A simplified method for estimating the seismic-induced displacements of gravity blocks was proposed by Newmark (1965). Newmark's approach is still recommended in recent guidelines, as for example PIANC (2001), so it seems necessary to review this method in the light of a simple approach presented in this paper. Newmark considers the permanent displacement of a rigid block resting on a moving support, the acceleration of which is presented as rectangular pulse, as shown in Fig. 11.

He derives a closed-form expression for permanent displacement of the block considered, using elementary mechanics, see Eq. (23) in his paper. Similar equation can be obtained using the method described in Section 2, assuming the zero

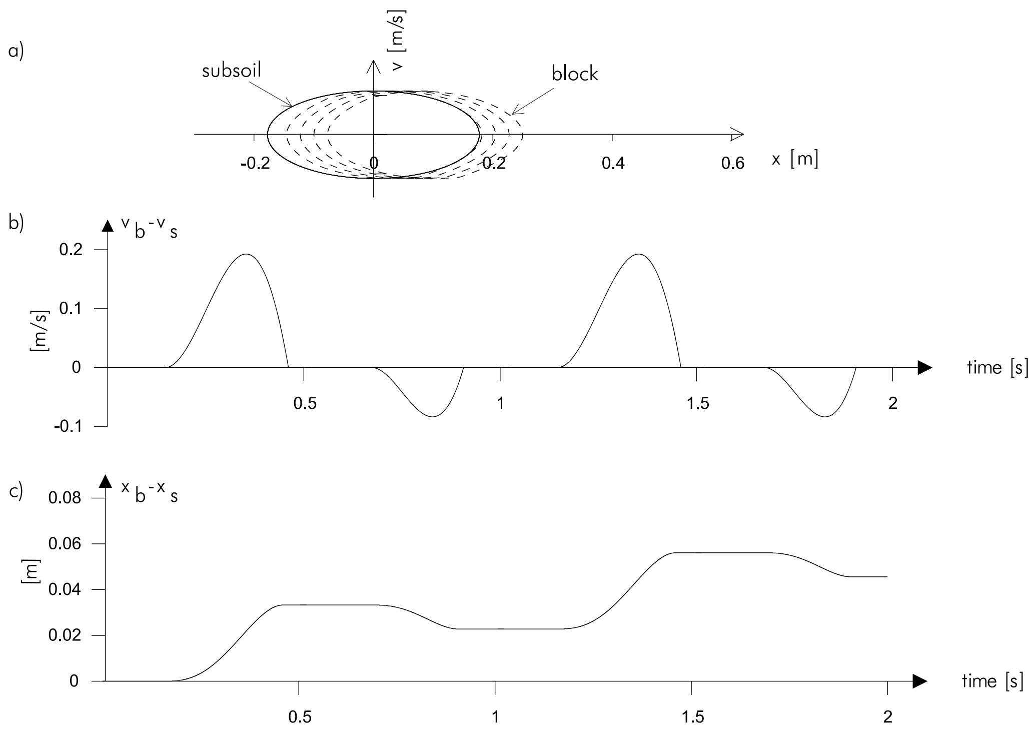

Fig. 9. Influence of the horizontal force on relative motion of the block with respect to the subsoil, for constant coefficient of friction: (a) motion in phase space; (b) relative velocity; (c) relative displacement

initial conditions, i.e. subsoil remains at rest before the acceleration pulse is applied. However, our derivation shows that the relative displacement of the block has the sign opposite to that presented by Newmark, i.e. it should be:

$$x_b - x_s = \frac{Agt_0}{2}\left(1 - \frac{A}{\mu}\right), \tag{14}$$

where A – non-dimensional magnitude of the pulse (see Fig. 11b), t_0 – duration of the pulse.

After deriving his equation, Newmark speculates about the possible application of this approach to determine permanent displacement of the block due to series of such pulses. It is rather hard to agree with Newmark's conclusion that "the result derived above is applicable also for a group of pulses when the resistance in either direction of possible motion is the same". Note that in the time interval $\langle 0, t_m \rangle$, where t_m – time corresponding to the beginning of common motion, the velocities of the block and subsoil differ, see Fig. 12. Therefore, if the second pulse is initiated in the time interval $\langle t_0, t_m \rangle$, Newmark's reasoning breaks down.

The other shortcoming of Newmark's approach is the simplistic shape of acceleration pulse shown in Fig. 11b. Such a shape of impulse is unrealistic as real

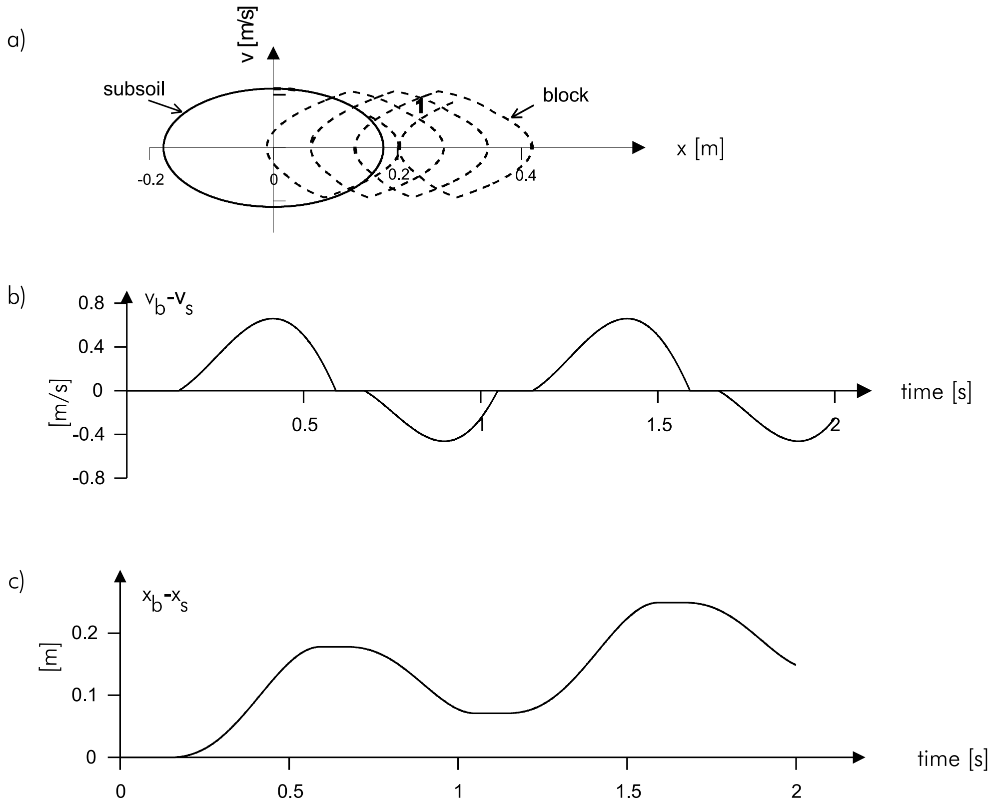

Fig. 10. Influence of the horizontal force on relative motion of the block with respect to the subsoil, for changing coefficient of friction: (a) motion in phase space; (b) relative velocity; (c) relative displacement

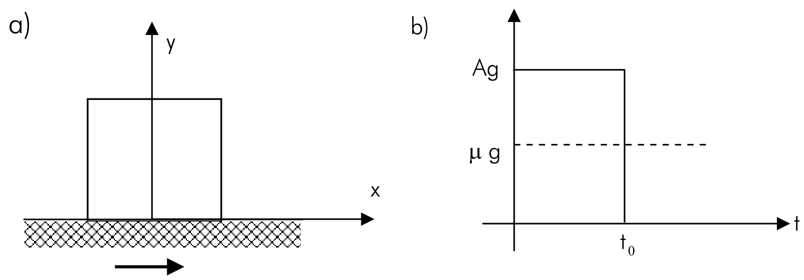

Fig. 11. (a) Rigid block on a moving support; (b) acceleration pulse, after Newmark (1965)

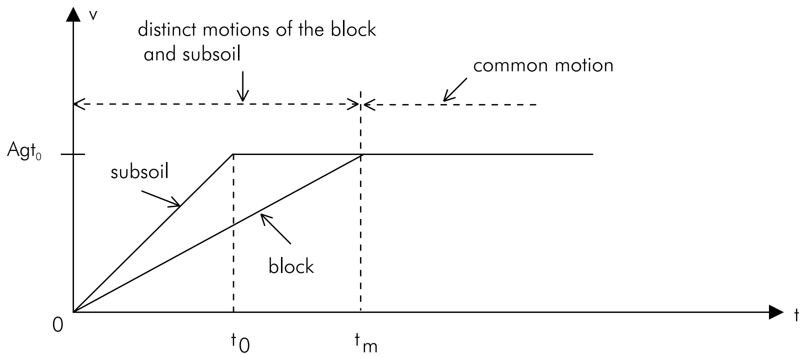

Fig. 12. Velocities of the block and subsoil caused by a single pulse from Fig. 11

records show rather sinusoidal or "wedge" – like shapes. The following examples will illustrate the influence of the shape of impulse on relative displacement of the block with respect to subsoil. Solutions were obtained analytically, using the method described in Section 2, and for the following data: $a_0 = 9$ m/s^2, $t_0 = 0.5$ s, $\mu = 0.6$ and the zero initial conditions.

Example 4: Rectangular Impulse

Eq. (14) gives immediately: $x_b - x_s = -0.595$ m.

Example 5: Triangular Impulse

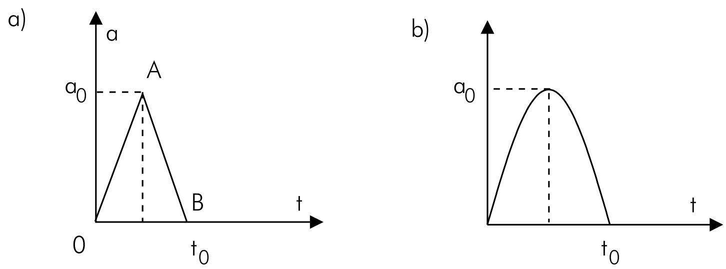

Fig. 13. Triangular (a) and sinusoidal (b) impulses

The motion of subsoil caused by triangular impulse (Fig. 13a) is given by the following equations:

– Interval OA (0 ≤ t ≤ $t_0/2$

$$a_s = 36t; \quad v_s = 18t^2; \quad x_s = 6t^3. \tag{15}$$

– Interval AB ($t_0/2 \le t \le t_0$)

$$\begin{aligned} a_s &= -36t + 18, \\ v_s &= -18t^2 + 18t - 2.25, \\ x_s &= -6t^3 + 9t^2 - 2.25t + 0.1875. \end{aligned} \tag{16}$$

The relative motion of the block and subsoil is initiated at $t = t^* = 0.1635$ s. The block moves according to the following equations:

$$\begin{aligned} a_b &= 5.886, \\ v_b &= 5.886t - 0.4812, \\ x_b &= 2.943t^2 - 0.4812t + 0.02622. \end{aligned} \tag{17}$$

The relative motion lasts until $t = t^{**} = 0.4588$ s, and then the block and subsoil move together again. The relative displacement is $x_b - x_s = -0.0453$ m.

Example 6: Sinusoidal Impulse (Fig. 13b)

The subsoil moves according to the following equations:

$$\begin{aligned} a_s &= 9 \sin 2\pi t, \\ v_s &= \frac{9}{2\pi}(1 - \cos 2\pi t), \\ x_s &= \frac{9}{2\pi}\left(t - \frac{1}{2\pi}\sin 2\pi t\right). \end{aligned} \tag{18}$$

The relative motion is initiated at $t = t^* = 0.1135 s$. Then, the block moves according to the following equations:

$$\begin{aligned} a_b &= 5.886, \\ v_b &= 5.886t - 0.319, \\ x_b &= 2.943t^2 - 0.319t + 0.0117. \end{aligned} \tag{19}$$

It can be checked easily that the sliding of the block lasts until the end of impulse and ceases at $t^{**} = 0.541$ s. In this case, the relative displacement of the block with respect to the subsoil is $x_b - x_s = -0.1332$ m.

The above examples show how drastically Newmark's method overestimates the relative displacement of the block with respect to the subsoil, caused by a single impulse, not to mention other shortcomings of this approach. Therefore, we cannot recommend the Newmark approach for practical purposes, in contrast to PIANC (2001) recommendations. We suspect that none of the twenty seven contributors to those recent guidelines has never applied the Newmark method in analysis of seismic-induced displacements of gravity structures.

6. Conclusions

a) A simple method of analysis of the motion of the system: rigid block–accelerating subsoil was presented. This method is based on principles of general mechanics and can be applied for predicting the seismic-induced displacements of gravity structures. The results presented in this paper can also be used in designing shaking table experiments.

b) Critical analysis shows that Newmark's approach, recommended in recent guidelines as geotechnical standard, has serious shortcomings.

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