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Wave-Induced Stresses and Liquefaction in Seabed According to the Biot-Type Approach

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Abstract

The problem of wave-induced stresses and liquefaction in the seabed according to the Biot-type approach is discussed. The first part of the paper deals with critical analysis of approaches presented in recent literature. It is shown that such approaches do not lead to proper description of the process of wave-induced pore-pressure generation and subsequent liquefaction of seabeds. The second part of this paper deals with the generalisation of the Biot-type approach to the case of shear modulus depending on mean effective stress.

Key words: seabed mechanics, water waves, liquefaction

1. Introduction

The problem of wave-induced stresses in seabed is very important in marine and coastal engineering, as these stresses play a key role in various practical problems, to mention just submarine slides, sinking/flotation of pipelines, bearing capacity of coastal structures, sediment transport, etc.

The seabed consists of the soil skeleton, the pores of which are filled with water, possibly containing some amount of gas. The behaviour of such a mixture is usually analysed using the methods of continuum mechanics, and particularly the methods of mechanics of multi-phase media. Reviews of these methods, with regard to their application to the problem of determination of wave-induced stresses in seabed, are presented in Sumer and Fredsøe (2002), and Sawicki and Mierczyński (2004).

Wave-induced stresses may cause liquefaction of seabed, i.e. such a state of the soil skeleton where the effective stresses vanish. Sumer and Fredsøe (2002) distinguish two different mechanisms of liquefaction, namely:

I. *Residual liquefaction*, which is caused by the progressive build-up of pore pressure.

II. *Momentary liquefaction*, that is caused by the upward vertical pressure gradient in the seabed during the passage of a wave trough.

They analyse both mechanisms within the framework of the Biot-type approach. In this paper, a critical discussion of such a method is presented, which shows that the Biot-type approach cannot properly describe the process of wave-induced pore-pressure generation and subsequent liquefaction of seabeds.

The second part of this paper deals with the generalisation of the Biot-type approach to a more realistic case, when the shear modulus depends on the mean effective stress. The governing equations are derived, and their numerical solution is compared with the solution obtained for a constant shear modulus. Both solutions differ significantly.

2. Biot Equations and Classical Solution

The Biot model is defined by the following system of differential equations (see Sumer and Fredsøe 2002):

$$G\nabla^2 u_x + \frac{G}{1-2\nu}\frac{\partial \varepsilon}{\partial x} = \frac{\partial u}{\partial x}, \tag{1}$$

$$G\nabla^2 u_y + \frac{G}{1-2\nu}\frac{\partial \varepsilon}{\partial y} = \frac{\partial u}{\partial y}, \tag{2}$$

$$G\nabla^2 u_z + \frac{G}{1-2\nu}\frac{\partial \varepsilon}{\partial z} = \frac{\partial u}{\partial z}. \tag{3}$$

$$\frac{k}{\gamma}\nabla^2 u = \frac{n}{K'}\frac{\partial u}{\partial t} + \frac{\partial \varepsilon}{\partial t}, \tag{4}$$

where:

$$\varepsilon = \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z} \tag{5}$$

is the volumetric strain, and: u_x, u_y, u_z – components of the displacement vector; u – pore water pressure; G – shear modulus of the soil skeleton; ν – Poisson ratio; k – coefficient of permeability, γ – specific weight of water, K' – bulk modulus of elasticity of water, n – porosity.

Yamamoto et al. (1978) obtained a closed-form analytical solution of these equations for the problem of infinitely deep seabed, loaded by water waves at its upper surface, see Fig. 1.

In the special case, when G/K' is a small number, they obtained the following formulae for wave-induced effective stress in seabed:

$$\sigma_x' = p_b \lambda z \exp(-\lambda z)\exp[i(\lambda x + \omega t)], \tag{6}$$

$$\sigma_z' = -p_b \lambda z \exp(-\lambda z)\exp[i(\lambda x + \omega t)], \tag{7}$$

$$\tau_{xz} = -i p_b \lambda z \exp(-\lambda z)\exp[i(\lambda x + \omega t)], \tag{8}$$

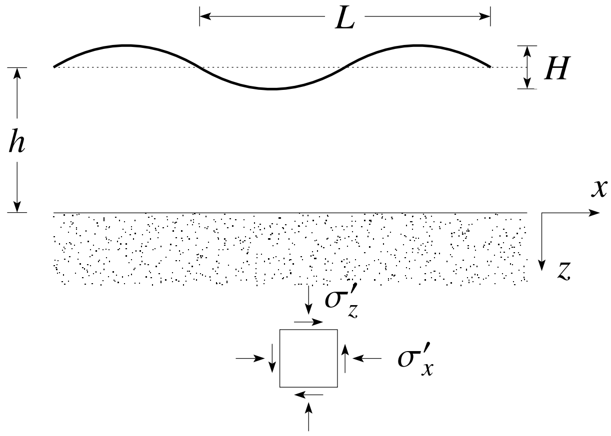


Fig. 1. Water waves propagating over permeable seabed

where:

$$p_b = \frac{\gamma H}{2} \frac{1}{\cosh(\lambda h)}, \tag{9}$$

$$\lambda = 2\pi/L - \text{wave number.} \tag{10}$$

The symbols appearing in the above equations have the following meaning: H – wave height; h – water depth; L – wave length; p_b – maximum value of the bed pressure; ω – angular frequency of waves; i – imaginary unit. The continuum mechanics sign convention is applied, where the plus sign denotes extension.

The wave-induced excess pore-pressure is the following:

$$u = p_b \exp(-\lambda z) \exp[i(\lambda x + \omega t)]. \tag{11}$$

3. Does the Classical Approach Predict Liquefaction?

Recall that the phenomenon of *momentary liquefaction* has been defined as the mechanism of liquefaction caused by an upward vertical pressure gradient in the soil during the passage of a wave trough. Also recall that liquefaction of saturated soil is defined by the condition that the mean effective stress is equal to zero. In the case considered, the mean effective stress is the following:

$$p' = \frac{1}{3}(\sigma'_x + \sigma'_y + \sigma'_z) = \frac{1+\nu}{3}(\sigma'_x + \sigma'_z). \tag{12}$$

It can be checked easily (see Eqs. 6 and 7) that always $p' = 0$ which means that the water waves cannot produce any liquefaction according to the Biot-type approach. Note that this is an "excess" mean effective stress, imposed on the geostatic state.

4. Modification of the Biot-Type Approach

As was shown above, the original Biot equations do not describe such process as the pore pressure generation in the seabed and subsequent liquefaction. In order to include this effect into the framework of Biot's equations an additional hypothesis has been adopted in Sumer and Fredsøe (2002), after McDougal et al. (1989). They have assumed that the pore pressure u generated in saturated soil by cyclic shearing is given by the following empirical formula:

$$u = \sigma_0' \frac{N}{N_l}, \tag{13}$$

where: σ_0' – initial mean effective stress; N – number of loading cycles; N_l – number of loading cycles to cause liquefaction. This latter quantity was also assumed as the empirical formula:

$$N_l = \left(\frac{1}{\alpha} \frac{\tau}{\sigma_0'} \right)^{1/\beta}, \tag{14}$$

where: τ – shear stress amplitude, α and β – constants.

Sumer and Fredsøe (2002) have included Eqs. (13) and (14) into the Biot equations using some formal mathematical manipulations for the uni-axial special case. They have reduced Eqs. (1)–(3) into a single equation

$$\frac{2G(1-\nu)}{1-2\nu} \frac{\partial^2 u_z}{\partial z^2} = \frac{\partial u}{\partial z}, \tag{15}$$

by assuming that the process is independent of the y-direction, and then neglecting the variations with respect to x. The storage equation (4) is then reduced to the following uni-axial form:

$$\frac{k}{\gamma} \frac{\partial^2 u}{\partial z^2} = \frac{n}{K'} \frac{\partial u}{\partial t} + \frac{\partial^2 u_z}{\partial z \partial t}. \tag{16}$$

Differentiation of Eq. (15) with respect to t, Eq. (16) with respect to z, and subsequent elimination of $\partial^3 u_z / \partial z^2 \partial t$ leads to the following single equation:

$$c_v \frac{\partial^3 u}{\partial z^3} = \frac{\partial^2 u}{\partial z \partial t}, \tag{17}$$

where

$$c_v = \frac{k}{\gamma} \frac{2G(1-\nu)}{(1-2\nu) + 2Gn(1-\nu)/K'} \tag{18}$$

is the coefficient of consolidation.

Integration of Eq. (17) gives:

$$\frac{\partial u}{\partial t} = c_v \frac{\partial^2 u}{\partial z^2} + c, \tag{19}$$

where c – integration constant.

Then Eq. (19) is averaged over a single loading cycle which leads to the following equation:

$$\frac{\partial \bar{u}}{\partial t} = c_v \frac{\partial^2 \bar{u}}{\partial z^2} + f, \tag{20}$$

where

$$\bar{u} = \frac{1}{T} \int_{t}^{t+T} u \, \mathrm{d}t, \tag{21}$$

and T – wave period, f – new constant.

According to Sumer and Fredsøe, f may be treated as a source term, and may generally be considered as a function of both space z and time t (page 471). Eq. (20) is the governing equation for the build-up of pore pressure, where the source term was assumed in the following form:

$$f = \frac{\bar{u}}{NT} = \frac{\sigma_0'}{N_l T}, \tag{22}$$

cf. Eqs. (13) and (14).

The above method, adopted probably after McDougal et al. (1989), was also extended to the 2D case, see Sumer and Cheng (1999).

5. Shortcomings of the Modified Biot-Type Model

5.1. Formal Errors

The original Biot equations (1)–(4) do not contain additional terms describing the effect of pore pressure generation due to cyclic loadings, as they have been derived on the basis of Hooke's law (linear elastic soil skeleton) and the Darcy law (diffusion of pore-water). These two laws define the physics of processes which can be described by Biot equations. These equations follow additionally from the basic physical principles, as conservation of momentum and mass, and they are formulated for the 3D case.

Therefore, the basic question was how to include into such a framework the effect of pore pressure generation due to cyclic loadings. Sumer and Fredsøe assumed the additional physical law (13), which is based on empirical observations, and formally included it into the diffusion equation as a source term (Eq. 20). It has been done for a simplified 1D case, and also extended to the 2D case (Sumer and Cheng 1999).

It is hard to accept the opinion that a source term f may be considered as a function of space – page 471 in (Sumer and Fredsøe 2002). The integration constant c appearing in Eq. (19) may depend only on time, i.e. $c = c(t)$, therefore, it is also inconsistent to treat the effective mean stress σ_0' as a function of space

(Eq. 10.86 in Sumer and Fredsøe) since this quantity appears in the source term f, cf. Eq. (22). Eq. (10.81) in Sumer and Fredsøe (Eq. 20 in this paper) may be considered formally proper only when $f = f(t)$ which means that there should be $\sigma_0' = \sigma_0'(t)$.

Formal consequences of the assumption $f = f(z)$ may be serious. Assuming $c = c(z)$ and differentiating Eq. (19) with respect to z gives:

$$c_v \frac{\partial^3 u}{\partial z^3} + \frac{\partial c}{\partial z} = \frac{\partial^2 u}{\partial t \partial z}, \tag{23}$$

which is obviously different from original Eq. (17).

The above result means that either the storage equation (16), or some other relation from which Eq. (15) was derived, is violated, which is inacceptable. Therefore, Eq. (20) cannot be applied for solving boundary-value problems. It is recommended to perform various formal exercises which would show the variety of physically inacceptable assumptions which may lead to Eq. (20).

5.2. Linear Elastic Soil Skeleton

The other shortcoming of the Biot-type approach is the assumption regarding the linear elastic response of the soil skeleton. Experimental data, also those presented by Peacock, Seed (1968), show that the cyclic shear strains increase with increasing pore pressure. This means that the shear modulus G (see Eqs. 1–3) is not constant, but depends on the mean effective stress, i.e. $G = G(\sigma') = G(\sigma_0' + u)$, where $\sigma_0' =$ const – initial effective mean stress for $z =$ const. The modulus G decreases as the pore pressure increases, which is sometimes designated as the "degradation of the shear modulus". This important feature of saturated soil behaviour has not been taken into account in the modified Biot theory.

5.3. Empirical Law as a Source Term

Sumer and Fredsøe assumed the source term f in Eq. (20) in the form of empirical law (22), where the number of cycles to liquefaction depends on a single component τ of the stress tensor, see Eq. (14). Recall that τ means the cyclic shear stress amplitude. Such an assumption is unacceptable from the point of view of applied mechanics, as the constitutive laws should be formulated in terms of the stress/strain invariants.

Water waves induce all the cyclic components of the effective stress tensor, not only the cyclic shear stress τ (nb. expressed in a particular system of co-ordinates), see Fig. 1. Physical laws should be independent of the co-ordinate system, and therefore they should be formulated in the invariant form. In the case of cyclic loading, the most simple measure of the cyclic shear stress is the second invariant of the stress deviator, defined as follows:

$$J = \frac{1}{2}\mathrm{tr}\left(\hat{\boldsymbol{\sigma}}'\right)^2, \tag{24}$$

where

$$\hat{\boldsymbol{\sigma}}' = \boldsymbol{\sigma}' - \frac{1}{3}\mathrm{tr}\boldsymbol{\sigma}'\,\mathbf{1} \tag{25}$$

is the stress deviator and **1** unit tensor.

In the case of the Yamamoto et al. (1978) solution for plane strain conditions (see Section 2), this invariant is the following:

$$J = \frac{1}{4}\left(\sigma_x' - \sigma_z'\right)^2 + \tau^2, \tag{26}$$

and such a quantity should appear in the physical law, instead of a single stress component τ.

It is interesting to note that substitution of the real parts of Eqs. (6–8) into Eq. (26) gives: $J = p_b^2(\lambda z)^2 \exp^2(-\lambda z)$ that is time independent. This result supports the conclusion that the Biot-type solution does not describe the cyclic shearing in seabed.

6. Biot-Type Approach for Varying Shear Modulus

The original Biot theory is based on the assumption of linear elasticity of the soil skeleton, which is a very crude approximation of the real behaviour of granular media. A more restrictive assumption, even within the framework of linear elasticity is that the soil shear modulus is constant. It is commonly accepted in soil mechanics, that the shear modulus strongly depends on the mean effective stress, which is supported by extensive experimental data. Therefore, the assumption that this modulus is constant leads to wrong predictions of wave-induced stresses and pore-pressures in seabeds. In this section, the modified Biot-type approach, that takes into account dependence of the shear modulus on the mean effective stress, is presented and applied to the analysis of wave-induced stresses in the seabed.

6.1. Governing Equations

Applying the classical procedure as, for example, that presented in Sumer and Fredsøe (2002), one can easily derive the Biot equations for the case $G = G(z)$. These equations take the following form, for the plane strain conditions:

$$\begin{aligned}\frac{\partial u}{\partial x} &= 2G\frac{1-\nu}{1-2\nu}\frac{\partial^2 u_x}{\partial x^2} + G\frac{\partial^2 u_x}{\partial z^2} + \frac{G}{1-2\nu}\frac{\partial^2 u_z}{\partial z \partial x} + \\ &+ \left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}\right)\frac{\mathrm{d}G}{\mathrm{d}z},\end{aligned} \tag{27}$$

$$\begin{aligned}\frac{\partial u}{\partial z} &= 2G\frac{1-\nu}{1-2\nu}\frac{\partial^2 u_z}{\partial z^2} + G\frac{\partial^2 u_z}{\partial x^2} + \frac{G}{1-2\nu}\frac{\partial^2 u_x}{\partial z \partial x} + \\ &\quad + 2\left(\frac{\nu}{1-2\nu}\frac{\partial u_x}{\partial x} + \frac{1-\nu}{1-2\nu}\frac{\partial u_z}{\partial z}\right)\frac{\mathrm{d}G}{\mathrm{d}z}.\end{aligned} \tag{28}$$

Note that additional terms $\mathrm{d}G/\mathrm{d}z$ appear in the above equations, in contrast to original Biot equations (1)–(3). Obviously, the system of governing equations contains also the storage equation (4).

The boundary conditions, in the case considered (see Fig. 1), are the following:

$$\begin{aligned}&\sigma_z'(z=0) = 0, \quad \tau_{xz}(z=0) = 0, \\ &u(z=0) = p_b \exp[i(\lambda x + \omega t)], \quad p_b \text{ by Eq. (9)},\end{aligned} \tag{29}$$

and

$$u_x = u_z = 0, \quad \frac{\partial u}{\partial z} = 0 \quad \text{at} \quad z = D. \tag{30}$$

6.2. Shear Modulus

In classical elasticity, the shear modulus is defined by the following relation:

$$\hat{\boldsymbol{\sigma}}' = 2G\hat{\boldsymbol{\varepsilon}}, \tag{31}$$

where $\hat{\boldsymbol{\sigma}}'$ – the effective stress deviator, $\hat{\boldsymbol{\varepsilon}}$ – strain deviator.

In the case of cyclic loadings, the shear modulus can be determined from tests performed in the triaxial apparatus. For this experimental configuration, the stress and strain deviators have the following form:

$$\hat{\boldsymbol{\sigma}}' = \frac{1}{3}q\begin{bmatrix}-1 & & \\ & -1 & \\ & & 2\end{bmatrix}, \quad q = \sigma_z' - \sigma_x', \tag{32}$$

$$\hat{\boldsymbol{\varepsilon}} = \frac{1}{3}(\varepsilon_z - \varepsilon_x)\begin{bmatrix}-1 & & \\ & -1 & \\ & & 2\end{bmatrix}, \tag{33}$$

where z denotes the vertical (axial) direction.

Therefore, in the case considered, Eq. (31) reduces to a single scalar relation:

$$q = 2G(\varepsilon_z - \varepsilon_x). \tag{34}$$

Fig. 2 shows a typical experimental record, corresponding to the "Eregli" gravelly sand (Sawicki et al. 2004).

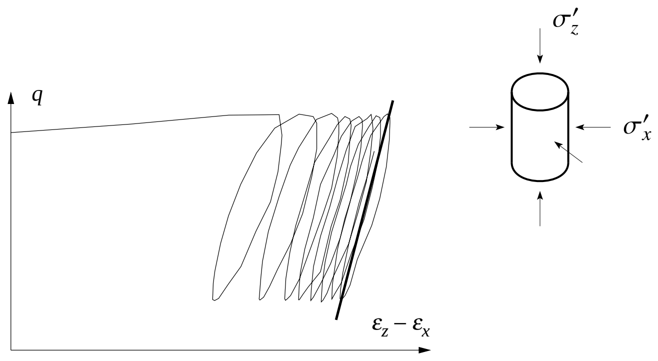


Fig. 2. Experimental determination of the cyclic shear modulus in the triaxial apparatus

The shear modulus is determined from the mean slope of the stress-strain loops. The experiments were performed for different values of the mean effective stress p', in order to determine the dependence of G on this variable. The results of these experiments are shown in Fig. 3, against some analytical approximations.

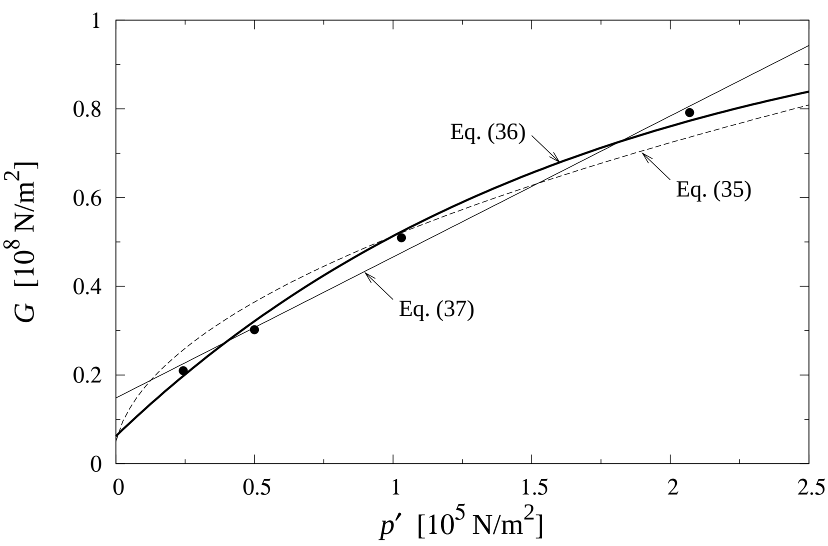


Fig. 3. Shear modulus as function of the mean effective stress. Experimental data (dots) against analytical approximations

Various analytical forms of these approximations are possible, none of them being specially privileged, for example:

$$G = \left[A_1^2 + \left(B_1^2 - A_1^2\right) p'\right]^{1/2}, \quad A_1 = 0.05, \quad B_1 = 0.51, \tag{35}$$

$$G = A_2 + 1 - (A_2 + 1 - B_2)^{p'}, \quad A_2 = 0.062, \quad B_2 = 0.51, \tag{36}$$

$$G = A_3 + (B_3 - A_3)p', \quad A_3 = 0.15, \quad B_3 = 0.47, \tag{37}$$

where $A_i = G(p' = 0)$ and $B_i = G(p' = 1)$. Note that p' should be substituted in stress unit 10^5 N/m^2. The shear modulus is expressed in unit 10^8 N/m^2.

The coefficients A_i and B_i were determined using the least squares method. Respective deviations are the following: 5.6% (Eq. 35), 2.3% (Eq. 36), 2.9% (Eq. 37).

6.3. Examples

The system of equations presented in Section 6.1 was solved numerically for the data adopted from Hsu and Jeng (1994, Fig. 6), for the subsoil of finite depth $D = 25$ m and permeability $k = 10^{-2}$ m/s. Depth of water $h = 70$ m, wave-length $L = 324$ m, wave period $T = 15$ s. Hsu and Jeng have found an analytical solution of the problem considered for constant shear modulus. The numerical algorithm was verified on the basis of this analytical solution, and then applied to the case of shear modulus depending on the mean effective stress.

In this paper, we will show the results of computations corresponding to constant shear modulus $G_0 = 10$ N/m^2 and $\nu = 1/3$, as well as the results corresponding to the shear modulus given by the following equation:

$$G = G_0(4p' + 1), \quad G_0 = 10^7 \,\text{N/m}^2, \tag{38}$$

and for $K' \gg G_0$, which corresponds to water containing a rather small amount of air.

Fig. 4 shows the amplitudes of effective stresses and wave-induced pore pressures, for constant shear modulus (Fig. 4a) and shear modulus given by Eq. (38) (Fig. 4b). The pore pressure changes are smaller in this second case than in the first and the amplitudes of the effective stresses differ significantly, perhaps with the exception of component τ_{xz} of the stress tensor.

Corresponding amplitudes of wave-induced strains are shown in Fig. 5. These strains differ greatly in the cases considered. It should be noted that the effective stresses shown in Fig. 4 are superimposed on the geostatic stress state.

7. Wave-Induced Stresses Against the Failure Criterion

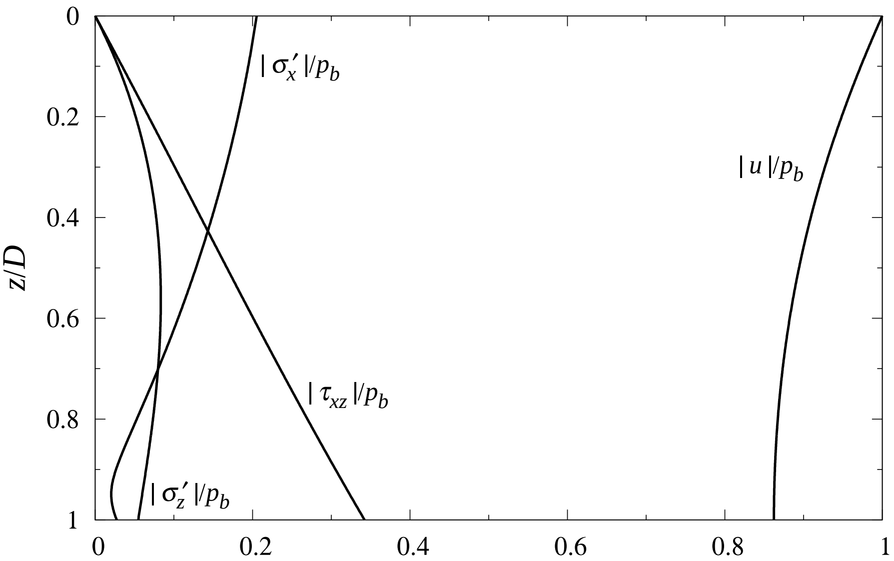
The global effective stresses in seabed consist of the geostatic part (distinguished later by superscript “s”) and wave-induced part (distinguished by superscript “c”):

$$\sigma'_z = \sigma'^s_z + \sigma'^c_z, \tag{39}$$

$$\sigma'_x = \sigma'^s_x + \sigma'^c_x, \tag{40}$$

$$\tau'_{xz} = \tau^c_{xz}, \tag{41}$$

a)



b)

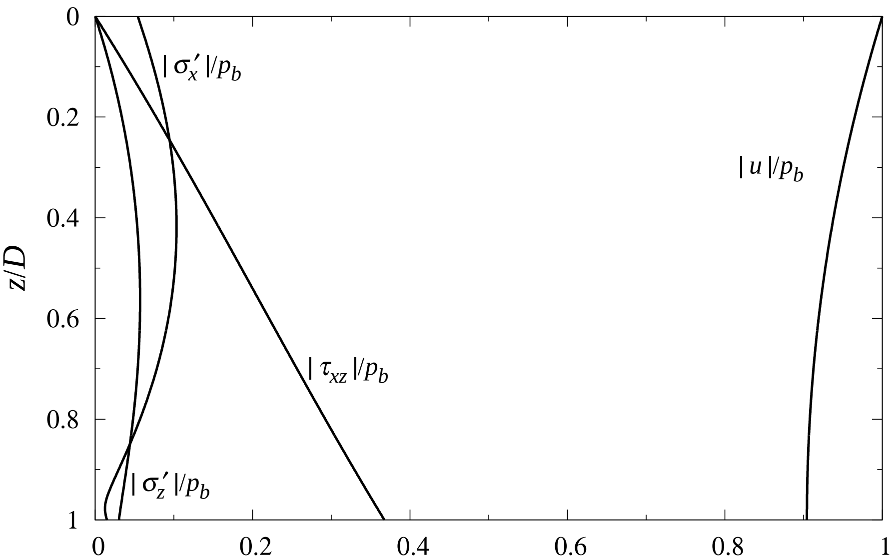


Fig. 4. Amplitudes of wave-induced effective stresses and pore pressures. Constant shear modulus (a); shear modulus given by Eq. 38 (b)

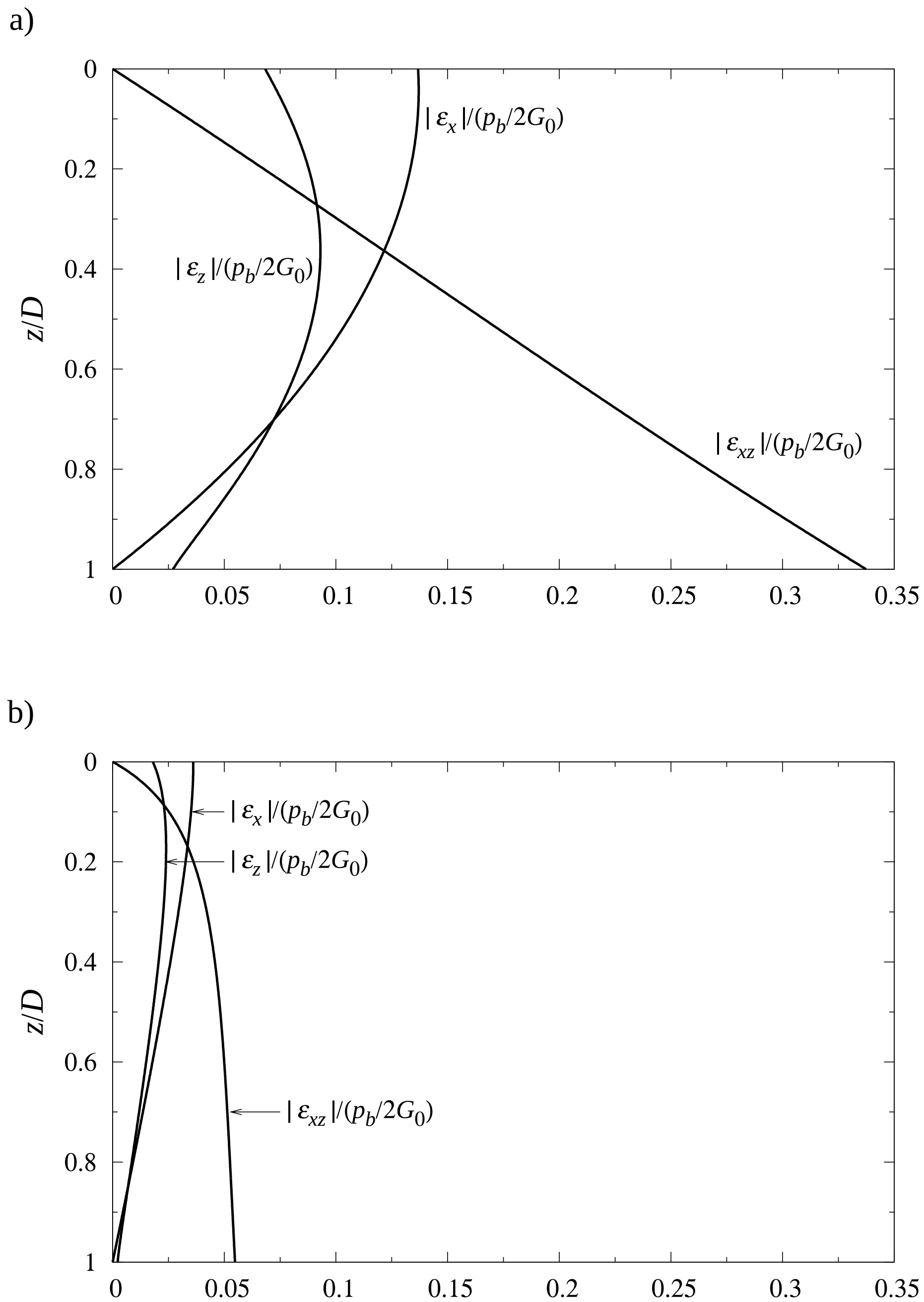


Fig. 5. Amplitudes of wave-induced strains in the seabed. Constant shear modulus (a); shear modulus given by Eq. 38 (b)

where:

$$\sigma_z'^s = \gamma' z' \quad (42)$$
$$\sigma_x'^s = K_0 \sigma_z', \quad (43)$$

γ' – submerged unit weight of granular subsoil; K_0 – coefficient of earth pressure at rest. Wave-induced stresses were determined in the previous section.

It is well known from basic soil mechanics that the effective stresses (39)–(41) should not exceed the Coulomb-Mohr failure criterion:

$$f_{CM} = \left(\sigma_z' - \sigma_x'\right)^2 - \left(\sigma_z' + \sigma_x'\right)^2 \sin^2\phi + 4\tau_{xz}^2 \leq 0, \quad (44)$$

where $\phi =$ angle of internal friction.

We have checked the failure criterion (44) for the following data: $L = 324$ m, $h = 70$ m, $D = 25$ m, $K_0 = 0.5$, $\phi = 30°$. Fig. 6 shows the changes of f_{CM} within a single cycle of loading, at the depth of $z = 1$ m, for constant shear modulus and various wave heights. In this case, the Coulomb-Mohr failure criterion will be exceeded for waves higher than 3.7 m.

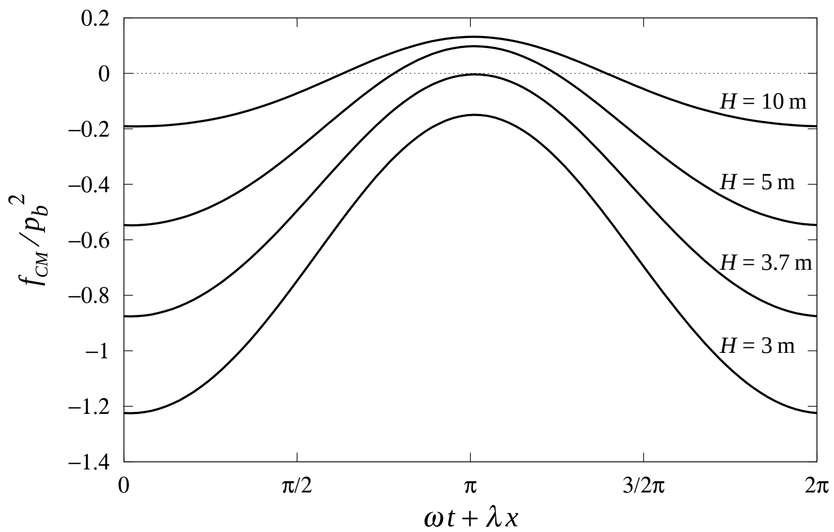


Fig. 6. Waves higher than 3.7 m produce “Biot-type” effective stresses which violate the Coulomb-Mohr failure criterion. The case of $G =$ const

Fig. 7 illustrates the depth of seabed D_{CM} in which the Coulomb-Mohr failure criterion will be exceeded, for two kinds of shear modulus, previously discussed, and for different wave heights. This example shows that the failure criterion will always be exceeded in the upper part of the seabed which is physically unacceptable.

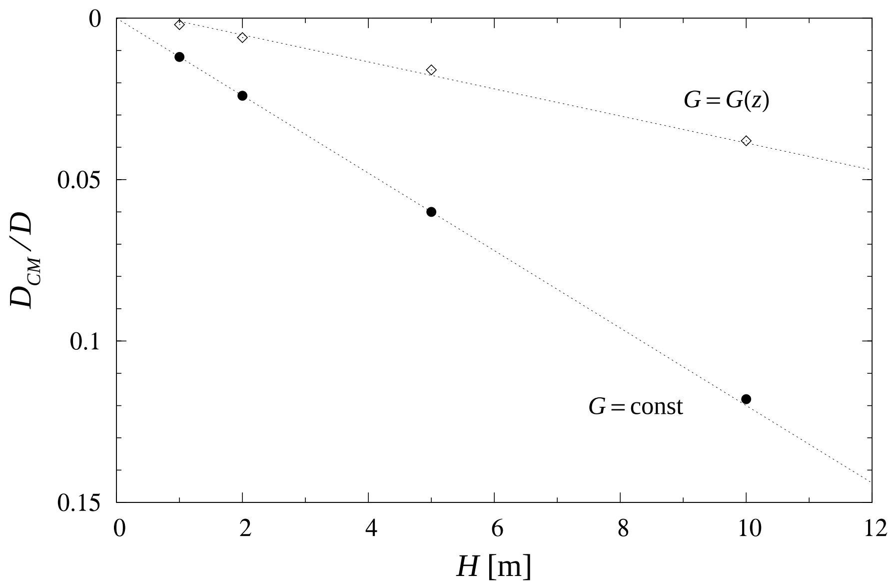


Fig. 7. Thickness of upper layer of seabed where the failure condition is violated as a function of wave height

8. Conclusions

The original results presented in this paper are the following:

a. Critical discussion of existing approaches, based on the Biot theory, to the analysis of wave-induced residual pore-pressures,
b. Solution of the problem of wave-induced stresses for the shear modulus depending on the mean effective stress.

It was shown that the Biot-type approach, in the form presented in many recent publications, cannot be applied to rational and realistic analysis of wave-induced stresses in seabeds. Some of the existing models contain even basic errors, as shown in previous chapters. Even minor modifications of this approach, as that presented in Section 6, lead to the results which significantly differ from classical solution, that is based on unrealistic assumption about a constant shear modulus. Certainly, the Biot-type models cannot predict wave-induced residual pore-pressures, i.e. pore-pressures generated in seabed by water-waves action, and subsequent liquefaction.

References

Hsu J. R. C., Jeng D. S. (1994), Wave-Induced Soil Response in an Unsaturated Anisotropic Seabed of Finite Thickness, *Int. Jnl Numerical and Analytical Methods in Geomechanics*, Vol. 18, 785–807.

McDougal W. G., Tsai Y. T., Liu P.l.-F., Clukey E. C. (1989), Wave-Induced Pore Water Pressure Accumulation in Marine Soils, *Jnl Offshore Mechanics and Arctic Engineering, ASME*, Vol. 111, 1–11.

Peacock W. H., Seed H. B. 1968, Sand Liquefaction under Cyclic Loading Simple Shear Conditions, *Jnl Soil Mechanics and Foundation Engineering Division, ASCE*, Vol. 94(SM3), 689–708.

Sawicki A., Świdziński W., Mierczyński J. (2004), Mechanical Properties of Soils from the Izmit Bay Coast (Turkey) and their Liquefaction Susceptibility, *II Problemowa Konferencja Geotechniki Współpraca budowli z podłożem gruntowym*, Białystok-Białowieża, 2004, 135–145.

Sawicki A., Mierczyński J. (2004), Developments in Modelling Liquefaction of Granular Soils, *Applied Mechanics Review*, in press.

Sumer B. M., Cheng N. S. (1999), A Random Walk Model for Pore Pressure Accumulation in Marine Soils, *The 9th International Offshore and Polar Engineering Conference ISOPE-99*, Brest, France, Vol. 1, 1999, 521–528.

Sumer B. N., Fredsøe J. (2002), *The Mechanics of Scour in the Marine Environment*, World Scientific, New Jersey/Singapore/London/Hong Kong.

Yamamoto T., Koning H. L., Sellmeijer H., Hijum E. van (1978), On the Response of a Poro-Elastic Bed to Water Waves, *Jnl Fluid Mechanics*, Vol. 87, Part 1, 193–206.