Comparison of Volume- and Surface-Detention-Time Distributions

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Abstract

The main process parameter for flow reactors (and in many cases also storage reservoirs) is the detention time. In technical practice the mean value of this parameter is commonly used. This version of the detention time results from the simplified plug-flow-model, which describes the real course of considered fluid-flow-phenomena only to a low degree. In order to obtain more precise characteristics of the system, one should make use of the so-called *extrinsic distribution of detention time*. Unfortunately, determination of such a curve for the whole reservoir (*volume-detention-time*) is rather expensive and time-consuming. Preceding this, a very attractive alternative is given by the *surface-detention-time*, which is determined for the upper layer of the liquid, close to the free surface. Investigation of the equivalence of both these possibilities is the subject of this paper.

Notation

The following symbols are used in the paper:

B	–	width of reactor,
c	–	concentration,
c_z	–	extrinsic distribution of detention time,
f	–	auxiliary function,
F	–	reservoir crosssection,
h	–	depth of channel,
H	–	depth of reactor,
k	–	reactionvelocity constant,
L	–	length of the reactor,
M_i	–	integral parameter,
n	–	instantaneous number of floats,

N –   total number of floats,
Q –   discharge,
r –   degree of pollutant reduction,
r_n –   required degree of reduction,
s –   distance along the mass trajectory,
t –   time,
t_p –   real detention time,
t_{ps} –   mean detention time,
t_{rn} –   time of reduction required,
v_c –   mean velocity in the channel,
v_r –   mean velocity in the reservoir,
V –   total volume of the reactor,
ε_c –   relative difference of concentration,
ε_t –   relative difference in time.

Additional indices:

a –   value determined for floats motion,
e –   value determined for tracer flow,
f –   value determined for real flow,
i –   number of the dissolved component,
j –   index of the mass element observed,
k –   terminal value,
m –   maximal value,
max   –   maximal value,
min   –   minimal value,
o –   initial value,
p –   value, determined for the plug-flow-model,
r –   after reactions,
w –   without reactions.

1. Introduction

Three-dimensional tanks can serve two main purposes – accumulation of fluid, without any change of its properties (*storage reservoir*) or providing facilities for chemical, biological and/or physical transformations of this fluid (*reactor*). In the second case, one of the most important parameters of the system is *detention time* t_p, which describes (more or less precisely) the time of flow through the reactor of the each fluid element.

The significance of this parameter in the process of designing of new (or during the control of already existing) flow reactors is evident – this value determining the real time of reactions and transformations, which proceed in the system considered. But even if we analyse a storage reservoir, which only accumulates the fluid, it is very useful to take into account the degree of this fluid exchange (one should tend to reach a uniform exchange, when the stagnation zones are minimal). In this case the detention time is also a basic parameter.

The velocity of each reaction, which proceeds in the reactor, is described by the kinetic equation. In practice, we very often assume that these reactions can be described by the model of the first order, when the kinetic equation has a particularly simple form:

$$\frac{dc_i}{dt} = -k_i c_i. \tag{1}$$

The solution to this case is given by an exponential function, which affords the possibility to express a degree of reduction as follows (Fig. 1a):

$$r_i(t) = 1 - \frac{c_i(t)}{c_{io}} = 1 - \exp(-k_i t). \tag{2}$$

For the required degree of reduction r_{in} (which is usually defined by the proper legal regulations) one can (using this expression) calculate the time t_{rn}, needed to attain this required reduction. In the next order the designer matches the characteristic dimensions of the reactor, in order that the real detention time be as close to the reduction time t_{rn}, as possible.

The time of flow through the reactor is almost always (in practice) represented by its mean value, obtained under the assumption, that the phenomenon can be described by the plug-flow-model, which means, that the time of flow of each element of the mass is the same and equal to the mean detention time (ASCE Manual 1992, Imhoff 1979):

$$t_{ps} = V/Q. \tag{3}$$

This relation results from the definition of the true velocity, under the assumption that for each mass element j we can take the same path $s_j = L$ and the same velocity, equal to the mean flow velocity ($u_j = v_r = Q/F$):

$$t_{pj} = \int_0^{s_j} \frac{ds}{u_j(s)} \approx \int_0^L \frac{dL}{v_r} = \frac{L}{v_r}. \tag{4}$$

However, the design method, based on this assumption, although widespread, is not precise. It does not take into account the dispersion of mass (Fisher et al. 1979), which is a very important feature of each real flow. As a consequence of this phenomenon, different elements of the same "mass slice" (which enters the reactor at one initial time t_o) passes through the reservoir in different periods of
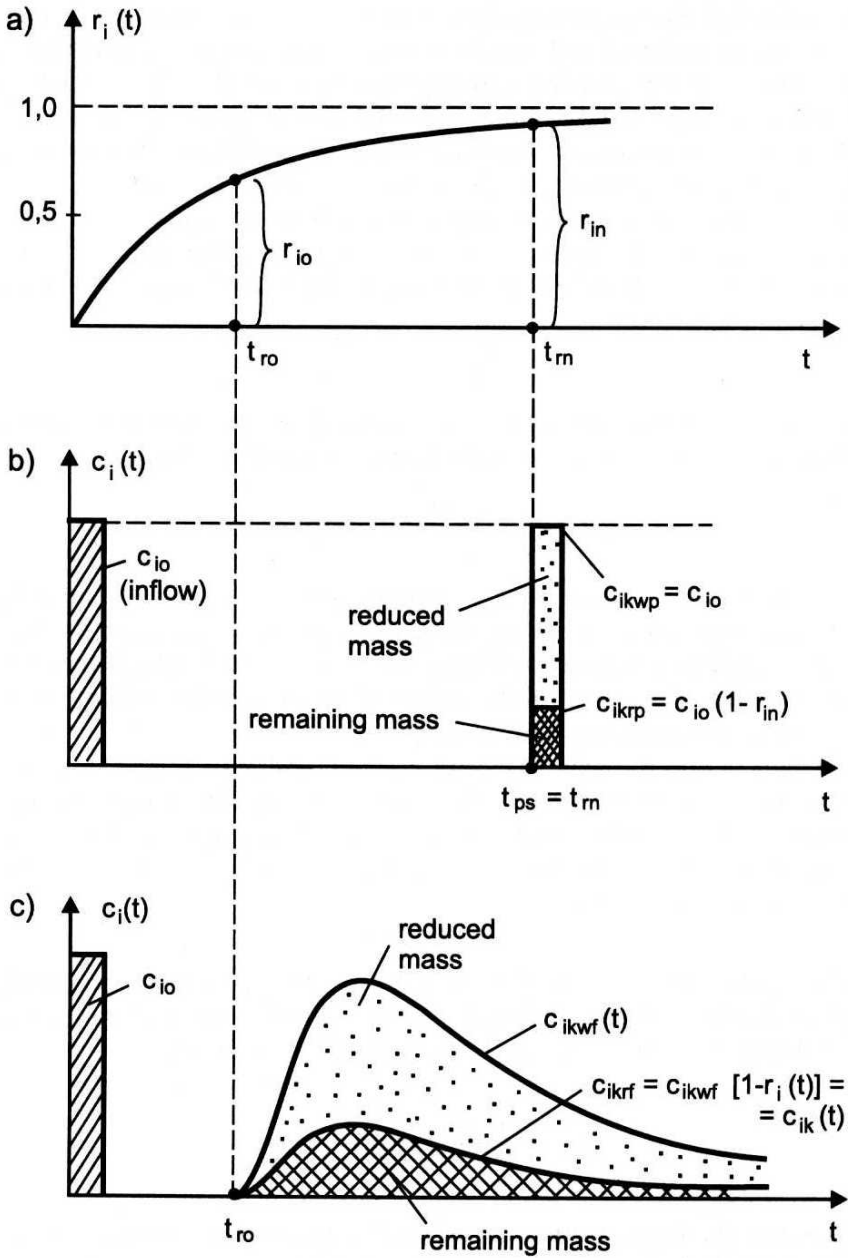
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Fig. 1. Determination of pollutants reduction
(a – velocity of reduction, b – plug-flow-model, c – real flow)

time. The most convenient form of description of this differentiation of time is probably *the extrinsic distribution of detention time.* Advantages, which redound from this kind of function, are illustrated in Fig. 1. According to the simplified plug-flow-model, the initial impulse of the i-th dispersed substance (pollutant, admixture or tracer) of the concentration c_{io} does not change, therefore the terminal concentration c_{ik} is equal to (see Fig. 1b):

– for each conservative component:

$$c_{ikwp} = c_{io} = \text{const},\tag{5}$$

– for each degradable component:

$$c_{ikrp} = c_{io}(1 - r_{in}).\tag{6}$$

However, applying the more accurate model, which takes into account the mass dispersion, we state that the initial concentration changes (Fig. 1c), and we can write:

– for each conservative component:

$$c_{io} \to c_{ikwf}(t),\tag{7}$$

– for each degradable component:

$$c_{io} \to c_{ikrf}(t) = c_{ikwf}(t)(1 - r_i(t)) = c_{ik}(t).\tag{8}$$

In the above relations the effect of dispersion was described by the terminal distribution of mass concentration. Typical extrinsic distribution of detention time has the following form:

$$c_{iz}(t) = c_{ik}(t)/M_i,\tag{9}$$

where:

$$M_i = \int_0^\infty c_{ik}(t)\, dt.\tag{10}$$

The function $c_{iz}(t)$ can be determined *theoretically* (with differing accuracy, to begin with the 1D model, matching the proper value of the coefficient of dispersion (Czernuszenko 1986, Czernuszenko 1990, Rutherford 1994), up to the full 3D model, according to which one has to determine the advective velocity field in the reactor and to describe the molecular or even the turbulent diffusion) or *experimentally.* The latter method is very convenient, as the results of measurements include all individual features of the investigated object, but may be very troublesome and expensive, especially when applied to real objects (e.g. waste water in a sewage treatment plant).

Considering the last statement it would seem very attractive to replace *the volume-detention-time* (determined experimentally for the whole reservoir, using the tracer mixed with the flowing fluid) by *the surface-detention-time* (also determined experimentally, with the exception of the surface layer of the fluid, using surface floats). Such a possibility is sometimes exploited in open-channel hydraulics (e.g. Masbernat et al. 1972), especially when the mean velocity of water must be measured, but formally it can also be applied to determine the simplified course of the extrinsic distribution of detention time. However, the question arises, concerning the equivalence of the exact method (with the use of tracer) and the approximated one (with the use of floats). The analysis of this problem is presented below.

2. Installation and Measuring Equipment

The starting point for the comparison of both methods analysed was given by the results of measurements, carried out in a laboratory glass channel with a width of $B = 0.385$ m. A rectangular reservoir $L = 1.00$ m long was formed in this channel, by means of some ceramic bricks (Fig. 2).

Two series of tests were performed – for the same bottom level in both the reservoir and the supply channel (TEST A) and where the channel bottom was placed 0.065 m above the reservoir bottom (TEST B).

The depth of water in reservoir H and both channels h was measured by means of a point-level-gauge, whereas the discharge Q – by means of a triangular Thomson weir.

In each experiment both investigated methods (*volume* and *surface*) were applied. In the exact version the concentrated solution of NaCl was used as a tracer, whereas in the simplified one – wooden cylindrical floats.

The concentration of dissolved tracer was measured at two points (initial P and terminal K – see Fig. 2) by means of two identical conductometers *Elwro TN 5721*.

The tracer was injected into the water in the initial cross section, through a rubber bulb about 10 cm ahead of probe P (volume of the sample – 200 ml, concentration – 200 g NaCl/dm^3). Observation of the dye (*potassium permanganate*) added to the tracer solvent enabled us to state that the injected impulse was uniformly distributed in the channel cross-section. The range of the electromagnetic field, induced by the detecting element, was broad enough, thus it could be assumed that the measured concentration was practically equal to its mean value.

The time-varying indications of conductometers were recorded by means of a standard video-camera, providing a simultaneous notation of time, and then – presented in the form of a diagram – "concentration versus time".

Such a methodology was convenient because of the second method of investigation, when the total number $N = 30$ wooden floats (15 mm in diameter and
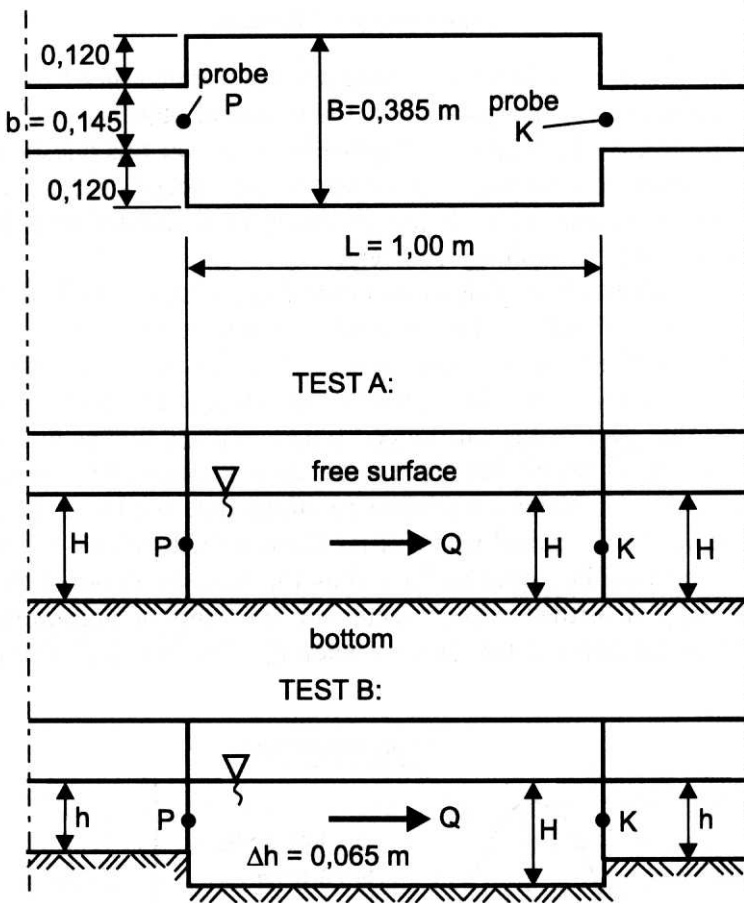
Fig. 2. Laboratory model

5 mm high) was suddenly ($t = 0$) introduced ("plunged" with one hand, so that they formed a compact, but irregular, "cloud" of material bodies) into the water in the initial cross-section P. The motion of this floats along the water free-surface was recorded (together with the passage of time) by the same video-camera. All the floats were recovered from water in the outflowing channel (when they left the reservoir, using a baffle made of wire netting). and used again in succesive tests.

In the course of results handling, the number of floats (from $n = 1$ to $n = N = 30$) crossing the terminal section K was counted, together with the actual time of flow. The results of this procedure afforded the possibility to present the course of each experiment in the form of a function:

$$n/N = f(t). \tag{11}$$

3. Discussion of Results

All the measurements described above have been performed on a laboratory scale, hence one should analyse the question of the technical objects, which have much greater dimensions. At this stage it is impossible to answer this question, but from the theoretical point of view there should not be any special discrepancies caused by the scale of the object it also being necessary to underline that the research presented is only of a preliminary character.

The extrinsic distribution of detention time was measured fivefold during each test (runs I, II, III, IV and V). Before further considerations, these results were averaged. An exemplary course of the curve $c_k(t)$ for the test A1 is shown in Fig. 3. As can be seen there, such a line proves a characteristic "tail", which iresults from the fact, that part of the introduced tracer remains longer in the reservoir (boundary layer, dead zones). For this reason, as a measure of the position of the back of this curve, it is highly convenient to afford time for the concentration to fall to a fraction of its maximal value. After some considerations it was assumed, that this fraction may be equal to 20% thus the time t_{20} denotes the moment, when $c = 0.2\, c_{max}$. A further, two characteristic moments of time were defined – $t_{min,e}$ (for the initial point of the "wave", when $c_k > 0$) and $t_{max,e}$ (for $c_k = c_{max}$).
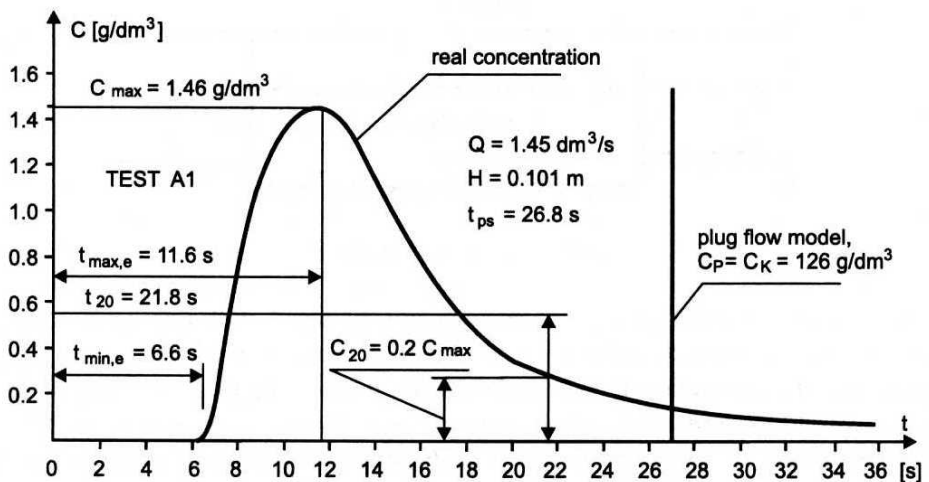

Fig. 3. Distribution of the tracer concentration

The results of the second part of each test are presented by the curve $n(t)/N$, which has already been discussed (Eq. 11). The exemplary shape of this line (test A1), together with 5 sets of experimental points, is shown in Fig. 4.

Both lines, $c_k(t)$ and $n(t)/N$, were also presented in the form of a normalized extrinsic distribution of detention time. The first graph was recalculated into the function $c_{ze}(t)$ according to Eqs. 9 and 10, whereas in the second case (floats)
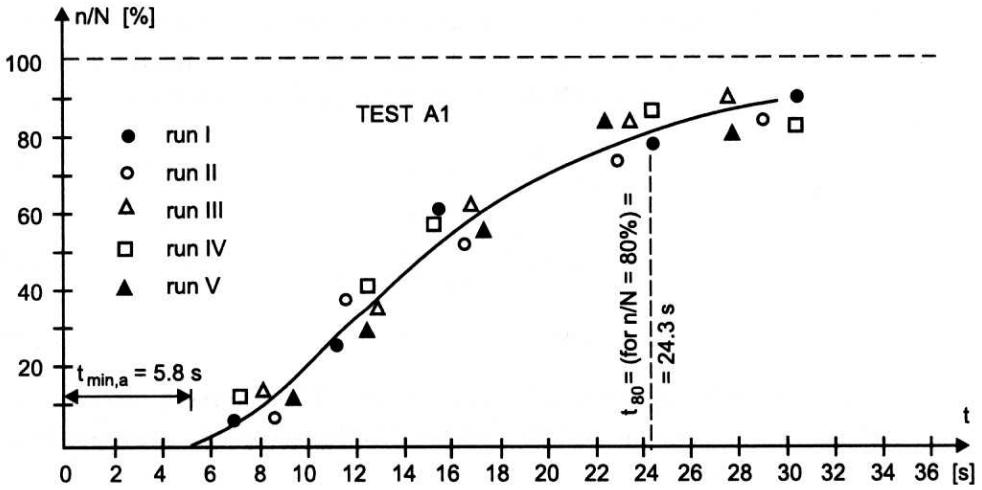
Fig. 4. Summation curve for floats motion

the line $c_{za}(t)$ was obtained by means of numerical differentiation of the function $n(t)/N$, as from definition we have:

$$c_{za}(t) = \frac{d[n(t)/N]}{dt}. \tag{12}$$

Obtained diagrams $c_{ze}(t)$ and $c_{za}(t)$ are shown in Fig. 5 (for the test A1). To be precise, one should note that the function $n(t)$ is random information. However, the main purpose of this paper is to compare two different experimental attitudes, not give a full mathematical discussion of this problem (which can be done, if necessary, by means of existing methods (Beckenbach 1961)).
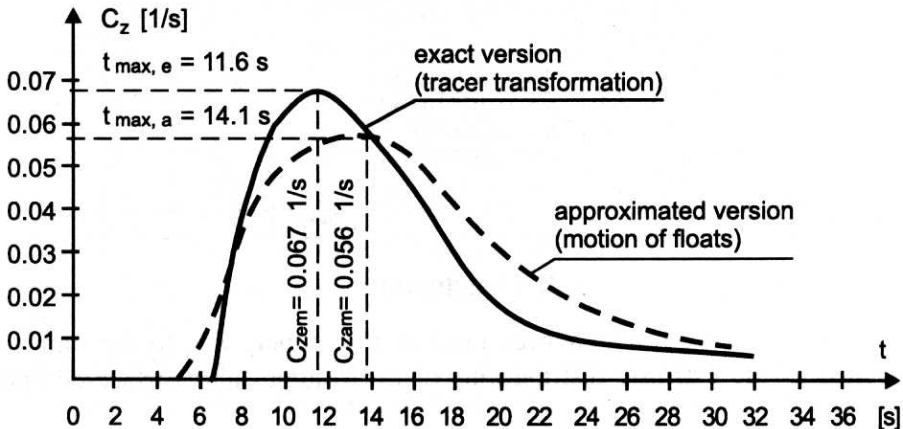

Fig. 5. Extrinsic distribution of detention time – exact and approximated

Together with the main parameters described above, for each test the mean detention time t_{ps} was calculated (Eq. 3). Differences between measures of the maximal time and maximal concentration were presented as the following relative parameters:

$$\varepsilon_{ta} = \frac{t_{max,a} - t_{max,e}}{t_{max,e}}, \tag{13}$$

$$\varepsilon_{ts} = \frac{t_{ps} - t_{max,e}}{t_{max,e}} \tag{14}$$

$$\varepsilon_c = \frac{c_{zam} - c_{zem}}{c_{zem}}. \tag{15}$$

All in all, 8 tests were performed (5 runs in each). The complete juxtaposition of the obtained characteristic parameters of the investigated phenomenon is given in Table 1 and Table 2.

Table 1. Results of experiments

Test	h (m)	H (m)	Q (dm³/s)	V (dm³)	t_{ps} (s)	min,e (m)	$t_{min,e}$ (s)	t_{20} (s)	c_{max} (g/dm³)	$t_{min,a}$ (s)	t_{80a} (s)	v_c (m/s)	v_r (m/s)
							Tracer				Floats		
A1	0.101	0.101	1.45	38.89	26.8	6.6	11.6	21.8	1.46	5.8	24.3	0.10	0.037
B1	0.041	0.106	1.45	40.81	28.1	5.8	10.9	19.6	1.23	5.4	26.4	0.24	0.036
A2	0.133	0.133	1.45	51.21	35.3	8.1	11.3	27.4	1.27	7.6	25.8	0.08	0.028
B2	0.088	0.153	1.45	58.91	40.6	7.6	11.2	24.4	1.08	7.2	28.2	0.11	0.025
A3	0.127	0.127	6.22	48.90	7.9	2.7	4.4	8.6	1.61	2.6	10.7	0.34	0.127
B3	0.086	0.151	6.22	58.14	9.3	2.4	5.0	9.7	1.20	2.2	7.8	0.50	0.107
A4	0.155	0.155	6.22	59.68	9.6	3.6	5.2	11.3	1.34	3.5	15.3	0.28	0.104
B4	0.099	0.164	6.22	63.14	10.2	3.0	5.1	12.3	0.82	2.8	7.2	0.43	0.099

Table 2. Comparison of investigated methods

Test	t_{ps} (s)	$t_{max,e}$ (s)	$t_{max,a}$ (s)	c_{zem} (1/s)	c_{zam} (1/s)	ε_{ta} (%) Eq. (12)	ε_{ts} (%) Eq. (13)	ε_c (%) Eq. (14)
A1	26.8	11.6	14.1	0.067	0.056	22	131	−16
B1	28.1	10.9	12.2	0.056	0.043	12	158	−21
A2	35.3	11.3	14.2	0.058	0.046	26	213	−21
B2	40.6	11.2	14.3	0.049	0.034	28	263	−31
A3	7.9	4.4	5.6	0.074	0.059	27	80	−20
B3	9.3	5.0	6.4	0.055	0.041	28	86	−25
A4	9.6	5.2	6.3	0.061	0.049	21	85	−20
B4	10.2	5.1	6.8	0.038	0.026	33	100	−32
Mean value						24.6	139.5	−23.5

4. Conclusions

The main goal of the research presented in this paper, was to determine the equivalence of the extrinsic distributions of the volume- and surface-detention-time.

As can be seen, even after qualitative comparison of two lines $c_{ze}(t)$ and $c_{za}(t)$, both discussed methods give very similar results. Existing discrepancies between

these two functions are rather regular and can be explained on the base of the methodology applied:

- the minimal time $t_{min,a}$ for the approximated line is always slightly shorter than the value $t_{min,e}$; this results from the fact that the floats move with a surface velocity, whereas the tracer – with the mean velocity, which is slightly less than the surface one;
- the latter remark is valid also when maximal times are compared ($t_{max,e} < t_{max,a}$);
- the maximal values of the approximated function c_{za} are always lower than the proper values for the exact version c_{ze} ($c_{zem} > c_{zam}$), but the curve $c_{ze}(t)$ steeper than the line $c_{za}(t)$; it is probably also the consequence of the float inertia;
- the same regularities appear in tests B (when the bottom levels in the channel and reactor are different).

Generally, one can state that the final level of conformity between both analysed methods is quite high and can be accepted for technical applications. As a matter of fact, the relative differences of maximal times and concentrations are not very small (on the average – 24.6% and 23.5% respectively), but one has to remember that:

- the correspondent differences for the popular simplified method (Eqs. 3 and 14) are much higher (139.5% on average);
- experimental determination of the curve $c_{ze}(t)$ is very expensive, whereas the distribution $c_{za}(t)$ can be obtained easily.

Taking into consideration all these circumstances, one can state that there exists some justification for the suggestion that the approximated function $c_{za}(t)$ can replace the exact function $c_{ze}(t)$, or at least – that it is purposeful to continue the problem investigations on a technical scale.

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