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Empirical Analysis of Virgin Compression of Sand

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Abstract

Experimental analysis of virgin compression of sand tested in the triaxial apparatus is presented. The experiments were performed on loose and dense specimens of "Lubiatowo" sand subjected to either isotropic or anisotropic compression. Experimental results are presented in simple analytical form useful in theoretical considerations. Mean trends of the sand behaviour and deviations from these means are determined. A simple method enabling assessment of the influence of the mean and deviatoric stresses on the volumetric and deviatoric strains, as well as identification of plastic and elastic strains, is proposed. The experimental results are also presented in standard geotechnical representation from which the compressibility and swelling constants are determined. Extensive discussion on this standard representation is enclosed, and some inconsistencies in classical literature pointed out.

1. Introduction

The natural state of granular soil is not precisely defined in soil mechanics, in contrast to the mechanics of solids where the natural state is understood as being stress free. A specimen of granular soil consists of thousand of grains, which separately, do not form a soil skeleton, until they are subjected to some compressive stresses which keep these grains together in such a way that a specimen is able to support additional loads. Therefore, the solid mechanics definition of natural state does not apply to granular soils, as in order to define this state, the initial stresses must be known. However, it is not sufficient to quote these stresses alone because the stress-strain history preceding the beginning of a specimen's investigation is also important.

In natural conditions, it is extremely difficult to define the initial (natural) state of soils, particularly the stress-strain history preceding the analysis of the soil site. This is an easier task in laboratory conditions, where the stress-strain history can be modelled and the initial state of stress is known, as, for example, in the triaxial compression tests. It is a common practice in geotechnical laboratories that the stress-strain history of soil specimen is modelled by a virgin compression, usually hydrostatic, see Atkinson (1993), Wood (1990), Tatsuoka et al. (1999). After that, a proper investigation takes place. In other apparatuses such as, for example, an oedometer or simple shear contraption, anisotropic virgin compression is applied (K_0 conditions).

In order to gain a better insight into soil behaviour, it is then necessary to analyse the strains and stresses during the virgin compression of the soil specimen. It has been done within a framework of wider experimental programme. The experiments were performed in a computer controlled hydraulic triaxial testing system from GDS Instruments Ltd, see Menzies (1988) and Świdziński (2000). The system has additionally been equipped with special gauges enabling the local measurement of both lateral and vertical strains, which enable a more precise measurement than traditional techniques. The experiments were performed on both loose and dense specimens of "Lubiatowo" sand, applying either isotropic (hydrostatic) virgin compression or anisotropic.

The results of experiments were approximated by analytical formulae from which compression was extracted, as well as some deviations from mean trends determined. An important feature of such a presentation is that the experiments provide some real numbers, which are important in validation of various soil models. Extensive discussion of the results obtained is presented, and some general conclusions formulated. The experimental results were also discussed in the context of standard geotechnical concepts.

2. Experimental Programme and Notation

The experiments were performed on "Lubiatowo" sand described in Sawicki and Świdziński (2002a), for various stress paths. In this paper only the behaviour of sand during the virgin compression is analysed. The first type of loading path corresponds to hydrostatic (isotropic) compression, when both the vertical (σ_1) and horizontal (σ_3) stresses increase simultaneously. This loading path is denoted as "I". The second loading path, denoted as "A", corresponds to anisotropic compression, when σ_1/σ_3 = const > 1. It is convenient to introduce the following quantities:

$$p = \frac{1}{3}(\sigma_1 + 2\sigma_3), \tag{1}$$

$$q = \sigma_1 - \sigma_3, \tag{2}$$

$$\varepsilon_v = \varepsilon_1 + 2\varepsilon_3, \tag{3}$$

$$\varepsilon_q = \frac{2}{3}(\varepsilon_1 - \varepsilon_3), \tag{4}$$

where ε_1 = vertical strain and ε_3 = horizontal strain.

Applied virgin loading paths are illustrated in p, q plane, see Fig. 1. Note that the anisotropic compression path cannot exceed the Coulomb-Mohr failure envelope, which is given by the following equation:

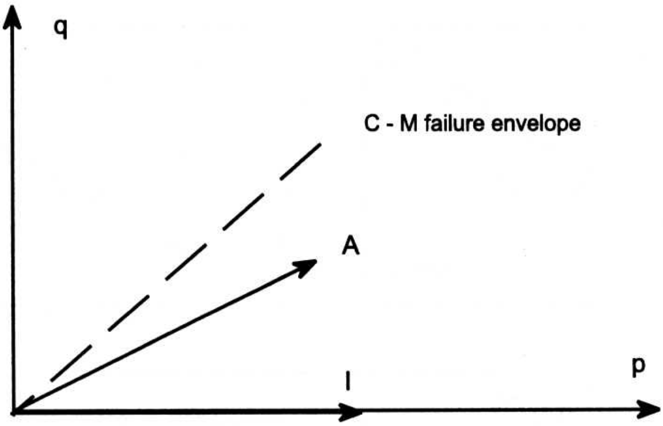

Fig. 1. Virgin loading paths

$$q = \frac{6 \sin \phi}{3 - \sin \phi} p, \quad (5)$$

where ϕ = angle of internal friction.

The experiments were performed on loose sand specimens, denoted as "L", for which the density index varied from $I_D = 0.095$ to 0.208, and dense specimens ("D"), for which I_D varied from 0.695 to 0.865. The following symbols are used in this paper:

LI = loose sand and isotropic compression,
LA = loose sand and anisotropic compression,
DI = dense sand and isotropic compression,
DA = dense sand and anisotropic compression.

The soil mechanics sign convention is applied where compression and compaction are regarded as positive. For the sake of convenience the following units are introduced: stress unit: 10^5 N/m^2 and strain unit: 10^{-3}. For example, the stress $\sigma = 4.2$ means 4.2×10^5 N/m^2, etc.

3. Isotropic Compression

The experimental records were plotted in the form shown in Fig. 2, and then approximated by analytical formulae. The strain paths ε_1, ε_3 can usually be nicely approximated by linear sectors, for both loose and dense sands. The plots ε_1, p, or other equivalent plots as ε_1, σ_1 etc., are either linear or non-linear, depending on the particular experiment. For the sake of simplicity, slightly non-linear plots were approximated by linear path, but the non-linearity was taken into account when evident. Examples of original plots and their approximations were presented in Sawicki and Świdziński (2002a, b).

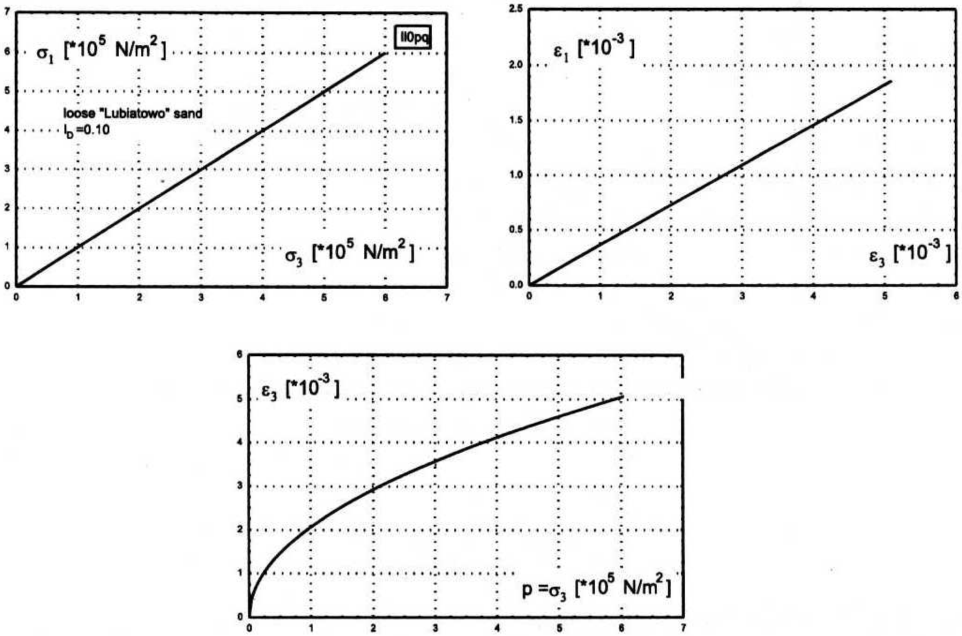

Fig. 2. Virgin hydrostatic compression of loose "Lubiatowo" sand ($I_D = 0.10$, $p_{\max} = 6 \times 10^5$ N/m^2). Approximated experimental record

The results presented in Fig. 2 can be described by the following analytical formulae: $p = 0.24\varepsilon_3^2$, $\varepsilon_1 = 0.365\varepsilon_3$, etc. from which the global volumetric and deviatoric strains can be determined:

$$\varepsilon_v = 4.83\sqrt{p} \quad \text{and} \quad \varepsilon_q = -0.865\sqrt{p}\ . \tag{6}$$

It should be remembered that strains and stresses are expressed in respective units: 10^{-3} and 10^5 N/m^2, respectively. Therefore, the coefficients appearing in Eq. (6) are expressed in unit: $10^{-5.5}$ m$/\sqrt{\text{N}}$. These units will be different if the shape of the respective equation is different than that in Eq. (6). For example, the data from Fig. 3 lead to the following formulae:

$$\varepsilon_v = 2.348p \quad \text{and} \quad \varepsilon_q = -0.15p\ , \tag{7}$$

and respective coefficients have unit: 10^{-8} m^2/N.

Fig. 4 shows some collated volumetric and deviatoric strains developed in loose sand during the virgin hydrostatic compression, drawn on the basis of five experimental records. The average values of $\varepsilon_v/p_{\max}$ and $\varepsilon_q/p_{\max}$ for chosen values of normalized mean stress $p/p_{\max}$ are plotted, and the bands illustrate the average deviations from the mean trend (Sawicki and Świdziński 2002a). For example, the mean value of $\varepsilon_v/p_{\max}$ at point $p/p_{\max} = 1$ is 2.185 with the average deviation of

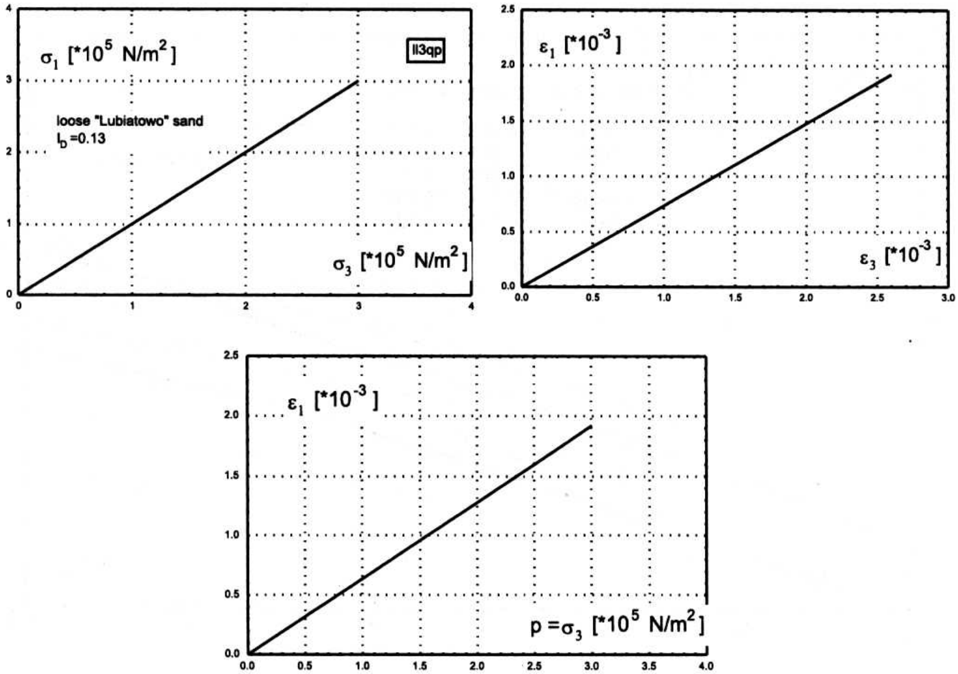

Fig. 3. Virgin hydrostatic compression of loose "Lubiatowo" sand ($I_D = 0.13$, $p_{\max} = 3 \times 10^5$ N/m^2). Approximated experimental record

0.386 (18%). The average value of $\varepsilon_q/p_{\max}$, at the same point, is 0.305 with the average deviation of 0.066 (22%). Certainly, more experimental data is necessary in order to perform a proper statistical analysis, but the results obtained already give some picture of strains developed during the virgin hydrostatic compression of loose sand.

In general, the strain-stress relationships are slightly non-linear, but a linear approximation is admissible in some cases. Some real experimental results, both linear and non-linear, are also plotted in Fig. 4. Note that the non-linearity is more pronounced for lower stresses, and the scatter of experimental data from mean trends is also greater for these stresses, and decreases with increasing stresses.

An important feature of the behaviour analysed is that the deviatoric strains develop during the virgin hydrostatic compression. The average ratio of $|\varepsilon_q/\varepsilon_v| = 0.14$, and $\varepsilon_q < 0$. This means that loose sand is not perfectly isotropic, as for ideally isotropic material it should be $\varepsilon_q = 0$. Similar features have been observed during the analysis of elastic response of "Lubiatowo" sand, Sawicki and Świdziński (2002a).

Similar qualitative behaviour is displayed by hydrostatically compressed dense sand (DI). Average stress-strain relations are the following in this case:

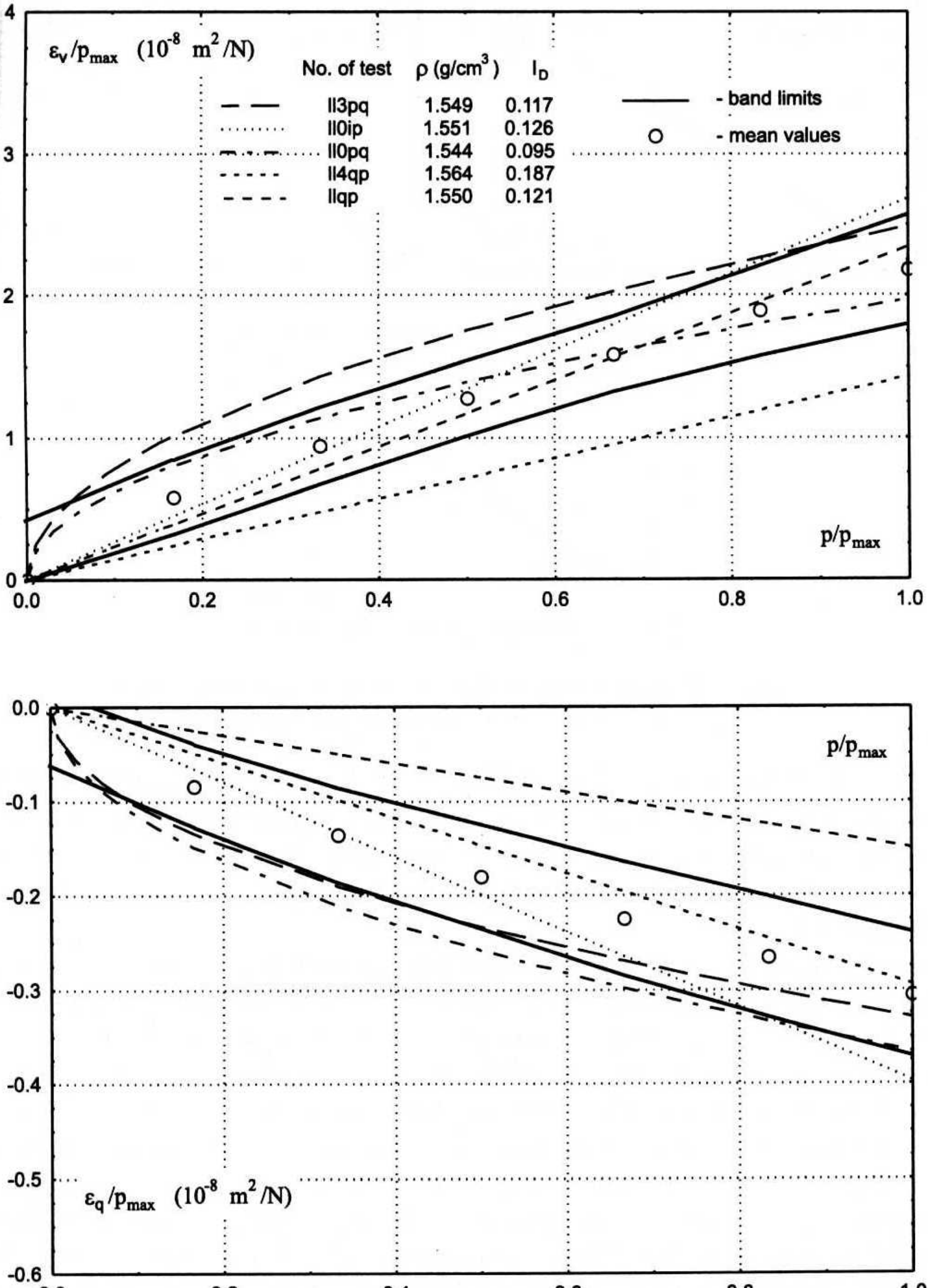

Fig. 4. Volumetric (a) and deviatoric (b) strains in loose "Lubiatowo" sand during virgin hydrostatic compression (LI)

$$\varepsilon_v = 1.673p \quad \text{and} \quad \varepsilon_q = -0.273p \, . \tag{8}$$

The absolute values of respective coefficients are smaller than those corresponding to loose sand, which is correct as dense sand is stiffer than loose.

4. Anisotropic Compression

Similar analysis was performed for anisotropic compression, as well as both loose and dense "Lubiatowo" sand. The stresses increased monotonically, according to $\sigma_1 = 1.6\sigma_3$, hence both the mean and deviatoric stresses were applied from the beginning of each experiment. A typical record of experimental data for dense sand is shown in Fig. 5.

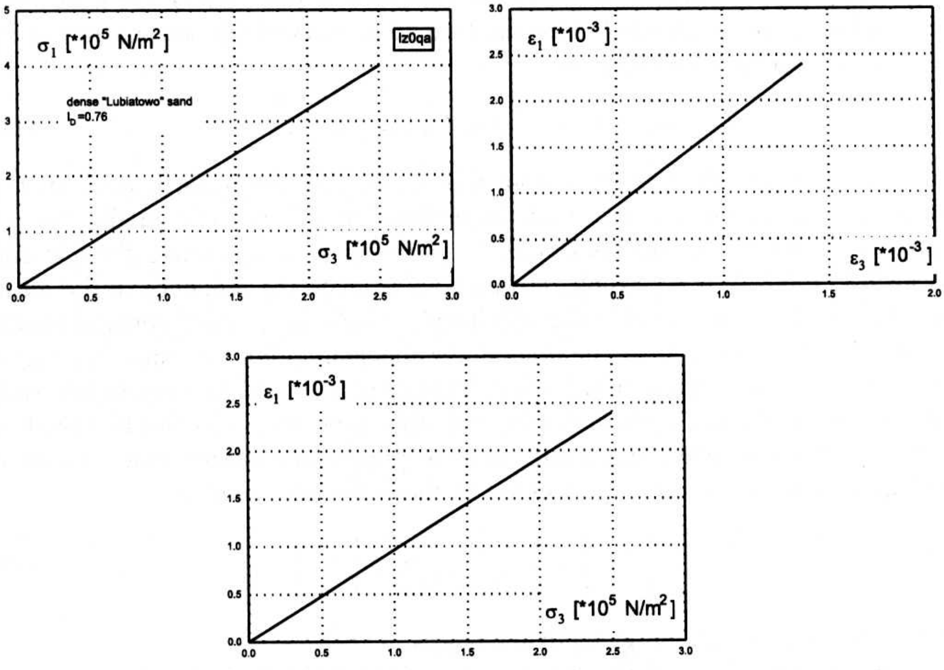

Fig. 5. Virgin anisotropic compression of dense "Lubiatowo" sand (DA) ($I_D = 0.74$). Approximated experimental results

The four experiments analysed display a similar, almost linear behaviour, which results in the following formulae:

$$\varepsilon_v = A\sigma_3 \quad \text{and} \quad \varepsilon_q = B\sigma_3, \tag{9}$$

where mean values of coefficients A and B are 1.972 and 0.317 respectively, with mean deviations of 0.09 (5%) and 0.056 (18%). An important feature of the behaviour of dense sand subjected to anisotropic compression is that the deviatoric strains change the sign as compared with hydrostatic compression, which displays stress induced anisotropy. In the case of loose sand we obtained: $A = 3.125$ and $B = 0.221$.

Note that exact comparison of experimental data from isotropic and anisotropic virgin compression tests is difficult, as in the latter case an important role is played by the stress ratio $a = \sigma_1/\sigma_3$ or the ratio:

$$\eta = q/p, \tag{10a}$$

or

$$\eta = \frac{3(a-1)}{a+2}. \tag{10b}$$

Note also that the strains in Eqs. (9) are expressed as functions of the horizontal stress which is an independent variable. Instead of this variable we may introduce some other stresses according to:

$$\sigma_3 = \sigma_1/a = 3p(a+2) = q(a-1). \tag{11}$$

It is thus not possible to extract from Eqs. 9 uniquely the strains caused by the mean and deviatoric stresses. Such decomposition was possible in the case of elastic response as the effects caused by p and q were separated, Sawicki and Świdziński (2002a). It is possible, however, to estimate the influence of the mean and deviatoric stresses during the anisotropic compression using collated results from "I" and "A" tests. It can be done under the assumption that there is a linear influence of these stresses on the soil deformation. It should be stressed that such an approach is the first approximation, and other hypotheses can also be tested on the basis of purely empirical results hitherto presented. Assume that volumetric and deviatoric strains can be presented by the following equation:

$$\left\{ \begin{array}{c} \varepsilon_v \\ \varepsilon_q \end{array} \right\} = \left[\begin{array}{cc} D_{vv} & D_{vq} \\ D_{qv} & D_{qq} \end{array} \right] \left\{ \begin{array}{c} p \\ q \end{array} \right\}, \tag{12}$$

where D_{ij}; $i, j = v, q$; are global compliances.

Compliances D_{vv} and D_{qv} can be determined from "I" tests, obviously with a certain accuracy. The average values of these compliances for loose and dense "Lubiatowo" sand were presented in Section 3, for example Eq. (8). During "A" tests, there is:

$$p = \frac{3}{3-\eta}\sigma_3 \quad \text{and} \quad q = \frac{3\eta}{3-\eta}\sigma_3. \tag{13}$$

Eqs. (9), (12) and (13) lead to the following formulae:

$$D_{vq} = \frac{1}{\eta}\left[\frac{(3-\eta)A}{3} - D_{vv}\right], \tag{14}$$

$$D_{qq} = \frac{1}{\eta}\left[\frac{(3-\eta)B}{3} - D_{qv}\right]. \tag{15}$$

For example, in the case of dense sand, we have: $D_{vv} = 1.673$, $D_{qv} = -0.273$, $A = 1.972$, $B = 0.317$ and corresponding $\eta = 0.5$. Therefore, $D_{vq} = -0.06$ and $D_{qq} = 1.074$.

A similar analysis can be carried out in the case of more complex analytical expressions describing the development of strains during virgin compression. The above simplified analysis is substantiated by empirical observations concerning the almost linear character of experimental results, at least in the case of dense sand.

5. Development of Elastic and Plastic Strains

The problem of determination of elastic and plastic strains in granular materials is still controversial. It seems that there continues to be a lack of sufficient empirical data, particularly from well designed and accurate experiments, which would help in testing various hypotheses. Sawicki and Świdziński (2002a) assumed that the behaviour of sand during unloading is elastic, and can be approximated by linear relationships, at least within the range of stresses applied in the experiments conducted. This assumption is supported by various experimental results, see also Sawicki and Świdziński (1998). It should be remembered that the empirical data always have a fuzzy character which should be taken into account in models of soil behaviour. Discussion as to the linear or non-linear character of some features of soil behaviour is sometimes irrelevant, as both approximations may be of the same value and accuracy, see Sawicki and Świdziński (1998). In this paper, the assumption as to the linear elastic response of sand will be retained, and the following relation assumed to be valid, Sawicki and Świdziński (2002a):

$$\left\{\begin{array}{c}\varepsilon_v^{el}\\ \varepsilon_q^{el}\end{array}\right\} = \left[\begin{array}{cc} M_{vv} & M_{vq}\\ M_{qv} & M_{qq}\end{array}\right]\left\{\begin{array}{c}p\\ q\end{array}\right\}, \tag{16}$$

where M_{ij} are elastic compliances. In the case of dense "Lubiatowo" sand the matrix of average elastic compliances is the following:

$$\mathrm{M} = \left[\begin{array}{rr} 0.479 & -0.088\\ -0.043 & 0.19\end{array}\right], \tag{17}$$

the coefficients of which are determined with respective accuracy taking into account the fuzzy character of the experimental data.

Assume that the total strain tensor can be decomposed into elastic (el) and plastic (pl) parts:

$$\varepsilon = \varepsilon^{el} + \varepsilon^{pl}. \tag{18}$$

The plastic strains can be determined from the following equation:

$$\left\{\begin{array}{c}\varepsilon_v^{pl}\\ \varepsilon_q^{pl}\end{array}\right\} = \left[\begin{array}{cc} D_{vv} - M_{vv} & D_{vq} - M_{vq}\\ D_{qv} - M_{qv} & D_{qq} - M_{qq}\end{array}\right]\left\{\begin{array}{c}p\\ q\end{array}\right\}, \tag{19}$$

In the case of dense "Lubiatowo" sand the matrix of plastic compliances assumes the following form:

$$\mathrm{L} = \begin{bmatrix} 1.194 & 0.028 \\ -0.23 & 0.884 \end{bmatrix}. \tag{20}$$

Therefore, for $\eta = 0.5$, we have:

$$\varepsilon_v^{pl} = 1.45\sigma_3 \quad \text{and} \quad \varepsilon_q^{pl} = 0.254\sigma_3, \tag{21}$$

$$\varepsilon_v^{el} = 0.522\sigma_3 \quad \text{and} \quad \varepsilon_q^{el} = 0.062\sigma_3, \tag{22}$$

$$\varepsilon_v = 1.972\sigma_3 \quad \text{and} \quad \varepsilon_q = 0.317\sigma_3. \tag{23}$$

It follows from this example, that plastic strains predominate during the virgin compression. The plastic volumetric strain constitutes about 74% of the total volumetric strain, and the plastic deviatoric strain is 80% of the total deviatoric strain. The ratio $\varepsilon_q/\varepsilon_v = 0.16$ which means that the volumetric strains dominate in the case analysed.

If the end of virgin compression is treated as the beginning of proper investigation of a soil specimen, it should be remembered that some "initial" elastic strains (and elastic energy) exist, whilst the pre-compression energy is dissipated. Probably the ratio of pre-compression η influences the structure of the specimen, but more general conclusions regarding this matter cannot be drawn due to insufficient experimental data.

6. Oedometric Virgin Compression

Oedometric virgin compression is a special kind of "A" tests, for a particular loading path $\sigma_3 = K_0\sigma_1$, where K_0 = coefficient of earth pressure at rest. The behaviour of sand in oedometric conditions was analysed elsewhere, Sawicki (1994), Sawicki and Świdziński (1998), and this section will indicate how the previous concepts can be applied to the analysis of virgin compression in this apparatus.

Fig. 6 shows the vertical stress-strain characteristic and the stress path followed during the virgin compression of loose "Lubiatowo" sand, Sawicki and Świdziński (1998). The stress-strain curve can be approximated by the following formula:

$$\varepsilon_1 = 5.303\sqrt{\sigma_1}. \tag{24}$$

There is also $\sigma_3 = 0.553\ \sigma_1$, $p = 0.714\ \sigma_1$ and $q = 0.484\ \sigma_1$.

The auxiliary set of data corresponds to isotropic compression of the same loose sand in the triaxial apparatus. Respective data are the following:

$$\varepsilon_v = 4.83\sqrt{p} \quad \text{and} \quad \varepsilon_q = -0.865\sqrt{p}\ . \tag{25}$$

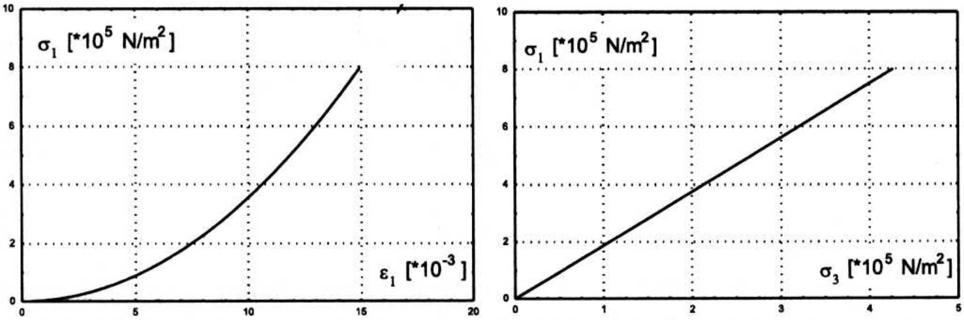

Fig. 6. Virgin compression in oedometric conditions. Loose "Lubiatowo" sand

Applying a similar procedure as before one obtains the following relation which describes both experiments:

$$\left\{ \begin{array}{c} \varepsilon_v \\ \varepsilon_q \end{array} \right\} = \left[\begin{array}{rr} 4.83 & 1.755 \\ -0.865 & 6.136 \end{array} \right] \left\{ \begin{array}{c} \sqrt{p} \\ \sqrt{q} \end{array} \right\}. \tag{26}$$

Eq. (26) shows the influence of the mean and deviatoric stresses on the volumetric and deviatoric strains.

The above example supplements procedure introduced in Section 4 as it describes the non-linear behaviour of loose sand.

7. Standard Geotechnical Characteristics

Experimental results are very often presented using some standard geotechnical characteristics, such as the voids ratio e and specific volume v, instead of the volumetric strain ε_v. It is therefore necessary to recall definitions of these quantities and their mutual relations. The specific volume is defined as (cf. Atkinson 1993):

$$v = V/V_s, \tag{27}$$

where V = volume of a sample containing a volume V_s of soil grains.

The voids ratio is defined as:

$$e = V_p/V_s, \tag{28}$$

where V_p = volume of pores (voids). Obviously, there is:

$$V = V_p + V_s. \tag{29}$$

The other quantity which describes the state of a soil is the porosity defined as:

$$n = V_p/V. \tag{30}$$

The above quantities are inter-related as follows (cf. Craig 1992):

$$e = \frac{n}{1-n}, \tag{31}$$

$$\upsilon = \frac{1}{1-n}, \tag{32}$$

$$\upsilon = 1 + e. \tag{33}$$

It is important to note that the above definitions include the actual volume of granular soil V. The volumetric strain is defined by Eq. (3), or alternatively as

$$\varepsilon_v = \frac{V_0 - V}{V_0}, \tag{34}$$

where V_0 = initial volume of granular soil. Note that the same volume of grains is contained in both V_0 and $V(V_s^0 = V_s)$, because the compressibility of grains can be neglected in comparison with the compressibility of the soil skeleton. Therefore, the volumetric strain is equivalent to the change of porosity. It follows from Eqs. (29), (30) and (34) that:

$$\varepsilon_v = \frac{V_p^0 - V_p}{V_0} = n_0 - \frac{V_p}{V_0}. \tag{35}$$

Note that the second member on the RHS of Eq. (35) differs from the porosity defined by Eq. (30). Rearranging of Eq. (34) gives:

$$V_0 = \frac{V}{1-\varepsilon_v}. \tag{36}$$

Eqs. (35) and (36) lead to the following expression:

$$\varepsilon_v = \frac{n_0 - n}{1-n}, \tag{37}$$

which displays a link between the porosity and volumetric strain. Combining Eqs. (31), (32) (33) and (37) gives:

$$\varepsilon_v = \frac{\upsilon_0 - \upsilon}{\upsilon_0} \quad \text{or} \quad \upsilon = \upsilon_0(1-\varepsilon_v). \tag{38}$$

The virgin compression curves $\varepsilon_v = \varepsilon_v\ (p)$ (see Eqs. 6 and 7) can be represented alternatively as $\upsilon = \upsilon(p)$. Note that in this case the volumetric strain should be substituted explicitly into Eq. (38). Differentiation of Eq. (38) gives:

$$d\varepsilon_v = -\frac{1}{\upsilon_0} d\upsilon. \tag{39}$$

This equation differs from Atkinson's (1993) Eq. (8.4) in which there is υ instead of υ_0 on the RHS. In order to check this discrepancy consider the other definition of volumetric strain than that given by Eq. (34):

$$\varepsilon_v^L = \frac{V_0 - V}{V}. \tag{40}$$

Here, the change in volume is related to the current, not the initial volume of the sample. Applying a similar procedure as before, one obtains the following relationships:

$$\varepsilon_v^L = \frac{n_0 - n}{1 - n_0}, \tag{41}$$

and

$$\varepsilon_v^L = \frac{\upsilon_0 - \upsilon}{\upsilon} \quad \text{or} \quad \upsilon = \frac{\upsilon_0}{1 + \varepsilon_v^L}. \tag{42}$$

Differentiation of Eq. (42) gives:

$$d\varepsilon_v^L = -\frac{\upsilon_0}{\upsilon^2} d\upsilon, \tag{43}$$

which differs from the equation provided by Atkinson (1993). The above analysis shows that Atkinson's Eq. (8.4) is wrong.

Note that the distinction between volumetric strain measures (34) and (39) is essential in the case of large porosity changes. In this paper definition (34) is adopted.

8. Standard Theoretical Concepts against Experimental Data

In basic soil mechanics literature of Schofield and Wroth (1968), Wood (1991) and Atkinson (1993), the virgin compression of soil is usually represented in the υ, ln p space, as shown in Fig. 7.

The line corresponding to first loading is designated as the normal consolidation line (NCL) and is given by:

$$\upsilon = N - \lambda \ln p, \tag{44}$$

where λ = slope of NCL, N = the value of υ at p = 1 kPa.

During the isotropic unloading one follows the swelling line (SL) given by:

$$\upsilon = \upsilon_K - \chi \ln p, \tag{45}$$

where χ = slope of SL, υ_k = the value of υ at p = 1 kPa. The parameters λ, N and χ are assumed as constants for a particular soil. According to Atkinson

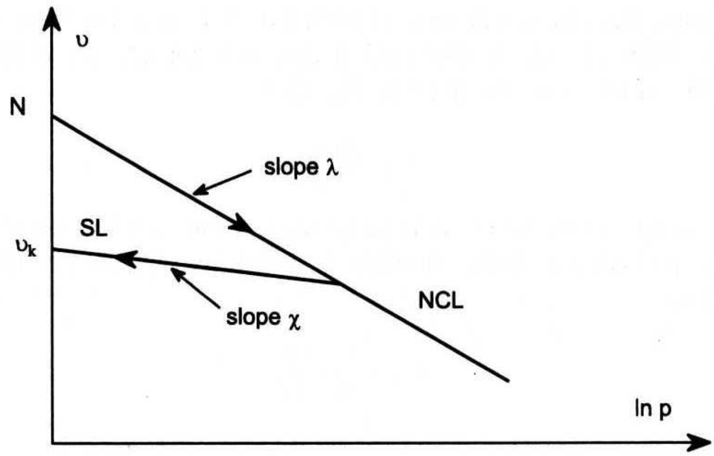

Fig. 7. Isotropic compression and swelling of soil – classical representation

(1993), typical values of these parameters are the following (see Table 9.1, page 111):

River sand: $\lambda = 0.16$, $N = 3.17$, $\chi/\lambda = 0.09$;
Carbonate sand: $\lambda = 0.34$, $N = 4.80$, $\chi/\lambda = 0.01$.

It will be shown later that these numbers are non-realistic.

It is interesting to check how our experimental data fit into the above classical concepts. The computational procedure is fairly simple if one remembers respective units. For example, values of p appearing in Eqs. (44) and (45) should be substituted in kilopascals (kPa). In the case of loose sand (LI) we have obtained the following average values of soil parameters: $\lambda = 0.00708$ and $N = 1.741$. The average deviations from these means were very small, namely 0.00025 (3.5%) and 0.00633 (0.4%), respectively. For dense sand (DI), we have obtained: $\lambda = 0.00455$ and $N = 1.568$. The characteristic of swelling, for loose sand, is $\chi = 0.00289$, and also $\chi/\lambda = 0.408$.

These results are very much different from typical data quoted by Atkinson (1993). Therefore, it is necessary to analyse the Atkinson's numbers in detail. It should be noticed first, that his values of N are quite large. Recall that they denote the specific volume at very low pressure p = 1 kPa. One may expect that these values of N would correspond to almost maximum specific volume (or porosity) of sand before proper investigation. In the case of sands and gravels, the specific volume varies from $\upsilon_{min} = 1.3$ to $\upsilon_{max} = 2$ which correspond to a porosity of from 0.23 to 0.5. In the case of spherical grains, we have $\upsilon_{min} = 1.348$ and $\upsilon_{max} = 1.91$, see Wiłun (1976). The values of N quoted by Atkinson are much higher. For example, $N = 3.17$ corresponds to $n = 0.685$ and $N = 4.80$ is equivalent to $n = 0.792$. These numbers have no meaning from the physical point of view. The data in Table 8.1 of Atkinson (1993) are also meaningless. Despite large values of N, they correspond to extremely large volumetric strains of about 43%.

Typical values of the ratio χ/λ quoted by Atkinson (0.09 and 0.01) also differ greatly from the respective numbers characterizing "Lubiatowo" sand (0.408).

Some comments are also necessary regarding the values of λ which define the slope of NCL. Again, the Atkinson's numbers are much larger than those characterizing "Lubiatowo" sand. It follows from Eqs. (38) and (44), assuming $N = \upsilon_0$, that:

$$\varepsilon_\upsilon = \frac{\lambda}{\upsilon_0} \ln p, \tag{46}$$

see also Eq. (39). This equation suggests that for each pair of corresponding values of ε_υ and p we should obtain the same value of λ if it is the soil constant indeed. As simple check of our experimental data shows that Eq. (46) gives different values of λ for different pairs of ε_υ and p. Large values of λ correspond to highly compressible soils. Sand's compressibility is not as great as suggested by Atkinson's numbers.

Atkinson also suggests that λ and N are constants for a given soil. He does not mention any relation of these numbers to relative density etc. Our data suggest that these numbers are different for loose and dense "Lubiatowo" sand. Recall, for loose sand, we have λ =0.00708 and N = 1.741, and λ =0.00455 and N = 1.568 for dense. The average values are: $\bar{\lambda}$ = 0.00582 and $\overline{N}$ = 1.655 with the average deviations of 22 and 5% respectively. It is a matter of subjective judgement whether these parameters can be treated as material constants. Perhaps in some low-resolution models of soils, such an assumption may be accepted. This problem is beyond the scope of this paper.

According to the above discussion, Eqs. (44) and (45) are not universal laws of Nature, and they form a rather weak foundation for more general soil theories. Also note that these equations lose some important information as, for example, regarding the shape of ε_υ, p curves.

9. Conclusions

a) Experimental results presented in this paper show that during virgin compression of sand, isotropic and anisotropic, both volumetric and deviatoric strains develop. Deviatoric strain is negative for isotropic compression, and positive for anisotropic.
b) Generally, the stress-strain curves are non-linear, but in the majority of experiments analysed, the linear approximation is quite good. A simple analysis of experimental results enables determination of mean trends and deviations. Generally, these deviations are less than 20% of the mean values, which is acceptable from the practical viewpoint. In the analysis, results of some 14 experiments were taken into account.
c) A simple method of dealing with experimental data was proposed. In particular, this method enables assessment of the influence of the mean and

deviatoric stresses on the volumetric and deviatoric strains, as well as identification of plastic and elastic strains. The method proposed is based on simple assumptions as regards soil behaviour. Some other hypotheses can also be tested on the basis of data presented in this and the other papers of Sawicki and Świdziński (2002a, b).

d) It was also shown how to collate experimental results from tests performed using different apparatus, e.g. triaxial system and oedometer.

e) The results presented provide more information as to the so-called natural state of the soil, i.e. the state preceding a proper investigation of soil specimens. For example, the initial strains can easily be estimated.

f) Some standard geotechnical parameters such as compressibility and swelling constants λ, N and υ were determined from experimental data. They differ substiantally from typical parameters quoted in classical geotechnical literature. Extensive discussion shows that there are some errors in standard text-books.

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