

## **Reliability Modelling of Water Distribution System (WDS) for Operation and Maintenance Needs**

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### **Abstract**

From the point of view of enterprises dealing with WDS operation and maintenance, parameter assessment of reliability of WDS functioning is one of the basics for the assessment of the quality of water-supply services afforded by enterprises. To make reliability assessment possible, a two-state model of WDS reliability has been proposed. The basic parameter of this model is the probability of satisfactory performance of the system as a function of its reliability. Other, no less important parameters characterizing the model are probability of part fault state, intensity of occurrence of part fault states, average time of satisfactory performance state and average time of part fault state of the system. The model has been used to estimate the function of a definite WDS reliability on the basis of operational research of reliability during 3 years of its functioning.

**Key words:** reliability, water distribution, modelling

### **1. Introduction**

Reliability of water supply to users by distribution system is perceived and understood in a slightly different way by services responsible for operation and maintenance of these systems and by the users themselves. The users would like to receive water in the appropriate quantity, quality and pressure, every time, irrespective of the conditions of functioning and operation of the system. The services realize that the water-supply network, like all technical objects, is prone to damage and requires organized maintenance to be able to fulfil its objectives. As a result, these two groups of people expect two different scopes of information concerning the reliability of the system's functioning.

- From the point of view of water consumer expectations – analysis and assessment of the consequences of unreliability, mainly faced water delivery cut-offs and their duration.

- From the enterprise's point of view – dealing with operation and maintenance of the WDS – a parameter assessment of WDS reliability as one of the basics of assessment of the quality of water-supply services carried out by enterprises; as well as an analysis and assessment of likely damage and analysis of reliability.

For the purpose of this paper, concentration will be placed on problems connected with modelling of distribution systems with complex reliability assessment of the system for the needs of waterworks supplying water to consumers.

## 2. Reliability Definition of Water Distribution System

A Water Distribution System is characterised by an explicit specification. This specification defines that the system operates in a definite area and should guarantee the delivery of water to the users in a defined quantity, quality, and pressure. Moreover, the water should be delivered at all times as required by the user. This multiplicity of functions of the system signifies that for a given area, the system should guarantee the following three defined standards or requirements simultaneously:

- quantity
- quality
- pressure

Considering this, we present the following definition:

*Reliability of a Water Distribution System is defined as the ability to deliver water to points of use in the required quantity, quality and pressure, and when required by the water user at any time during the systems operation (Kwietniewski 1999).*

In accordance with this definition, occurrences of Water Distribution System failures at the water user will be as follows:

- lack of water
- reduced water quantity and water pressure, at the required quality
- and/or inadequate quality, at the required water quantity and water pressure

In the above definition, reliability of the WDS must be considered in 3 aspects; namely, it must simultaneously provide the required quantity, quality and pressure of water. For the purposes of further analysis, it is assumed that the requirements of pressure and quality are satisfied with a probability approaching “1”. In this way, the reliability assessment and the modelling of the WDS reduces to the aspect of quantity. The guarantee of water quantity by the system is a precondition to the assessment of correct system functionality in terms of water quality and pressure.

### 3. Water Distribution Systems Reliability Model for Operational Needs

On the whole, the hitherto presented divagations on water-supply systems reliability modelling have concentrated on solving problems connected with designing of such systems. They are most often: verifying hydraulic calculations taking into consideration the criterion of water-supply reliability, modelling of the system and optimisation tasks as well as the choice of optimal configuration of the system, including categories of water users. Normally, the necessary element for solving these problems is the geometrical structure of the distribution system, including separating of its components, e.g. pipeline sections, nodes etc. (Abramow 1984, Germanopoulos et al 1986, Ilin 1987, Knapik 1990, Kwietniewski et al 1993, Vreeburg et al 1993, Wieczysty 1990).

In the case of existing water distribution systems, it is necessary to assess their reliability from the point of view of the enterprise's needs. Reliability is in this case, the measure of the level of service afforded in the field of water supply to users. In the case of a WDS, the essential component of reliability assessment is taking into consideration the consequences of non-functionality in a representative area, for example, water delivery deficiency in a certain system area (Vreeburg et al 1993, Slipper and Whipp 1993, Kwietniewski 1999). Long-term reliability tests have proved that a non-functional WDS is related to a specific area (sub-domain) of the delivery system; while in the remaining areas (sub-domains) the system fulfils its function (is usable). This means that, for example, if in a specific sub-domain a total lack of water occurs (totally non-operational), while other parts of the system work at full efficiency (Kłoss-Trębaczkiwicz et al 1990, Kwietniewski 1991).

In order to create a model of reliability for the exploitation of the WDS we represent it as a test object, which functions in an area  $D$ . The physical state of this object at time,  $t \geq 0$ , is characterised by a random vector  $X'(t)$  whose values are related to functional parameters of the system and in this way identify its state of reliability. This takes into account a three-dimensional reliability state<sup>1</sup> in part of the area (sub-domain) of the system of operation or in the whole area of operation of the system i.e.  $S(1, 2, 3)$  where:

1. means full satisfactory performance
2. means partially satisfactory/part fault
3. means total fault

In connection with this, the state of the system may be written as  $\langle A \rangle$ , relative to any non-void (non-empty) sub-domain (fragment) of system  $A \subset D$  at time  $t \geq 0$  in the form of:

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<sup>1</sup> The concept of "reliability state of the WDS" is understood as: 'event or process describing the momentarily working quality of the system'. The reliability state is a component of the operational process, which is defined by random events and processes such as satisfactory performance, unsatisfactory performance (failure).

$$(A, A', i, j) \equiv A_{i;j} \quad \text{where: } i \in \{1, 2, 3\}; j \in \{1, 2, 3\}$$

This notation means that subset  $A$  is located in state  $i$ , and simultaneously its complement  $A' = D - A$  is in state  $j$ .

According to this, the states of the system can be written for modelling purposes in the following way:

- State 1.  $A_{11} \Leftrightarrow D_1$  i.e. Fully Satisfactory Performance State in the whole area ( $D$ ) of operation of the system (FSPS)
- State 2.  $A_{22} \Leftrightarrow D_2$  i.e. Partially Satisfactory State in the whole area ( $D$ ) of operation of the system (PSS)
- State 3.  $A_{33} \Leftrightarrow D_3$  i.e. Totally Fault State in the whole area ( $D$ ) of operation of the system (TFS)
- State 4.  $A_{13}$  (or  $A_{31}$ ) i.e. Fully Satisfactory Performance State in sub-domain  $A$  or  $A'$  and simultaneously Total Fault State in its complement ( $FSPS \wedge TFS$ )

Using these states we can build some reliability models for the exploitation of a WDS. As a result of the analysis and verification of reliability states a two-state model is developed. This describes system behaviour at time  $t \geq 0$  with the aid of the function  $E$ , which takes on values of  $e_1$  and  $e_2$  according to the particular state of the system.

1.  $E = e_1 \Leftrightarrow A_{11} = D_1$   
i.e. Fully Satisfactory Performance State in the whole area  $D$  (FSPS), which means that the whole system works at full productivity, completely satisfying the demand for water.
2.  $E = e_2 \Leftrightarrow A_{13} \wedge A_{31}$   
i.e. Fully Satisfactory Performance State in the sub-domain  $A$  (or  $A'$ ) and simultaneously Total Fault State in its complements ( $FSPS \wedge TFS$ ), which means that the system works with full productivity only in part of the area of the settlement unit.

For simplification in further considerations in the scope of the assessment of the functionality of the WDS, the following symbols and names are assumed: for state 1 (Satisfactory Performance State – SPS) and for state 2 (Partly Faulty State – PFS).

A chart of the described reliability model is presented in Fig. 1.

The basic objective in using this model for assessment of the level of water distribution system functioning as needed by an enterprise is assessing the probability of the time of satisfactory performance of the system  $P_{SPS}(t)$  interpreted as a function of reliability of the system. Apart from this key index, to assess the reliability using the proposed model, a set of other parameters is defined in Table 1 (Kwietniewski 2002).

**Table 1.** Indices for Water Distribution System reliability model for operational needs

Reliability indices	Definition	Describing Formulas
Probability of a Satisfactory Performance State for system $P_{SPS}(t)$	The probability of an event occurring, such that, in the time interval $(0, t)$ the system will not fail, assuming that at the beginning of this time interval the system was fit for use. It also means that time $t_{SPS}$ will be no shorter than given time $t$	a) $P_{SPS}(t) = P(t_{SPS} \geq t)$ b) $P_{SPS}^*(t) = 1 - [n_{SPS}(t)]/N_{SPS}$
Probability of a Part Fault State for system – $F_{PFS}(t)$	The probability of an event occurring, such that, in the time interval $(0, t)$ the system will go into the part fault state (will fail) assuming that at the beginning of this time interval the system was in a state of satisfactory performance. It also means that up-time $t_{SPS}$ will be shorter than given time $t$	a) $F_{PFS}(t) = F_{PFS}(t_{SPS} < t)$ b) $F_{PFS}^*(t) = [n_{SPS}(t)]/N_{SPS}$
Intensity of occurrence of the Part Fault State of system – $\lambda_{PFS}(t)$	The number of occurrences of the Part Fault State during a unit of time	a) $\lambda_{PFS}(t) = \{dE[v_{PFS}(t)]\}/dt$ b) $\lambda_{PFS}^* = n_{PFS}/\Delta t$
Mean Satisfactory Performance Time – $T_{SPS}$	The expected value of the random variable which describes the length of time in which the system is in the Satisfactory Performance State	a) $T_{SPS} = \int_0^{\infty} t \cdot f_{SPS}(t) dt$ b) $T_{SPS}^* = (1/N_{SPS}) \sum_i^{N_{SPS}} t_{SPSi}$
Mean Part Fault Time – $T_{PFS}$	The expected value of the random variable which describes the length of time in which the system is in the Part Fault State	a) $T_{PFS} = \int_0^{\infty} t \cdot f_{PFS}(t) dt$ b) $T_{PFS}^* = [1/N_{PFS}] \sum_k^{N_{PFS}} t_{PFSi}$

**Denotation:**

Formulas: a) basic, b) for empirical indices on the basis of tests;

$P_{SPS}^*(t)$ ,  $F_{PFS}^*(t)$ ,  $\lambda_{PFS}^*$ ,  $T_{SPS}^*$ ,  $T_{PFS}^*$  – estimators of indices  $P_{SPS}(t)$ ,  $F_{PFS}(t)$ ,  $\lambda_{PFS}$ ,  $T_{SPS}$ ,  $T_{PFS}$  respectively, which are determined based on operational data;

$n_{SPS}(t)$  – number of events for which the Satisfactory Performance Time value  $t_{SPS}$  is less than a given value  $t$  ( $t_{SPS} < t$ );

$N_{SPS}$  – number of all recorded values of the Satisfactory Performance Time for the system during the period of test observation;

$E[v_{PFS}(t)]$  – expected value of the random variable  $v_{PFS}(t)$ , which describes the number of Part Fault States which occurred up to time  $t$ ;

$n_{PFS}$  – number of Part Fault States recorded during time interval  $\Delta t$ ;

$\Delta t$  – length of time interval into which the observation period is divided;

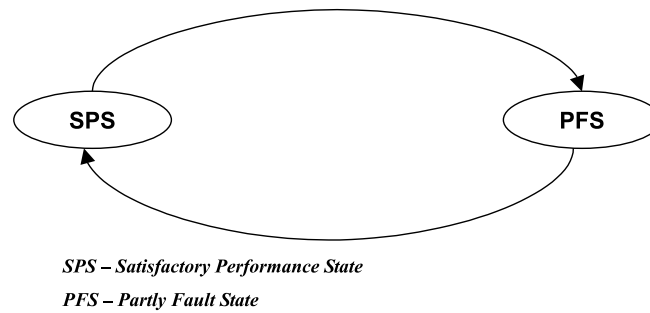
$f_{SPS}(t)$  – time probability density of duration in the Satisfactory Performance State;

$t_{SPSi}$  – value of the  $i$ -th time period of Satisfactory Performance State;

$t_{PFSi}$  – value of the  $i$ -th time period of Fault State;

$f_{PFS}(t)$  – time probability density of duration in the Part Fault State;

$N_{PFS}$  – number of all recorded values of the Part Fault Time for the system during the period of test observation.



**Fig. 1.** A chart of the reliability distribution system functioning

#### **4. Estimation of the Function of Water Distribution System Reliability on the Basis of Selected System**

To estimate the function of water distribution system reliability operational research on reliability has been conducted on a selected distribution system during 3 years of its functioning. The system studied comprised 600 km of pipes, supplying water to 300,000 inhabitants.

On the basis of the data presented on the realization of random variables:

- Satisfactory Performance Time ( $t_{SPS}$ )
- Part Fault State Time ( $t_{PFS}$ )

the function of system functioning reliability has been assessed as probability of satisfactory performance time.

In the research period, 302 realizations of satisfactory performance state time  $t_{SPS}$ , and the same number of realizations of part fault state  $t_{PFS}$  have been observed.

From the analysis of the observed values of satisfactory performance time and fault state time, it follows that they fall into very wide scopes. Satisfactory performance time is in the scope of 1 to 738 h, and fault state time between 2 to 344 h. These wide scopes of variability of random variables are connected mainly with various types of failures, kinds of failing elements (mainly pipes of different diameters) and the number of simultaneous failures. It may also be observed that the vast majority of satisfactory performance and fault state time values are found in the initial areas of the scopes of their variability. On the whole, about 82% of the value of satisfactory performance time falls in the 1 to 75 h scope and about 97% of the value of fault state time falls in the 1 to 150 h scope. This influences the average values of these times. The average time of satisfactory performance of the researched water distribution system (WDS) is  $T_{SPS} = 46.9$  h (1.96 days) and within the 95 per cent probability, and falls in the  $s$ -confidence interval  $46.9$  h  $\pm$  7.3 h. The average time of part fault state is  $T_{PFS} = 38.7$  h (1.56 days) and falls in the  $s$ -confidence interval  $38.7$  h  $\pm$  5 h within the 95 per cent probability.

To assess the function of distribution system reliability, the hypothesis about exponential distribution satisfactory performance time has been verified by  $\chi^2$  Pearson's test with significance level  $\alpha = 0.05$ . The formula for the achieved reliability function is as follows:

$$P_{SPS}(t) = P(t_{SPS} \geq t) = \exp\left(-\frac{t}{46.9}\right) \quad (1)$$

where:  $P(t_{SPS} \geq t)$  – probability of satisfactory performance state time; probability that satisfactory performance state time  $t_{SPS}$  of the distribution system will be not shorter than given period  $t$ , [h].

The course of the above reliability function is presented in Fig. 2.

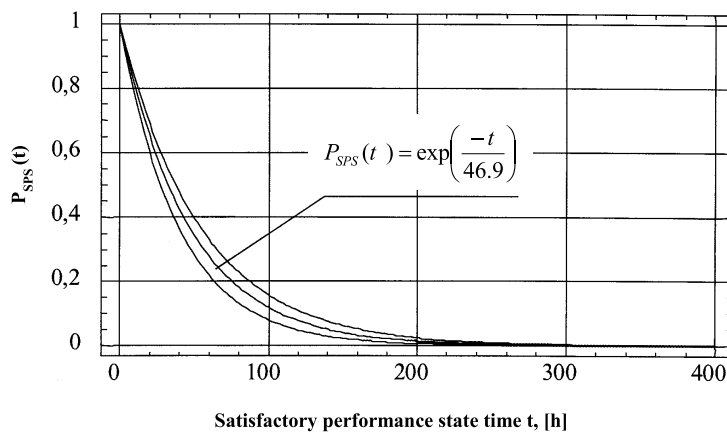


Fig. 2. The function of reliability of the researched water distribution system within the 95%  $s$ -confidence interval

The probabilistic model of the researched system reliability, achieved thanks to statistical verification, enables us to assess the probability of satisfactory performance state of this system. Satisfactory performance state means in this case functioning of the system on the whole area at full efficiency fulfilling water users' needs.

## 5. Conclusions

The measure of the assessment of the quality of water-supply services done by water-supply enterprises is reliability of water supply to the users. The proposed two-state model of reliability of water distribution system functioning enables to assess this reliability from the point of view of an enterprise.

The basic parameter of the model is the probability of Satisfactory Performance State (SPS) in the system as the function of reliability of its functioning. This

function represents the probability of an event occurring, such that, in the time interval  $(0, t)$  the system will not fail. Other, no less important parameters characterising the model are: probability of part fault state, intensity of occurrence of part fault states, the average time of satisfactory performance state and the average time of part fault state.

The model is designed to be applied mainly at the stage of operation of WDS. Thus, the basis for obtaining reliable data (realisation of random variables of 'satisfactory performance stage time' and 'part fault stage time') for the model to assess the reliability of water distribution system functioning is operational research on reliability, i.e. research done in natural operation conditions.

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