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Vertical Velocity Distribution in Open Channels with Moveable Bottom under the Influence of Wind

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Abstract

Water motion in lower courses of large rivers is generated by series of different factors, including also wind. Hitherto existing solutions of wind influence on vertical velocity distribution were obtained by assuming a fixed bottom. However, series of field measurements indicate that the problem of the moveable bottom should not be neglected. Under conditions of acting wind, high velocities are generated close to the bottom. They create so-called slip velocity at the border between mediums, which has an important impact on changes in bottom position. An attempt to solve this problem has been presented in this article.

Notations

A – area of a channel cross-section,
B – channel width at a water surface,
C – velocity coefficient in Chezy formula,
g – acceleration of gravity,
H – channel depth,
H_{sr} – average depth at a cross-section,
i_b – bottom slope,
i_w – water surface slope caused by wind,
i_{zw} – water surface slope,
K_o – turbulent viscosity coefficient steady at a vertical,
n – roughness coefficient in Manning formula,
q – flow through a unit width,
R_h – hydraulic radius,
V_o – flow velocity average at a vertical,
W_{10} – wind velocity at a level of 10 m above water surface,

x – "horizontal" axis of rectangular co-ordination system – directed along a bottom slope at bottom level,

z – "vertical" axis,

ν – coefficient of kinematical viscosity,

χ – wetted perimeter,

λ – ratio of wind stresses at water surface and stresses tangent at the bottom,

ρ – water density above a moveable bottom,

ρ_b – water density within a moveable bottom,

τ_b – stresses tangent to bottom,

τ_w – wind stresses tangent to water surface,

1. Introduction

Vertical velocity distribution is one of characteristic properties of flow in open channels. In a selected transverse cross-section a flow velocity vector is assigned to each point. The ends of these vectors create a surface called the velocity surface. Cross-section points where flow velocity vectors have the same value, create lines of equal velocity. It is usually accepted that at the wetted perimeter flow velocity is equal to zero. The envelope of ends of flow velocity vectors that are situated at one vertical line on a surface of transverse cross-section, presents vertical distribution of flow velocity from the bottom to the water surface. The shape of vertical distribution of flow velocity changes with the change of transverse cross-section and at selected transverse cross-sections differs for different vertical profiles. However, it always has two essential properties:

- it is usually described with the same equation,
- it is assumed that the velocity component at the bottom in the direction of the main flow is equal to 0; $V_b = 0$.

As regards wide channels – of large width – it is usually assumed that average velocity distribution in the plane is almost rectangular and one characteristic vertical distribution of flow velocity that represents a whole cross-section of a channel is used.

Wind influencing water surface can change typical shapes of vertical distributions of flow. At the same time, while taking into account natural channels, there is always a smaller or bigger layer of moveable bottom depending on its type. The longitudinal component at the level of the so-called moveable bottom – the border between typical water and moveable bottom – cannot therefore be assumed to be $V_b = 0$.

It is therefore important not only to state changes, but also to have formulae allowing definition of these changes. Such a problem is analysed further in the paper.

2. Physical Assumptions of the Model

Mathematical model of vertical distribution of flow velocity in channels with acting wind and moveable bottom assumed in the further part of this paper is based on the following assumptions and simplifications:

- uniform and steady movement in a wide prismatic channel is assumed; the channel depth is H, unit flow q = const., average velocity $V_o = \frac{q}{H}$ = const.,
- channel flow is treated as a flat issue in a vertical cross-section along the channel axis and along the moveable bottom, $V_b \neq 0$,
- there are tangent stresses at the water surface level, caused by wind friction; the wave phenomenon that can influence vertical change of momentum, especially in subsurface layers, and transfer of wind stresses was omitted,
- water motion is caused by a component of a weight force in the direction of movement and wind friction on the flow surface (τ_w); there are tangent stresses at the bottom (τ_b) as a reaction to forces causing movement,
- turbulent transfer of momentum in a vertical direction occurs inside a stream,
- the one-dimensional equation of vertical flow distribution arises from the fact that only vertical transfer of momentum is taken into account,
- the Boussinesq hypothesis on turbulent stresses is valid,
- bank influence being insignificant, is omitted.

Remaining assumptions and simplifications have been given further in the text.

3. Mathematical Model of Vertical Distribution of Flow Velocity Including Wind Influence

The solution of the phenomenon of vertical distribution of flow velocity in channels subjected to wind actions has been presented in Buchholz's works (Buchholz 1989, 1990).

Starting with general equations of turbulent movement written with components of turbulent stresses and using the Boussinesq hypothesis, the general equation of vertical distribution of flow has been obtained:

$$V_x(z) = \int \frac{\tau_b \left[1 - (1+\lambda) \cdot \frac{z}{H}\right]}{\rho \cdot K_z(z)} dz \; + \text{const.} \tag{1}$$

The solution of this equation depends on the choice of turbulence model – i.e. turbulent viscosity coefficient $K_z(z)$. Seceral authors, including (Elsner 1987), assume that in wide and relatively deep rivers, this coefficient is constant:

$$K_z(z) = \text{const} = K_o. \tag{2}$$

It was shown in previous works that Eq. 2. can be written as:

$$K_z(z) = \text{const} = K_o = \aleph_o \cdot V_o \cdot H \tag{3}$$

where: $\aleph_o$ – coefficient.

The assumption of a turbulent viscosity coefficient steady in the vertical and on a fixed bottom with velocity component at the bottom equal to zero, gives the following Eq. 4:

$$V_x(z) = \frac{\tau_b \cdot H}{\rho \cdot K_o}\left(\frac{z}{H}\right)\left[1 - \frac{1+\lambda}{2}\left(\frac{z}{H}\right)\right], \tag{4}$$

whereas the average velocity in the vertical is given by:

$$V_o = \frac{1}{2} \cdot \frac{\tau_b \cdot H}{\rho \cdot K_o}\left(\frac{2-\lambda}{3}\right). \tag{5}$$

The relation between surface stresses (wind) and bottom stresses is as follows:

$$\left.\begin{array}{l} 2\tau_b - \tau_w = 6\rho\dfrac{V_o \cdot K_o}{H} \\ \text{or} \\ \tau_b = \dfrac{3\rho \cdot K_o}{H} \cdot V_o + \dfrac{1}{2}\tau_w \end{array}\right\}, \tag{6}$$

whereas the vertically constant turbulent viscosity coefficient is written as:

$$\left.\begin{array}{l} K_z(z) = K_o = \dfrac{g}{3C^2} V_o \cdot H \\ \text{where} \\ C = \sqrt{\dfrac{g}{3\aleph_o}} \\ \aleph_o = \dfrac{1}{3} g \cdot n^2 \cdot R_H^{-1/3} \end{array}\right\} \tag{7}$$

All relationships presented here have been obtained on the assumption that of a fixed bottom and that the velocity component at the bottom is $V_b = 0$.

4. Mathematical Model of the Vertical Distribution of Flow Velocity Including a Moveable Bottom

Water flow in natural channels is characterised by a fact that the phenomenon of a moveable bottom and another movement of sediments accompanying it. Assuming that there is a moveable bottom phenomenon in channels, it is obvious that the velocity at the contact point of water and moveable bottom will not be equal to zero: $V_b \neq 0$.

Zero velocity – $V_{b1} = 0$ – will move "deeper" into the bottom by the value defined here as Z_b (Fig. 1).

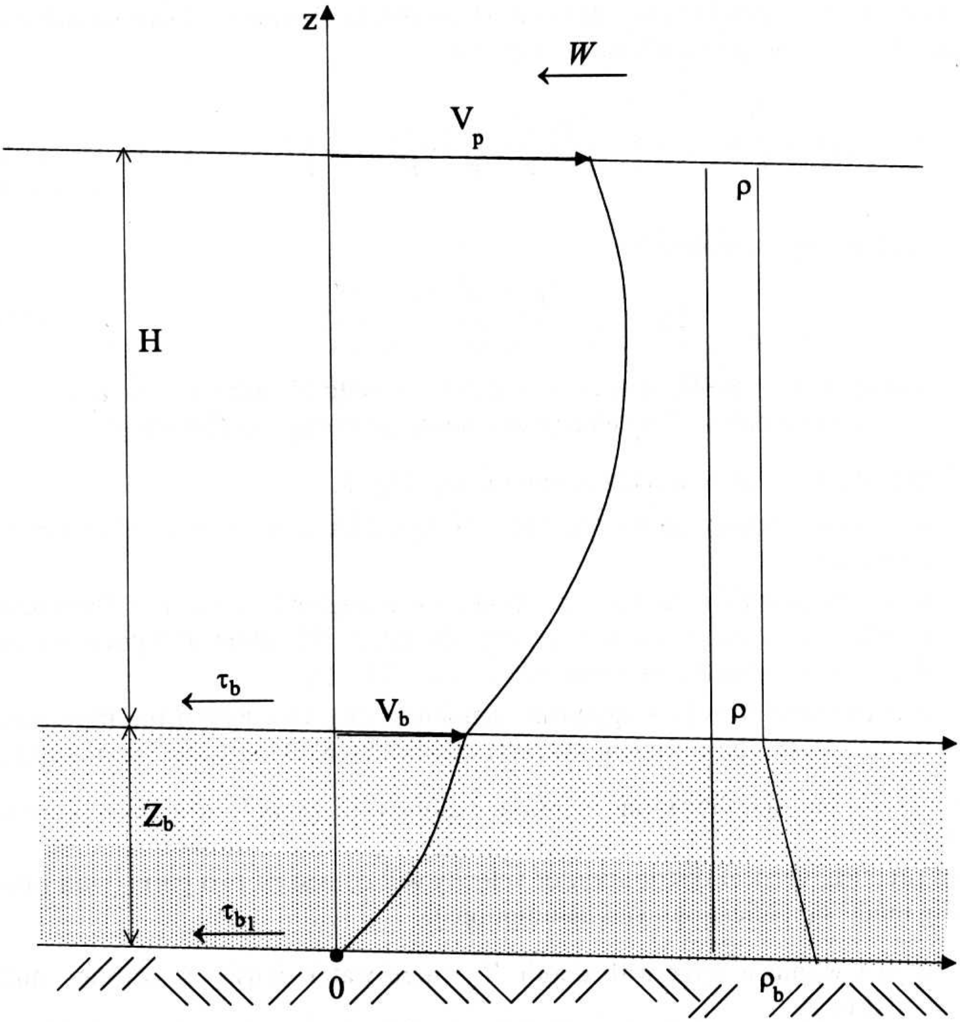

Fig. 1. Calculation zone

With such assumptions the curve of vertical distribution of flow velocity cannot be described by the previously presented Eq. 4.

Assuming the boundary condition (Fig. 1):

$$V_x(z) = V_b|_{z=Z_b} \tag{8}$$

and taking this into account in Eq. 1. (general equation of vertical distribution of flow velocity) the solution to Eq. 1. is obtained:

$$V_x(z) = \frac{\tau_b \cdot H}{\rho \cdot K_o}\left(\frac{z - Z_b}{H}\right)\left[1 - \left(\frac{1+\lambda}{2}\right)\left(\frac{z - Z_b}{H}\right)\right] + V_b. \tag{9}$$

This means, that the boundary condition mentioned does not change the vertical distribution of velocity but only moves it by a constant. Average velocity calculated on that basis in the perpendicular is equal to:

$$V_o = q\,H = \int_0^H V_x(z)\,d\left(\frac{z - Z_b}{H}\right) = \frac{1}{6}\frac{\tau_b \cdot H}{\rho \cdot K_o}\left(\frac{z - Z_b}{H} - \frac{\tau_w}{\tau_b}\right) + V_b \tag{10}$$

and the resulting relationship

$$2\tau_b - \tau_w = \frac{6\,(V_o - V_b)}{H} \cdot \rho \cdot K_o. \tag{11}$$

The most important problem here is to define velocity V_b and value of Z_b.

An attempt to define this velocity was made assuming the following:

- that the layer of moveable bottom is Z_b (Fig. 1),
- that water density above the layer of moveable bottom is constant; $\rho =$ const (Fig. 1),
- that density within the moveable bottom is changing linearly from the water density $\rho =$ const. to the soil density, saturated with water at a point where the bottom particles are immobile $\rho = \rho_b$ (Fig. 1),
- it is assumed (this is a significant simplification), that within the moveable bottom layer, the turbulent viscosity coefficient $K_z(z) =$ const. as in water.

Assumptions and simplifications mentioned along with symbols have been presented in Fig. 1.

In practice, vertical distribution of velocity will consist of two parts (with previously made simplifications and assumptions):

- a part within a layer with depth H, where water density is constant and equal to ρ,
- a part within a layer with depth Z_b, where water density changes – from ρ toρ_b with steady bottom, whereas both these values are treated as known.

There is the same common velocity V_b and the same tangent stresses τ_b at the border of these two distributions. It is obvious that due to the different character of these curves, they will not have a common tangent at the level of velocity V_b, as tthe result of previous simplifications and assumptions.

The change of density within the Z_b layer can be defined as follows:

$$\rho(z) = \frac{\rho_b - \rho}{Z_b}(Z_b - z) + \rho \tag{12}$$

or after transformations:

$$\left.\begin{aligned} \rho(z) &= \frac{Z_b \cdot \rho_b + z(\rho - \rho_b)}{Z_b} \\ &\text{or} \\ \rho(z) &= \frac{Z_b \cdot \rho_b - z(\rho_b - \rho)}{Z_b} \end{aligned}\right\} \tag{13}$$

It is easy to notice that:

$$\left.\begin{aligned} &\text{for } z = 0 \rightarrow \rho(z) = \rho_b \\ &\text{for } z = Z_b \rightarrow \rho(z) = \rho \end{aligned}\right\}. \tag{14}$$

Substituting varying $\rho(z)$ into the general equation of vertical distribution of velocity Eq. 1. and using symbols as in Fig. 1., one can obtain the equation of flow distribution within layer Z_b:

$$V_{x_1}(z) = \int \frac{\tau_{b_1}\left[1 - (1 + \lambda_1)\frac{z}{Z_b}\right]}{\rho(z) \cdot K_o} dz \tag{15}$$

where:

τ_{b_1} – stresses at the level of a steady bottom,

$\lambda_1 = \frac{\tau_b}{\tau_{b_1}}$.

After substituting Eq. 13. into Eq. 15. one obtains:

$$\left.\begin{aligned} V_{x_1}(z) &= \int \frac{Z_b \cdot \tau_{b_1}\left[1 - (1 + \lambda_1)\frac{z}{Z_b}\right]}{K_o\left[Z_b\rho_b + z(\rho - \rho_b)\right]} dz \\ &\text{or} \\ V_{x_1}(z) &= \int \frac{Z_b \cdot \tau_{b_1}\left[1 - (1 + \lambda_1)\frac{z}{Z_b}\right]}{K_o\left[D + m \cdot z\right]} dz \end{aligned}\right\} \tag{16}$$

After differentiating and assuming conditions $V_{x1}(z) = 0$ for $z = 0$, the equation of vertical distribution of flow velocity in Z_b layer is:

$$V_{x_1}(z) = \frac{\tau_{b_1} \cdot Z_b}{K_o \cdot m}\left[\left(1 + \frac{1 + \lambda_1}{\beta}\right)\ln\left(1 + \beta\frac{z}{Z_b}\right) - (1 + \lambda_1)\frac{z}{Z_b}\right] \tag{17}$$

where:

$m = \rho - \rho_b$,

$D = Z_b \cdot \rho_b,$
$\beta = \dfrac{Z_b \cdot m}{D}.$

Taking into account condition $V_{x_1}(Z_b) = V_b$ in the analysis, one obtains:

$$V_b = \frac{\tau_{b_1} \cdot Z_b}{K_o \cdot m}\left[\left(1 + \frac{1+\lambda_1}{\beta}\right)\ln(1+\beta) - (1+\lambda_1)\right], \tag{18}$$

whereas the relationship for average velocity with condition:

$$V_{o_1} = \frac{1}{Z_b}\int_o^{Z_b} V_{x_1}(z)\,dz \tag{19}$$

gives the form:

$$V_{o_1} = \frac{\tau_{b_1} \cdot Z_b}{K_o \cdot m}\left\{\frac{1}{\beta}\left[1 + \frac{1+\lambda_1}{\beta}\right] \cdot \left[(1+\beta)\ln\left(\frac{1+\beta}{e}\right) + 1\right] - \frac{1}{2}(1+\lambda)\right\}. \tag{20}$$

By transforming Eq. 11, one obtains the formula for velocity V_b from within H layer in the following form:

$$V_b = V_o - \frac{(2\tau_b - \tau_w)}{6\rho \cdot K_o} \cdot H \tag{21}$$

and comparing it with Eq. 18. we obtain:

$$V_o - \frac{(2\tau_b - \tau_w)}{6\rho \cdot K_o} \cdot H = \frac{\tau_{b_1} \cdot Z_b}{K_o \cdot m}\left[\left(1 + \frac{1+\lambda_1}{\beta}\right)\ln(1+\beta) - (1+\lambda_1)\right]. \tag{22}$$

Assuming a rectangular channel and that $R_H \approx H$, and by analysing the equilibrium of stream forces (Buchholz 1989, 1990), values of τ_b and τ_{b1} can be defined as:

$$\left.\begin{aligned} \tau_b &= \rho \cdot g \cdot H \cdot i_b - \tau_w \cdot \frac{B}{\chi} \\ \tau_{b_1} &= \rho_b \cdot g \cdot Z_b \cdot i_b - \tau_b \cdot \frac{B}{\chi_1} \end{aligned}\right\} \tag{23}$$

and:

$$\left.\begin{aligned} \lambda_1 &= \frac{\rho \cdot g \cdot H \cdot i_b - \tau_w \cdot \frac{B}{\chi}}{\rho_b \cdot g \cdot Z_b \cdot i_b - \left(\rho \cdot g \cdot H \cdot i_b - \tau_w \cdot \frac{B}{\chi}\right) \cdot \frac{B}{\chi_1}} \\ &\text{and} \\ \lambda &= \frac{\tau_w}{\rho \cdot g \cdot H \cdot i_b - \tau_w \cdot \frac{B}{\chi}} \end{aligned}\right\} \tag{24}$$

where:

$\tau_w = \aleph_w \cdot W_{10}^2$ – measurement data,

$\aleph_w$ – wind roughness coefficient.
Assuming further that unit flow q and depth H are known (from measurements), the only unknown in Eq. 22. will be the value of Z_b. The value of K_o can be taken after (Buchholz 1989):

$$\left.\begin{aligned} K_o &= \frac{g}{3C^2} V_o \cdot (H + Z_b) \\ C &= \sqrt{\frac{g}{3\aleph_o}} \\ \aleph_o &= \frac{1}{3} g \cdot n^2 \cdot (H + Z_b)^{-\frac{1}{3}} \end{aligned}\right\} \tag{25}$$

The following equation may also be formed:

$$q = q_1 + q_2 = \left(V_o \cdot H + V_{o_1} \cdot Z_b\right) \cdot 1_m \tag{26}$$

which would unfortunately complicate calculations.

After defining the Z_b value, the remaining values in Eq. 17–25 can be calculated, while taking into account previous simplifications and assumptions.

Analyses presented here indicate that one can assume the existence of a certain velocity V_b (slip velocity) at a level of moveable bottom and thickness of moveable layer. The presented method which bases on previous solutions (Buchholz 1989, 1990) for a fixed bottom and assuming that certain values $(Q, q, H, B, \chi, \aleph_w, n, \rho, \rho_b, W_{10})$ are known, enables us to define Z_b and V_b both on conditions of wind acting on a water surface in a river and without wind $(\tau_w = 0)$.

5. Conclusions

The solution presented is an attempt at the analytical definition of the layer of moveable bottom Z_b and slip velocity V_b. A solution can be obtained with significant simplifications and having series of very good quality measurement data. The solution would allow for approximate definition of Z_b value, depending on flow and external conditions (wind). It seems that the greatest simplification is the "broken" vertical distribution of density $\rho(z)$ at the Z_b level. As a result, the curve of vertical distribution of flow velocity is not smooth at that point, which definitely does not correspond to a real phenomenon.

It is therefore proposed that in further investigations the continuous distribution of density $\rho(z)$ at a significant distance from the water surface in the direction of the bottom which would be close to the steady one (Fig. 2), without the broken point at the level of Z_b should be assumed.

The equation of $\rho(z)$ distribution can be obtained by detailed measurements of vertical distributions of velocities at selected base cross-sections with and without

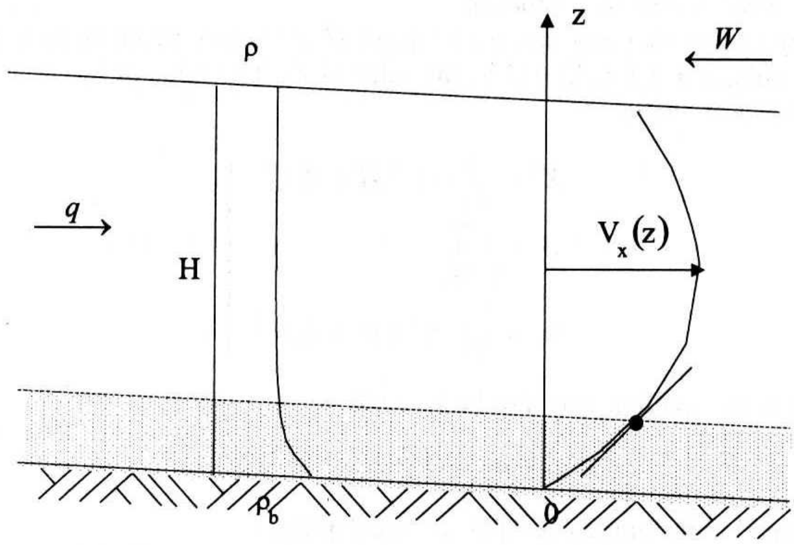

Fig. 2. Vertical distribution of density and velocity

wind and for different flows. For these purposes using an ADCP (Acoustic Doppler Current Profiler) e.g. "Rio Grande" seems appropriate. This would allow the defining of a more real course of phenomenon.

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