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Quasi 3D Model of Wave-induced Currents in Coastal Zone

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Abstract

The present model allows for simultaneous computations of depth-variable longshore currents and undertows, being the two most important components of coastal flows, at arbitrarily chosen locations of the nearshore zone, with the assumption of parallel isobaths. In the model, the flow velocities are calculated as functions of energy dissipation due to wave breaking. In the description of the wave-breaking, the effect of a roller has been taken into account. As a result of interaction of the cross-shore and longshore currents, a spiral-like resultant velocity vector appears, variable over water depth. The comparisons of calculated and measured vertical distributions of water flow velocity for conditions of the campaigns "Lubiatowo 87" and "Lubiatowo 96" have shown good conformity between experimental data and the present model.

1. Introduction

Coastal currents differ from the flows occurring in deep sea by the scale of magnitude and the origin. Such processes as refraction, bottom friction and – most of all – wave breaking cause the decrease in wave height. The energy dissipated in these processes induces the appearance of the characteristic coastal currents, often called in literature energetic currents, wave-driven currents or simply wave currents. The scale of these currents depends on the severity of waves, angle of wave incidence and, in particular, the intensity and number of waves breaking. In storm conditions, the wave-driven currents have both high velocities and large ranges. In general one can assume that in such conditions their range approximately amounts to double the width of the surf zone. During calmer wave conditions, these currents occur only in the close vicinity of the shore and their velocities are small.

The coastal currents are a very compound phenomenon and arise from a number of overlapping processes, non-stationary in time and heterogeneous in space. The analysis of individual components of the wave current systems shows that longshore currents and undertows are of predominant importance. These two flow systems imply the sediment transport quantities, the rate of cross-shore profile evolution and the change of shoreline position. Because of the above, the knowledge of these two flow types is necessary to design any hydro-technical structures and assess the influence of existing structures on sea shores.

The activities aimed at the theoretical description of mechanisms of wave-driven current generation commenced in the 'sixties when a radiation stress concept was presented in the works by Longuet-Higgins and Stewart. Within the existing mathematical background of models of these currents, an assumption is made that a change in the radiation stresses in a coastal zone is a driving force of the progressive water flow.

The studies by Bowen (1969), Longuet-Higgins (1970) and Thornton (1970), published almost simultaneously, described the first models enabling computation of the distributions of longshore currents, averaged over depth and wave period, as functions of offshore distance. In the following years, numerous papers appeared focused on longshore currents above a multi-bar bottom and for multiple wave-breaking. Those papers were also aimed at explaining of the displacement of flow velocity maximum with respect to the location of a wave breaker. This phenomenon is observed in nature. Some studies yielded increasingly improved models ensuring calculation of the longshore currents for actual coastal conditions, e.g. Kraus & Sasaki (1979), Visser (1984), Thornton & Guza (1986), Szmytkiewicz & Skaja (1993) and Kuriyama (1994).

The mathematical description of the undertow started in the 'eighties. The approach by Dally (1980) was one of the first complete models of these currents. Next, the works by a team of Svendsen (1984, 1986, 1987, 1988) and Stive & Wind (1982, 1986) were published. In the 'nineties, the term of stresses $\bar{\tau} = \rho \cdot \overline{\tilde{u} \cdot \tilde{w}}$, related to orbital velocities, started to be taken into account. Such complex models of return flow were presented – inter alia – by Deigaard et al. (1991), Stive & De Vriend (1994) and Rivero & Arcilla (1995). As a result of intensive research carried out at various scientific institutions, including IBW PAN (see Szmytkiewicz 2002), the contemporary models of return currents enable calculation of their vertical distributions at arbitrary locations of the multi-bar sea bed, for multiple wave breaking.

In the next stage of investigations, the research activities dealt with the theoretical description of the mechanisms of interaction between cross-shore and longshore wave-driven currents. The mathematical description of this interaction should be considered in a three dimensional system, thus constituting a new model, called by De Vriend & Stive (1987) the *fully 3D* model. However, difficulties associated to formulation and solution of equations accounting for non-linear interaction of various currents have led to the development of much less sophisticated models, called in literature the *quasi 3D* models.

These models are based on the natural extension of horizontal circulation analyses and seeks to establish both the depth-averaged flow and the vertical structure that was ignored in horizontal circulation analyses. This approach utilises the fact that the horizontal length scale in the nearshore circulation is far greater than the vertical one. It means that the contribution of the vertical turbulent transfer of horizontal momentum ($\rho \cdot \overline{u' \cdot w'}$) is the dominant term in the mean horizontal momentum balance.

In the quasi 3D models, only the two most important flows are considered, i.e. the longshore currents and the return currents, generated by breaking waves. As a result of interaction between these currents, a spiral and depth-variable resultant velocity vector arises, presented for the first time by Svendsen & Lorenz (1989).

The intensive works directed towards mathematical description of the interaction between longshore currents and undertows were commenced in the 'nineties and resulted in publications e.g. by Svendsen & Putrevu (1990), Deigaard (1993), Dongeren et al. (1994), Lee & Wang (1994) and Kuroiwa et al. (1998).

Similar to the above-mentioned papers, the present study deals with cross-shore and longshore currents only. Other components of the coastal currents system, not related to the wave breaking phenomenon, are not considered

In the present study, a quasi three-dimensional model called CUR-3DQ is described, which under the assumption of mutually parallel isobaths enables computation of depth-variable velocities $V(z)$ and $U(z)$ of the longshore current and undertow, respectively. These two basic components of coastal flows can be determined by the model at arbitrarily chosen locations of the multi-bar sea bed profile, accounting for multiple wave-breaking. In the computations of wave motion, it is assumed that the waves are random and that their heights in the entire coastal zone can be described by Rayleigh distribution. In the model, a root-mean-square wave height (H_{rms}) has been assumed as a statistically representative wave height. A so-called "roller effect" is also taken into consideration. This means that the lag between wave-breaking and appearance of currents is represented in the equations of momentum and energy by a rotating roller of water, located on the crest of the breaking wave. According to this concept, the wave energy lost during wave-breaking is first transferred for roller induction, then the water flows appear.

The velocities calculated by the present model have been compared with the corresponding quantities measured on the Coastal Research Station in Lubiatowo, located on the south Baltic shore.

2. Mathematical Description of the Model

The quasi three-dimensional model of wave-generated currents in the coastal zone has been formulated with the following assumptions:

- isobaths are approximately parallel to the shoreline,
- shear stresses inside the liquid in the cross-shore direction play the predominant role and can be determined according to the Boussinesq hypothesis,

- water flow velocities related to circulations of the open sea are negligibly small in comparison to the wave-induced currents in the nearshore zone,
- variability of the water flow velocity $\overline{U}(z)$ in the cross-shore direction is definitely smaller than its variability over depth,
- dissipation of wave energy due to bottom friction is so small in comparison to dissipation due to wave-breaking, that it can be neglected,
- there is a fully developed roller just in front of the breaking wave crest.

The first of the above assumptions means in practice that the velocities are calculated along an individual cross-shore profile for an arbitrary angle of wave approach. The other assumptions have been formulated on the basis of observations and conclusions from numerous measurements of waves and turbulent flows carried out in many coastal zones of the world, particularly in the surf zones.

In the mathematical description of the current, the following coordinate system has been assumed:
axis x – directed perpendicularly onshore from the open sea,
axis y – directed along averaged shoreline and located at the offshore boundary,
axis z – directed vertically upwards from the still water level.

The surf zone represents a region in which broken waves have many similar features with periodic bore-type waves. In this area, the occurrence of the roller in front of the breaking wave is the basic characteristic feature. The roller can approximately be represented as a rotating mass of water, moving shoreward in front of the breaking wave. The first mathematical model of the roller was presented by Svendsen (1984). In that model, the roller was defined as the water mass moving between crest and trough of the breaking wave with a phase velocity C in the direction of wave propagation. The roller energy E_r and momentum M_r are described respectively by the following relationships:

$$E_r = \frac{1}{2}\rho \cdot A \cdot \frac{C^2}{L} = \frac{1}{2}\rho \cdot A \cdot \frac{C}{T}, \tag{1}$$

$$M_r = \rho \cdot C\frac{A}{T}, \tag{2}$$

where A denotes the roller surface, ρ – water density and C, L and T stand for phase velocity, length and period of the wave, respectively.

The surface A was determined by Svendsen as a function of the wave height using the following empirical formula:

$$A = k_r \cdot H^2 \tag{3}$$

in which H denotes wave height and k_r is the proportionality coefficient.

When the wave approaches the shore obliquely, accounting for the function of a transition between the system (r,n) (where: r – coordinate along the wave ray, n – coordinate along the wave crest) and the system of coordinates (x, y), the relationships determining the roller momentum in the cross-shore and longshore directions are as follows:

$$M_{xx} = \rho \cdot C\frac{A}{T}\cos^2\theta = M_r \cdot \cos^2\theta, \tag{4}$$

$$M_{xy} = M_{yx} = \rho \cdot C\frac{A}{T}\sin\theta \cdot \cos\theta = M_r \cdot \sin\theta \cdot \cos\theta, \tag{5}$$

$$M_{yy} = \rho \cdot C\frac{A}{T}\sin^2\theta = M_r \cdot \sin^2\theta, \tag{6}$$

where θ is the angle between the wave ray and the axis x.

Deriving the value of pressure p from the momentum equation in the direction of the axis z, and next inserting it to time-averaged (over wave period) momentum equations in the directions of x and y axes, accounting for the assumptions and relationships given previously, one transforms these equations to the following form:

$$\frac{\partial}{\partial x}\left[\rho \cdot \left(\overline{\widetilde{u}^2} - \overline{\widetilde{w}^2}\right)\right] + \frac{\partial}{\partial x}(\rho \cdot g \cdot \overline{\eta}) + \frac{\partial}{\partial z}(\rho \cdot \overline{\widetilde{u} \cdot \widetilde{w}}) + \frac{\partial}{\partial x}\left(\frac{1}{h} \cdot M_r \cdot \cos\theta^2\right) = \\ = \frac{\partial}{\partial z}\left(\rho \cdot \nu_{Tz}\frac{\partial \overline{U}(z)}{\partial z}\right), \tag{7}$$

$$\frac{\partial}{\partial x}[(\rho \cdot \overline{\widetilde{u} \cdot \widetilde{w}})] + \frac{\partial}{\partial z}[(\rho \cdot \overline{\widetilde{v} \cdot \widetilde{w}})] + \frac{\partial}{\partial x}\left(\frac{1}{h} \cdot M_r \cdot \sin\theta \cdot \cos\theta\right) + \\ -\frac{\partial}{\partial x}\left(\rho \cdot \nu_{Tx}\frac{\partial \overline{V}(z)}{\partial x}\right) = \frac{\partial}{\partial z}\left(\rho \cdot \nu_{Tz}\frac{\partial \overline{V}(z)}{\partial z}\right) \tag{8}$$

where:

- ν_{Tx}, ν_{Tz} – turbulent viscosities in the horizontal and vertical direction, respectively,
- $\widetilde{u}$, $\widetilde{v}$, $\widetilde{w}$ – orbital velocities in the directions of the axes x, y and z, respectively,
- g – gravity acceleration,
- h – water depth,
- $\overline{U}(z)$, $\overline{V}(z)$ – time-averaged (in the wave period) flow velocities in the cross-shore and longshore direction, respectively, at an arbitrary level z,
- $\overline{\eta}$ – mean elevation of the free surface above still water level.

Deigaard & Fredsoe (1989) showed for the first time that for a wave propagating above a sloped bed, accounting for the energy dissipation, a phase shift between the horizontal component ($\widetilde{u}$) and vertical component ($\widetilde{w}$) of the orbital velocity (also for the linear wave theory) differs from 90°. Thus the time-averaged term $\overline{\widetilde{u} \cdot \widetilde{w}}$ in Eq. (7) is not zero in the entire vertical water column. The authors cited considered two separate cases. In the first, the wave energy dissipation results from the bottom friction, while in the second, the energy dissipation is related to the wave-breaking process.

For the latter case Deigaard & Fredsoe (1989) showed that the presence of the roller at the crest of the breaking wave causes a two-way modification of the horizontal orbital velocities. Namely, the roller causes a local additional elevation of the free surface η^* with respect to adjacent coastal regions where this elevation η (set-up) is described by the model of Longuet-Higgins (1964). This additional roller-related water surface elevation η^* and associated additional pressure gradient $\partial p^*/\partial x = \rho \cdot g \cdot \partial \eta^*/\partial x$ modifies horizontal orbital velocities beneath the roller. Secondly, the presence of the roller generates additional shear stresses at the interface between the roller and the water layer located below. These stresses do the work while moving the fluid particles, which in turn leads to further modification of the horizontal orbital velocities. The work is equal to the wave energy dissipation in the boundary layer between the roller and the underlying water.

Much simpler than the Deigaard & Fredsoe (1989) method of determining the correlation term $(\overline{\widetilde{u} \cdot \widetilde{w}})$ was presented by Rivero & Arcilla (1995). These authors considered a general case of the two-dimensional wave in which the motion of fluid particles is characterised by an elementary displacement related to instantaneous orbital velocities $\widetilde{u}$and $\widetilde{w}$, as well as an elementary rotation associated to the angular frequency $\widetilde{\omega} = \partial\widetilde{u}/\partial z - \partial\widetilde{w}/\partial x$. Calculating the quantity $\widetilde{w} \cdot \widetilde{\omega}$:

$$\widetilde{w} \cdot \widetilde{\omega} = \widetilde{w} \cdot \frac{\partial \widetilde{u}}{\partial z} - \widetilde{w} \cdot \frac{\partial \widetilde{w}}{\partial x} = \frac{\partial}{\partial z}(\widetilde{u} \cdot \widetilde{w}) - \widetilde{u} \cdot \frac{\partial \widetilde{w}}{\partial z} - \widetilde{w} \cdot \frac{\partial \widetilde{w}}{\partial x} \tag{9}$$

and determining the value $\partial\widetilde{w}/\partial z$ from the continuity equation, averaged over wave period, and assuming the potentiality of the wave motion in a surf zone, the latter authors presented the term $(\overline{\widetilde{u} \cdot \widetilde{w}})$ in the following ultimate form:

$$\frac{\partial}{\partial z}(\overline{\widetilde{u} \cdot \widetilde{w}}) = -\frac{1}{2}\left(\frac{\partial}{\partial x}(\overline{\widetilde{u}^2} - \overline{\widetilde{w}^2})\right). \tag{10}$$

Inserting the above relationship to the momentum equation in the direction of x axis (Eq. (7)), using the linear wave theory in description of the orbital velocities, assuming locally flat bed and implementing the shallow water approximation ($\sinh(2kh) \approx 2kh$, $\cosh(k \cdot h) \to 1$), the first and third components of Eq. (7) can be rewritten in the form:

$$\frac{\partial}{\partial x}\rho\cdot\left(\overline{\widetilde{u}^2}-\overline{\widetilde{w}^2}\right)+\frac{\partial}{\partial z}\left(\rho\cdot\overline{\widetilde{u}\cdot\widetilde{w}}\right)= \\ =\frac{\rho\cdot g}{16\cdot h}\left[(\cosh(k\cdot\varsigma)^2\cdot\cos^2\theta-\sinh(k\cdot\varsigma)^2\right]\frac{dH^2}{dx} \tag{11}$$

where:

$\varsigma = z + h$,

k – wave number.

The component $\partial(\rho\cdot g\cdot\overline{\eta})/\partial x$ in Eq. (7), describing the gradient of the free surface slope, was determined from the momentum equation in the x direction, averaged over time and depth. From this equation, neglecting the bed shear stresses and assuming $(\overline{\eta}+h)\approx h$, one obtains:

$$\frac{\partial\overline{\eta}}{\partial x}=-\frac{1}{\rho\cdot g\cdot h}\frac{\partial S_{xx}}{\partial x} \tag{12}$$

where:

S_{xx} – component of the radiation stress tensor in the cross-shore direction.

Ultimately, the momentum equation in the x axis direction (Eq. (7)) can be written in the form:

$$R_x=\frac{\partial}{\partial z}\left(\rho\cdot\nu_{Tz}\frac{\partial\overline{U}(z)}{\partial z}\right), \tag{13}$$

where R_x, using Equations (2), (3), (4), (11) and (12) is determined by the following relationship:

$$R_x=\frac{\rho\cdot g}{16\cdot h}\left[(\cosh(k\cdot\varsigma)^2\cdot\cos^2\theta-\sinh(k\cdot\varsigma)^2\right]\frac{dH^2}{dx}-\frac{1}{h}\frac{\partial S_{xx}}{\partial x}+ \\ +k_r\cdot\rho\cdot\frac{C}{h\cdot T}\cdot\cos^2\theta\cdot\frac{dH^2}{dx}. \tag{14}$$

For small angles of wave approach ($\cos^2\theta\approx 1$), the above equation has the form:

$$R_x=\frac{1}{16\cdot h}\cdot\rho\cdot g\cdot\frac{dH^2}{dx}-\frac{3}{16\cdot h}\cdot\rho\cdot g\cdot\frac{dH^2}{dx}+k_r\cdot\rho\cdot\frac{C}{h\cdot T}\cdot\frac{dH^2}{dx}. \tag{15}$$

Outside the surf zone, the last terms on the right hand sides of Equations (14) and (15), determining the roller-related shear stresses, are neglected.

The first two terms on the left hand side of the momentum equation in the y axis direction (Eq. (8)), using the linear wave theory in description of the orbital velocities, and – as before – assuming locally flat bed and the conditions of the shallow water approximation, can be determined by the following formulas:

$$\frac{\partial(\rho \cdot \overline{\tilde{u} \cdot \tilde{w}})}{\partial x} = \frac{\rho \cdot g}{8 \cdot h} \cdot \sin\theta \cdot \cos\theta \cdot \frac{dH^2}{dx}, \tag{16}$$

$$\frac{\partial(\rho \cdot \overline{\tilde{v} \cdot \tilde{w}})}{\partial x} = 0. \tag{17}$$

The determination of the stresses $\frac{\partial}{\partial x}\left(\rho \cdot \nu_{Tx} \cdot \frac{\partial \overline{V}}{\partial x}\right)$, being the last term on the left hand side of the momentum equation (8), is a separate problem.

Svendsen & Lorenz (1989), basing on perturbation methods, showed that these stresses in the first approximation can be replaced by the bed shear stresses $\frac{\partial}{\partial x}\left(\rho \cdot \nu_{Tx} \cdot \frac{\partial \overline{V}_b}{\partial x}\right)$. Svendsen & Putrevu (1990) proposed to determine the nearbed velocity $\overline{V}_b$ along the cross-shore profile from the approach by Longuet-Higgins (1970), i.e. from the model for depth-averaged longshore current. This implies that the velocity $\overline{V}$ in the term $\frac{\partial}{\partial x}\left(\rho \cdot \nu_{Tx} \cdot \frac{\partial \overline{V}}{\partial x}\right)$ is replaced by the depth-averaged velocity $\overline{V}_m$.

In the present study, the approach by Svendsen & Putrevu (1990) has been followed, however, with the velocity $\overline{V}_m$ determined from the model presented by Szmytkiewicz (2002). This model, called LONG-CUR, enables the computation of the time- and depth-averaged longshore current distributions as functions of the offshore distance for an arbitrary sea bed configuration and an arbitrary number of wave-breakings.

Additionally, accounting for the roller-related stresses in the surf zone, determined by Equations (2), (3) and (5), the left hand side of the momentum equation in the y axis direction (Eq. (8)) can be presented in the following form:

$$\begin{gathered} R_y = \frac{1}{8h}\rho \cdot g \sin\theta \cos\theta \frac{dH^2}{dx} + \\ + k_r \cdot \rho \cdot \frac{C}{h \cdot T} \cdot \sin\theta \cdot \cos\theta \cdot \frac{dH^2}{dx} - \frac{\partial}{\partial x}\left(\rho \cdot \nu_{Tx} \frac{\partial \overline{V}_m}{\partial x}\right). \end{gathered} \tag{18}$$

Ultimately, the momentum equation in the y axis (Eq. (8)), as in the case of its counterpart in the x axis direction, can be written in the form:

$$R_y = \frac{\partial}{\partial z}\left(\rho \cdot \nu_{Tz} \frac{\partial \overline{V}(z)}{\partial z}\right). \tag{19}$$

3. Boundary Conditions

Assuming depth-invariable turbulent viscosity, momentum equations (13) and (19) after double integration have the form:

$$\overline{U}(\varsigma) = \frac{1}{2}\frac{1}{\rho \cdot \nu_{Tz}} R_z \cdot \varsigma^2 + \frac{C_1}{\rho \cdot \nu_{Tz}} \varsigma + C_2, \tag{20}$$

$$\overline{V}(\varsigma) = \frac{1}{2}\frac{1}{\rho \cdot \nu_{Tz}} R_y \cdot \varsigma^2 + \frac{C_3}{\rho \cdot \nu_{Tz}} \varsigma + C_4. \tag{21}$$

The integration constants C_1 and C_2 in the equation describing vertical distribution of the cross-shore current, have been determined from the following conditions:

- the generally approved assumption that the resultant onshore flow Q in the region between wave crest and trough is equal to the offshore flow between the bed and the wave trough, namely:

$$\int_{o}^{d_{tr}} \overline{U}(\varsigma) d\varsigma = \overline{U}_m d_{tr} = -Q, \tag{22}$$

  where:

  $\overline{U}_m$ – depth averaged, between the bed and wave trough, undertow velocity,

  Q – resultant onshore flow between the wave crest and trough,

  d_{tr} – distance from the bed to the wave trough.

- from the postulate of the continuity of the velocity at the top of the bed boundary layer, which denotes that the undertow velocity at the bed is equal to the nearbed velocity $\overline{U}_b$.

The determination of the onshore flow Q is closely associated with the liquid mass transport of water. Although this flow is generally minor hydro- and lithodynamic importance, it is extremely important for vertical distribution of return currents.

Stokes was the first to postulate in 1847 the existence of the flows of this kind. Therefore, liquid mass transport in literature is often called Stokes drift. These flows result from the fact that the horizontal components of the orbital velocity increase with the distance from the bed. As a result, water particles beneath the wave crest move onshore faster than the particles around the trough move offshore.

In the surf zone, one should also account for the influence of the roller arising on the crests of the breaking waves. Its presence causes the additional generation of water flow in this region. As a result, the onshore flow Q in the surf zone between wave crest and trough is the sum of two components:

$$Q = Q_s + Q_r, \tag{23}$$

where:

- Q_s – flow corresponding to Stokes drift,
- Q_r – roller-related flow.

Similarly, the mean velocity of undertow $\overline{U}_m$ can be described in the form of:

$$\overline{U}_m = \overline{U}_s + \overline{U}_r, \tag{24}$$

where $\overline{U}_s$ and $\overline{U}_r$ are the undertow mean velocities below the wave trough related to the mass transport Q_s and to the roller-induced flow Q_r, respectively.

For the linear wave theory, the distribution of the mass transport velocity averaged over wave period can be presented as (see e.g. Craik 1982 or Van Rijn 1993):

$$\overline{U}_s(z) = \frac{1}{8} \cdot \omega \cdot k \cdot H^2 \frac{\cosh[2k(z+h)]}{\sinh^2(kh)}, \tag{25}$$

where $\omega = 2\pi/T$.

Below the wave trough, the depth-averaged discharge of the mass transport is zero. In the region between the wave crest and trough, onshore water flow occurs, due to asymmetry of the horizontal components of the orbital velocity. This flow is described by the relationship:

$$Q_s = \int_{-h}^{0} \overline{U}_s(z) dz = \frac{E}{\rho \cdot C} = \frac{g \cdot H^2}{8 \cdot C} \tag{26}$$

from which the mean undertow velocity beneath the wave trough, resulting from the existence of mass transport Q_s, amounts to:

$$\overline{U}_s = -\frac{Q_s}{h} = -\frac{1}{8} \cdot \frac{g \cdot H^2}{C \cdot h} \tag{27}$$

For actual wave conditions, the assumption of a constant sinusoidal wave shape in the surf zone is rather doubtful. Svendsen (1984) assumed that the wave shape, different from sinusoidal, can be described in this region by an empirical coefficient B_0. Thus, rearranging Eq. (27), one can obtain:

$$\overline{U}_s = -B_0 \cdot \frac{g \cdot H^2}{C \cdot h} = -B_0 \cdot C \cdot \left(\frac{H}{h}\right)^2 \tag{28}$$

where $B_0 = \frac{1}{T}\int_0^T \left(\frac{\eta}{H}\right)^2 dt$ is the wave shape parameter, which for a sinusoidal wave is $B_0 = \frac{1}{8} = 0.125$.

On the basis of laboratory measurements, Svendsen (1984) and Svendsen & Buhr-Hansen (1987) estimated the value of the B_0 parameter, in the range of $B_0 \approx 0.06 \div 0.09$, while Stive (1980) and Sive & Wind (1982) and Szmytkiewicz (2002), also on the basis of wave flume experiments, assessed the value of B_0 as $B_0 \approx 0.1$.

The mean velocity $\overline{U}_r$, resulting from the roller, according to the definition presented by Svendsen (1984), is described by the formula:

$$\overline{U}_r = -\frac{Q_r}{h} = -\frac{A}{T \cdot h}, \tag{29}$$

where A is determined by Eq. (3).

Actually, roller surface A is not constant. It increases from zero at the initial location of wave-breaking up to the maximum value in the inner part of the surf zone. Indeed, estimation of the roller surface is difficult. The literature, e.g. Okayasu et al. (1988) or Deigaard et al. (1991) offers a number of suggestions providing more precise determination of roller surface A. All the proposals are based on laboratory measurements and concern only the inner part of the surf zone. In the present study, the computations have been made assuming the coefficient k_r in Eq. (3) as $k_r \approx 0.5$. This value has been established by Szmytkiewicz (2002) on the basis of comparison between the calculated velocities $\overline{U}_m$ and the wave flume data.

Inserting the formulas (28) and (29) into Eq. (24), one obtains the mean undertow velocity $\overline{U}_m$ below the wave trough in the surf zone:

$$\overline{U}_m = \overline{U}_s + \overline{U}_r = -C \cdot \frac{H^2}{h^2}\left(B_0 + \frac{A \cdot h}{L \cdot H^2}\right) \tag{30}$$

Outside the surf zone, where roller doesnot occur, the second component of the above equation is zero.

The second boundary condition, necessary to solve Eq. (20) describing the vertical distribution of undertow, is associated with the determination of the velocity $\overline{U}_b$ in the sea bed vicinity. For the oscillating-current bed boundary layer, the velocity $\overline{U}_b$ is the sum of two actions: in the bed boundary layer – time-averaged onshore flow, in the outer region (between wave trough and upper limit of the bed boundary layer) – offshore flow.

Due to lack of a theory describing wave field in the surf zone, similar to the determination of the onshore flow Q, so far there are no correct theoretical solutions for the description of the velocity $\overline{U}_b$.

In most of the undertow models, the nearbed velocity $\overline{U}_b$ is determined at the top of the bed boundary layer. To do this, the authors of these models refer to the studies of Longuet-Higgins (1953 and 1960), in which the velocity $\overline{U}_g$ was determined just outside the oscillatory turbulent boundary layer. Next, with the assumption that the turbulence in the surf zone is mainly generated by breaking waves (not nearbed eddies), numerous modifications of the Longuet-Higgins formula appeared (e.g. the works by Svendsen's team in 1984-1987, Dally 1980 and Stive & Wind1982). According to these modifications, the nearbed velocity $\overline{U}_b$ is described by the following semi-empirical formula:

$$\overline{U}_b = -f_b \cdot \left(\frac{H}{h}\right)^2 \cdot C \tag{31}$$

in which f_b is the proportionality coefficient.

It should be pointed out that the coefficient f_b is not universal. Its value depends on the hydrodynamic conditions occurring at the individual specific location in the coastal zone. On the basis of comparisons between calculated return currents and the wave flume measurements, Szmytkiewicz (2002) determined the proportionality coefficient f_b for the surf zone to be in the range of: $f_b \approx \frac{1}{4} \div \frac{1}{16}$. Previously, this quantity was approximately determined as 3/16 and 1/8 by Svendsen (1984) and Dally (1980), respectively.

Because of the lack of definite criteria providing the coefficient f_b *a priori* (without support of measurements), Kaczmarek & Szmytkiewicz (1993) proposed an alternative method for determination of the nearbed velocity $\overline{U}_b$. Bearing in mind that eddy viscosity in the bed boundary layer is much smaller than its counterpart beyond this layer, the quantity $\overline{U}_b$ can be determined assuming that the shear stresses at the top of the bed boundary layer are zero, which implies:

$$\left.\frac{d\overline{U}(z)}{dz}\right|_{z=-h} = 0. \tag{32}$$

Inserting in the above equation the right hand side of Eq. (20) as the velocity $\overline{U}(z)$ and taking advantage of Eq. (22) one obtains:

$$\overline{U}_b = \overline{U}_m - \frac{1}{6}\frac{1}{\rho \cdot \nu_{Tz}}\frac{dR_x}{dx}d_{tr}^2. \tag{33}$$

It can be seen from the form of the above relationship that the velocity $\overline{U}_b$ can be determined without introduction into the model of any new empirical coefficients.

The integration constants C_3 and C_4 in Eq. (21) describing the vertical distribution of the longshore current velocity, are determined from the following conditions:

– depth-averaged distribution of the velocity $\overline{V}(z)$ equals the mean longshore current velocity $\overline{V}_m$, i.e.:

$$\int_{-h}^{0} \overline{V}(z)dz = \overline{V}_m \cdot h, \tag{34}$$

– the bed shear stresses are the same as for the considerations of the depth-averaged longshore currents, i. e.:

$$\rho \nu_{Tz} \frac{\partial \overline{V}(z)}{\partial z}\bigg|_{z=-h} = \overline{\tau}_{by}. \tag{35}$$

In the present study, as mentioned, the depth-averaged longshore current velocity $\overline{V}_m$ is determined using the numerical model LONG-CUR by Szmytkiewicz (2002).

Adapting the classical relationship between the shear stresses at the top of the bed boundary layer with the flow velocities beyond this layer, and then assuming so small a wave approach angle θ that practically $V \perp \widetilde{u}_b$, and that the longshore current is small with respect to orbital velocity ($V << \widetilde{u}_b$), the time-averaged linear form of the bed shear stresses in the longshore direction, for linear wave theory and the shallow water approximation, has the form:

$$\vec{\tau}_{by} = \rho \cdot \frac{1}{2} f_w \frac{H \cdot g}{\pi \cdot C} \cdot \overline{V}_m. \tag{36}$$

In the above formula, the friction coefficient f_w in most of the longshore current models is determined on the basis of measurements and assumed constant for the entire coastal zone. Its mean value is estimated by many researchers for various nearshore regions as about 0.01. For practical calculations of the longshore current velocities, the value of the friction coefficient should be determined for specific hydro- and litho-dynamic conditions occurring in the analysed area. For instance, for the multi-bar coastal zone of the south Baltic, Szmytkiewicz (2002) determined the friction coefficient as $f_w \approx 0.015$. This value was found on the basis of a great number of field measurements, mainly at the Coastal Research Station of Lubiatowo.

Taking advantage of the above boundary conditions, one obtains the following final forms of the equations describing the depth-variable velocities of the return flow $\overline{U}(z)$ and the longshore current $\overline{V}(z)$ at an individual location of the cross-shore profile:

$$\begin{aligned}\overline{U}(z) = & \frac{1}{\rho \cdot \upsilon_{Tz}} \cdot R_x \cdot \frac{1}{2} \cdot (z+h)^2 + \\ & +(z+h)\left[\frac{2(\overline{U}_m - \overline{U}_b)}{d_{tr}} - \frac{1}{\rho \cdot \upsilon_{Tz}} \cdot R_x \cdot \frac{1}{3} \cdot d_{tr}\right] + \overline{U}_b.\end{aligned} \tag{37}$$

$$\overline{V}(z) = \frac{1}{2}\frac{1}{\rho \cdot \nu_{Tz}} \cdot R_y \cdot \frac{1}{2}(z+h)^2 + \\ +(z+h)\left[\frac{2(\overline{V}_m - \overline{V}_b)}{h} - \frac{1}{\rho \cdot \nu_{Tz}} \cdot R_y \cdot \frac{1}{3} \cdot h\right] + \overline{V}_b, \tag{38}$$

$$\overline{V}_b = \overline{V}_m - \frac{1}{2 \cdot \rho \cdot \nu_{Tz}} \cdot \overline{\tau}_b \cdot h - \frac{1}{6} \cdot \frac{1}{\rho \cdot \nu_{Tz}} \cdot R_y \cdot h^2. \tag{39}$$

For small angles of wave approach, the quantity R_x is determined by Eq. (15), while for an arbitrary angle Eq. (14) should be used.

4. Turbulence Viscosity

According to Boussinesq hypothesis, turbulent stresses can be expressed by the mean parameters of flow. In such an approach, after Prandtl, the turbulent viscosity can generally be presented in the following form:

$$\nu_T = -\overline{|u_i' \cdot l_i'|} \tag{40}$$

where u_i' is the measure for turbulent scale of velocity and l_i' the measure for turbulent scale of the mixing length.

The problem of choice of the characteristic scales for velocities and lengths in the description of turbulence in the surf zone is the most sophisticated task in computations of coastal currents. In spite of a number of measurements of velocity profiles, both in laboratory and field, there are no clear-cut justifications for any choice of scales characterising the turbulence.

For the description of wave-driven currents, it is particularly important to determine the viscosity ν_{Tz} in the vertical. The mathematical background of depth-variable longshore and cross-shore currents comprises an assumption of linear relationship between the time-averaged shear stresses $\overline{\tau}_z$ and the gradients of time-averaged velocities $\partial\overline{U}(z)/\partial z$ and $\partial\overline{V}(z)/\partial z$, see Equations (13) and (19).

Because of the complex mechanism of generation and motion of the turbulent eddies in the surf zone, no clear-cut theoretical circumstances exist which can provide proper determination of turbulent viscosity varying in time and space. In all existing models of the wave-driven currents, a constant value of ν_{Tz} in time is assumed, while various approaches exist to determine its variability over depth. Generally, two separate groups of the models can be distinguished.

In the first group, a constant depth-averaged turbulent viscosity is assumed. This group of models comprises works by such authors as: Dally (1980), Svendsen (1984), Stive & Wind (1986), Svendsen et al. (1987) and Mckee Smith et al. (1992). In these models, it is assumed that the wave phase velocity is a measure for the turbulent scale of velocity, while the water depth in the surf zone is a measure for the turbulent length scale. As a result, the turbulent viscosity in vertical is described by the formula of the following type:

$$\nu_{Tz} = f_t \cdot h \cdot \sqrt{g \cdot h} \tag{41}$$

where f_t is a coefficient.

On the basis of numerous measurements, mainly in the laboratory, the coefficient f_t can be estimated to lie within the range of from 0.01 to 0.03. For instance, Szmytkiewicz (2002), basing on the comparisons of measured velocities in twelve verticals, for each of four wave flume experimental series, found this coefficient to be: $f_t = 0.025$.

The second group of models constituted those which account for variability of the turbulent viscosity over water depth. The authors of these models are right to claim that the turbulent energy approaching the sea bed reduces, due to dissipation processes in the surf zone. Such models of wave-driven currents were presented by, inter alia, Madsen & Svendsen (1983), Yamashita et al. (1990, 1991), Cox et al. (1994) and Rattanapitikon & Shibayama (2000).

The variability of ν_{Tz} over depth in these models is most often described within one- and two-equation models of transport of the turbulent kinetic energy. The necessity of additional determination of proportionality coefficients in the final forms of respective formulas is the shortcoming of this kind of description.

Because of major difficulties associated with determination of the above-mentioned proportionality coefficients, one can alternatively assume the linear increase of the turbulence from the sea bed towards water surface. This assumption corresponds well with the observations of the turbulent eddies occurring in various regions of the surf zone, as presented e.g. in the studies of Cox et al. (1994, 1996). Examples of such determination of the viscosity ν_{Tz} were given e.g. by Rattanapitikon & Shibayama (2000) and Szmytkiewicz (2002). In the latter study, with the assumption that the total energy dissipation of breaking waves is a measure of the turbulent scale of velocity, the following formula for the turbulent viscosity is postulated:

$$\nu_{Tz} = \nu_{Tz_0} + C_D \cdot \left(\frac{D_{total}}{\rho}\right)^{1/3} \cdot \varsigma \qquad \text{for} \quad 0 \le \varsigma \le d_{tr}, \tag{42}$$

in which d_{tr} is the distance from the sea bed to wave trough, D_{total} denotes energy dissipation of the breaking wave accounting for the roller's presence and ν_{tz_0} – the turbulent viscosity immediately outside the bed boundary layer. The proportionality coefficient C_D and turbulent viscosity ν_{tz_0} were estimated from the wave flume measurements.

The above proposals of determination of the turbulent viscosity ν_{Tz} show that due to the complexity of the processes which generate and transport turbulent eddies, there are significant difficulties not only in the theoretical description of this quantity, but also in its determination from measurements. Therefore, in models of wave-driven currents, this parameter boasts a certain degree of freedom. The ultimate value of ν_{Tz} is found from the calibration of a mathematical model using available experimental data of depth-variable velocities. This implies that it is difficult to judge whether the assumption of a constant depth-averaged turbulent viscosity is more effective than the assumption of its vertical variability, in order to achieve undertow and longshore distributions compliant to actual values.

In the present paper, the viscosity ν_{Tz} is assumed constant, averaged over water depth. Its value is calculated from Eq. (41).

The determination of the turbulent viscosity ν_{Tx}, being present in Equations (18) and (39), is a separate problem. The results of measurements, from both literature and the Author's data base, show that the variability of undertow over depth is much greater than the horizontal variability of longshore currents. This denotes that the turbulent viscosity ν_{Tx} is much greater than ν_{Tz}. The above-mentioned difference between ν_{Tx} and ν_{Tz} results from different geometrical scales in the directions related to these values.

On the basis of measured longshore currents, Svendsen & Putrevu (1990) suggested this viscosity to be determined from the relationship:

$$\nu_{Tx} \approx \frac{\nu_{Tz}}{\tan\beta}, \tag{43}$$

in which β is the mean bottom slope in the cross-shore direction.

While describing theoretically the longshore currents, in determination of the horizontal turbulent viscosity ν_{Tx}, Longuet-Higgins (1970) proposed for the first time to take the maximum nearbed velocity as the characteristic scale of velocity and the product of the offshore distance and von Karman constant – as the characteristic scale of the length of turbulent eddies. For the linear wave theory, shallow water approximation and the assumption of evenly sloped bottom, this formula yields:

$$\nu_{Tx} = N \cdot x \cdot \sqrt{g \cdot h}, \tag{44}$$

where N is the proportionality coefficient, estimated by Longuet-Higgins as $0.0 \div 0.016$.

By dividing the turbulent viscosity ν_{Tx} determined by the above relationship by Eq. (41) determining the viscosity ν_{Tz} one obtains:

$$\frac{\nu_{Tx}}{\nu_{Tz}} = \frac{N \cdot x \cdot \sqrt{g \cdot h}}{f_b \cdot h \cdot \sqrt{g \cdot h}} = \frac{N \cdot x}{f_b \cdot h} \approx O\left(\frac{x}{h}\right). \tag{45}$$

The last term of Eq. (45) is true for N approaching its maximum value.

It can be seen that Equations (43) and (45) have similar forms. Therefore, it can be assumed that:

$$\nu_{Tx} \approx m \cdot \nu_{Tz}, \tag{46}$$

where m is a factor ($m \approx \tan\beta$), the value of which should be determined from measurements.

Assuming that the quantity x can be approximately treated as the surf zone width and the quantity h as water depth in the location of wave-breaking, on the basis of the comparisons between modelled and measured velocity distributions, Szmytkiewicz (2002) estimated the value of the parameter m for the south Baltic coastal zone as $m \approx 50$.

5. Nearshore Wave Field

The knowledge of wave parameters at all locations of the coastal zone (wave approach angle θ, wave number k and wave height H) is necessary for the solution of Equations (38) and (39), describing the vertical distributions of the velocities $\overline{U}$ and $\overline{V}$.

In the present study, assuming the linear wave refraction, the variability of the wave angle approach is calculated from the Snell law, while the wave number k is determined from the dispersion relationship for the linear wave theory. Assuming that there are no wave reflections from the shore and that there is no interaction between waves and current, the wave height H is computed from the equation of the energy flux conservation, which accounting for the roller has the following form:

$$\frac{\partial}{\partial x}(E \cdot C_g \cdot \cos\theta) + \frac{\partial}{\partial x}(E_r \cdot C \cdot \cos\theta) = -D, \tag{47}$$

where:

E	–	total wave energy,
E_r	–	kinetic energy of the roller by Eq. (1),
C, C_g	–	phase and group velocity of waves, respectively,
θ	–	wave approach angle,
D	–	wave energy dissipation due to wave-breaking.

Observed in nature for most wave situations by many researchers (inter alia by Kuriyama 1994 and Szmytkiewicz 1997), the shoreward displacements of the maximum flow velocities with respect to wave-breaking locations denote that a certain lag exists between breaking wave and wave-induced current. At present, this phenomenon is explained by the appearance of the roller on the breaking wave crest. The presence of the roller in the above equation denotes that the wave energy dissipation first causes the appearance and development of the roller then, when the roller attains a fully developed shape, the water flow is generated. As a result, a certain delay exists between the starting moment of wave energy dissipation and the moment of appearance of the wave-driven currents.

The dissipation on the right hand side of Eq. (47) has been calculated on the assumption that the wave energy dissipation is related to the wave breaking process only. Actually, the bottom friction is the second source of energy dissipation. However, this factor has much less influence on reduction of wave height approaching shore, than the wave-breaking process. For typical storm conditions on the Pacific Ocean, as shown by Thornton & Guza (1983), the energy dissipation due to bottom friction is less than 3% of that due to wave-breaking. For storm conditions in the Baltic coastal zone, Szmytkiewicz & Skaja (1993) estimate that the dissipation due to bottom friction does not exceed 1%.

Assuming the narrow spectrum of random waves in the coastal zone and Rayleigh distribution of the wave height, the energy dissipation of breaking waves has been described by the formula of Battjes & Janssen (1978), i.e.:

$$D = \frac{\alpha}{4} Q_b \cdot f_p \rho g H_m^2, \tag{48}$$

in which the factor Q_b, characterising the percentage of broken and breaking waves at any individual point of the coastal zone, is described by the relationship:

$$\frac{1 - Q_b}{\ln Q_b} = -\left(\frac{H_{rms}}{H_m}\right), \tag{49}$$

in which α is an empirical coefficient of the order O(1), f_p – the wave spectrum peak frequency $((f_p = 1/T_p)$, H_m denotes the maximum possible wave height at the considered location of the coastal zone and H_{rms} – the actual sought (root-mean-square) wave height.

The maximum possible wave height H_m for a given location in the coastal zone can be found from the criterion formulated by Miche:

$$H_m = 0.88 \cdot k_p^{-1} \tan h(\gamma \cdot k_p \cdot h/0.88), \tag{50}$$

where k_p is the wave number calculated from the dispersion relationship for the linear wave theory with wave spectral peak f_p and γ the wave breaking coefficient.

On the basis of the long-term experience of the Author associated with participation in a number of field measurements and execution of many computations for various shore segments of the south Baltic, the value of the wave-breaking coefficient is hereby suggested as $\gamma \approx 0.8$ for intensive wave conditions (distinct multiple wave-breaking). For weaker wave conditions, this coefficient can be calculated using the formula by Battjes & Stive (1985):

$$\gamma = 0.5 + 0.4 \cdot \tanh(33 \cdot s_0), \tag{51}$$

where s_0 is the deep water wave steepness.

6. Comparison of Measured and Calculated Wave Driven Currents

In the numerical model CUR-3DQ, the computations are carried out in three stages. In the first, the wave field parameters (wave height, angle of incidence), as well as mean free surface elevation and energy dissipation due to wave-breaking are calculated. In the second stage, the depth-averaged longshore current velocities for the entire cross-shore profile are computed. These computations are carried out using the LONG-CUR model, presented by Szmytkiewicz (2002). In the third stage, according to Equations (37) and (38), the depth-variable longshore and cross-shore currents are determined. The computations are carried out for arbitrarily assumed verticals located in the cross-shore transect.

The correctness of the CUR-3DQ model has been using the flow velocities measured in the multi-bar shore profile during two field campaigns: "Lubiatowo 87" and "Lubiatowo 96".

Fig. 1 shows the exemplary comparison of two series of velocity distributions $\overline{V}(z)$ and $\overline{U}(z)$ computed and measured during the "Lubiatowo 87" campaign. The full set of data used for the model tests was presented by Szmytkiewicz (1994). The figure depicts the assessment of agreement between calculated and measured wave heights and velocities at chosen locations of the multi-bar coastal zone of the south Baltic. The data presented in Fig. 1 were obtained for storm conditions, during which strong winds from the west with velocities attaining 20 m/s, generating mean deep water waves up to 1.4 m high occurred.

Similar comparisons of computed and measured velocity distributions during the "Lubiatowo 96" campaign are shown in Fig. 2.

Shown in Figures 1 and 2 the locations of the measuring verticals with respect to the surf zone width imply that the first offshore vertical was located, depending on wave conditions, either beyond or just at the beginning of the surf zone. The second measuring vertical was always located in the surf zone. According to the theoretical influence of the sign of the gradients $\partial(H)^2/\partial x$ and $\partial\overline{\eta}/\partial x$ in the surf zone and just seawards of it, as postulated by Kaczmarek & Szmytkiewicz (1993), two different shapes of undertow distributions were obtained for these regions.

In Figures 1 and 2, relatively good conformity between calculated and measured velocities can be seen, particularly in the surf zone. Greater discrepancies visible in the vertical B can be explained by the fact that this vertical could actually be located at the offshore limit of the surf zone while in computations it was treated as being located beyond this zone. For the verticals located close to each other, particularly for the situations depicted in Fig. 2, small inaccuracy in the computations of wave transformation could result in unrealistic location of the measuring vertical with respect to the wave-breaking line.

Due to the limited amount of data, it should be pointed out that the comparisons presented cannot be treated as the complete and ultimate verification of the model.

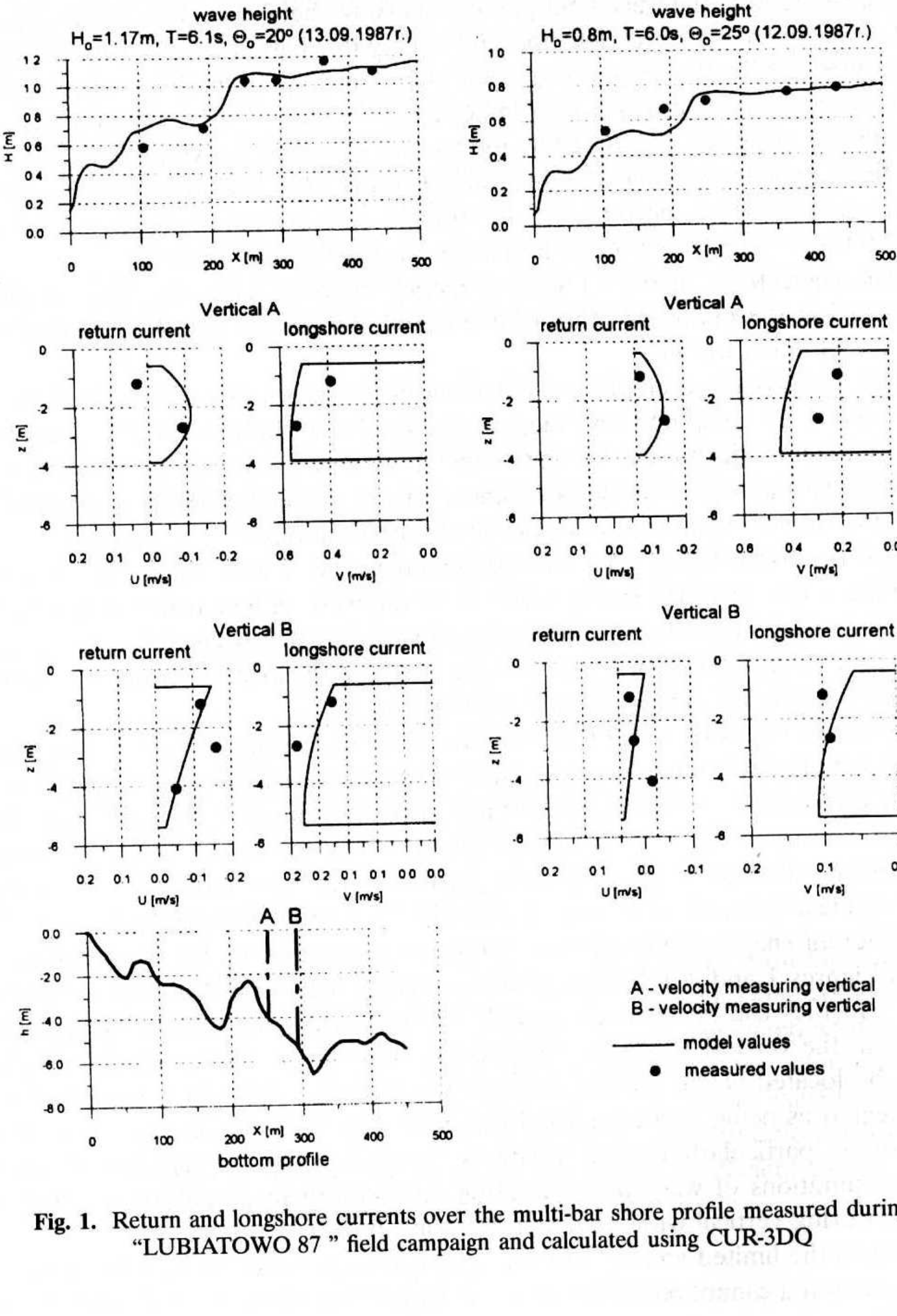

Fig. 1. Return and longshore currents over the multi-bar shore profile measured during "LUBIATOWO 87 " field campaign and calculated using CUR-3DQ

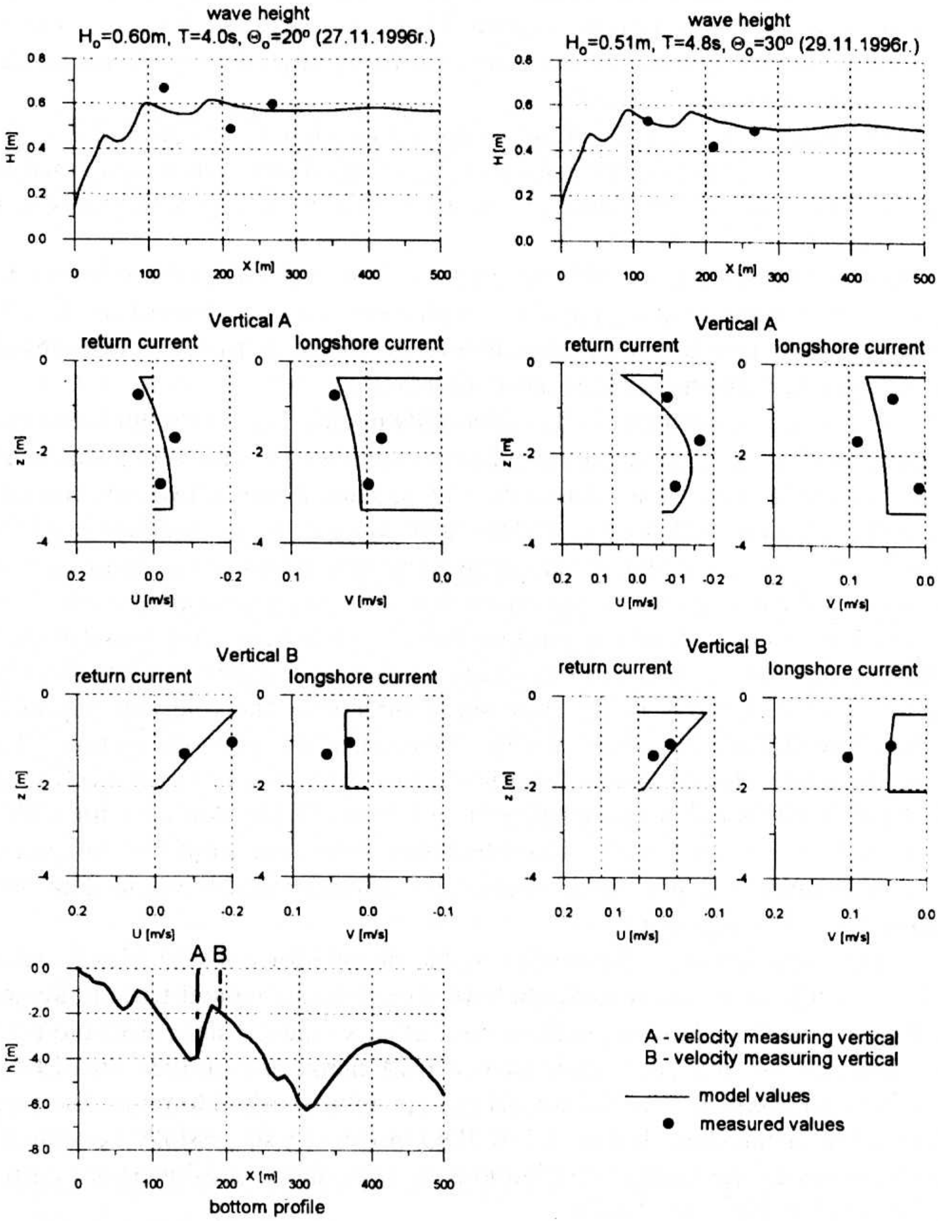

Fig. 2. Return and longshore currents over the multi-bar shore profile measured during "LUBIATOWO 96 " field campaign and calculated using CUR-3DQ

7. Conclusions

The quasi-three-dimensional model CUR-3DQ enables the computation of depth-variable velocities of longshore current $\overline{V}(z)$ and undertow $\overline{U}(z)$, i.e. the two most important components of the nearshore flows, at arbitrarily chosen verticals located in the cross-shore profile.

In the model, the water flows are computed as a function of dissipation of the wave energy flux. In the determination of the energy dissipation it is assumed that the wave-breaking with the roller on the wave crest is the only factor causing the wave energy loss.

The calculations imply that between wave crest and trough the resultant flow exists which is directed along the shore, with a slight turning towards it. This flow becomes almost parallel to the shoreline about the wave trough, whilst below it the flow gradually turns in an offshore direction.

There are eight constants in the CUR-3DQ model. The first, namely the wave-breaking coefficient γ, characterises the surf zone width. This coefficient is used in all models describing wave-driven currents in coastal zones. Two next constants, i.e. the bed friction coefficient f_w and turbulent viscosity ν_T are used in the LONG-CUR model, which is employed in necessary computations of $\overline{V}_m$. Four constants are used in the description of the return flows. There are two coefficients k_r and B_0, necessary to determine the onshore flow between wave crest and trough, the turbulent viscosity ν_{Tz} and proportionality coefficient f_b in the formula describing the nearbed velocity $\overline{U}_b$ in the cross-shore direction. The role and importance of these coefficients have been widely discussed in the previous sections. Their values have been found mainly from laboratory data. The last eigtht constant in the CUR-3DQ model is the coefficient $m \approx \tan\beta$ in the Author's formula for the turbulent viscosity ν_{Tx}. Its value has been estimated from the comparisons between calculated depth-variable longshore velocities and the field data of the "Lubiatowo 87" campaign.

Comparisons between the depth-variable distributions of velocities calculated by CUR-3DQ and experimental data have shown the agreement to be quite good.

Because of the relatively small amount of experimental data available (in the computations use has been made of two field campaigns of 1987 and 1996), it is difficult to assess the model correctness unambiguously. However, the results obtained so far indicate that the CUR-3DQ model can successfully be applied in computations of the vertical distributions of cross-shore and longshore currents in the south Baltic coastal zone.

References

Battjes J. A., Janssen J. P. F. M. (1978), Energy loss and set-up due to breaking of random waves, *Proc. 16th Conf. Coastal Eng.*, Vol. I, 569-587.

Battjes J. A., Stive M. J. F. (1985), Calibration and verification of a dissipation model for random breaking waves, *J. Geophysical Research*, Vol. 90, No. C5, 9159–9167.

Bowen A. J. (1969), The generation of longshore currents on a plane beach, *Journal of Marine Research*, 27, 206–215.

Cox D., Kobayashi N., Okayasu A. (1994), Vertical variations of fluid velocities and shear stress in surf zone, *Proc. 24th Coastal Eng. Conf., ASCE*, 98–112.

Cox D., Kobayashi N. (1996), Undertow profiles in the bottom boundary layer under breaking waves, *Proc. 25th Coastal Eng. Conf., ASCE*, 3194–3206.

Craik A. D. D. (1982), The drift velocity of water waves, *J. Fluid Mech.*, Vol. 116, 187–205.

Dally W. R. (1980), *A numerical model for profile evolution*, A thesis submitted to the faculty of the University of Deleware in partial fulfilment of the requirements for the degree of Master of Civil Engineering.

Deigaard R., Fredsoe J. (1989), Shear stress distribution in dissipative water waves, *Coastal Engineering* 13, 357–378.

Deigaard R., Justesen P., Fredsoe J. (1991), Modelling of undertow by a one-equation turbulence model, *Coastal Engineering*, 15, 431–458.

Deigaard R. (1993), A note on the three-dimensional shear stress distribution in a surf zone, *Coastal Engineering*, 20, 157–171.

Kaczmarek L. M., Szmytkiewicz M. (1993), Mechanizmy generacji prądów powrotnych w strefie przybrzeżnej morza, *Rozprawy Hydrotechniczne*, R. 56, 67–88.

Kraus N. C., Sasaki T. O. (1979), Effects of wave angle and lateral mixing on the longshore currents, *Coastal Engineering in Japan*, Vol. 22, 59–73.

Kuriyama Y. (1994), Numerical model for longshore current distribution on a bar-trough beach, *Proc. 24th Coastal Engineering Int. Conference*, ASCE, Kobe, Japan, 2237–2251.

Kuroiwa M., Hideaki N., Yuhei M. (1998), Applicability of a quasi-three dimensional Kuroiwa numerical model to nearshore currents, *Proc 26th Coastal Engineering Conference ICCE*, 815–842.

Lee J., Wang H. (1994), A quasi-3D surf zone model, *Proc 24th Coastal Engineering Conference ICCE*, 2267–2281.

Longuet-Higgins M. S. (1953), Mass transport in gravity waves, *Philos. Trans. R. Soc. London*, Ser A, 245.

Longuet-Higgins M. S. (1960), Mass transport in the boundary layer at a free oscillating surface, *J. Fluid Mechanics*, 8, 293–306.

Longuet-Higgins M. S., Stewart E. W. (1964), Radiation Stress In Water Waves, A Physical Discussion With Applications, *Deep Sea Res.*, Vol 11, 529–562.

Longuet-Higgins M. I. (1970), Longshore currents generated by obliquely incident sea waves, *J.Geophys. Res.*, Vol. 75, No. 33.

Madsen P. A., Svendsen I. A. (1983), Turbulent bores and hydraulic jumps, *J. Fluid Mech.*, 129, 1–25.

Mckee Smith J., Svendsen I., Putrevu U. (1992), Vertical structure of the nearshore current at Delilah: measured and modelled, *Proc. 23rd Coastal Eng. Conf., ASCE*, 2825–2837.

Okayasu A., Shibayama T., Horikawa K. (1988), Vertical variation of undertow in the surf zone, *Proc. 21st Coastal Eng. Conf., ASCE*, 478–491.

Rattanapitikon W., Shibayama T. (2000), Simple model for undertow profile, *Coastal Engineering Journal*, Vol. 42, No. 1, 1–30.

Rivero F. J., Arcilla A. S. (1995), On the vertical distribution of $< \widetilde{u} \cdot \widetilde{w} >$, *Coastal Engineering*, 25, 137–152.

Stive M. J. F. (1980), Velocity and pressure field of spilling breakers, *Proc. 17th Coastal Eng. Conf. ASCE*, 547–566.

Stive M. J. F., Wind H. G. (1982), A study of radiation stress and set-up in the nearshore region, *Coastal Engineering*, 6, 1–25.

Stive M. J. F., Wind H. G. (1986), Cross-shore mean flow in the surf zone, *Coastal Engineering*, 10, 325–340.

Stive M. J. F., De Vriend H. (1994), Shear stresses and mean flow in shoaling and breaking waves, *Proc. 24th Int. Conf. Coastal Eng.*, 594–605.

Svendsen I. A. (1984), Mass flux and undertow in a surf zone, *Coastal Engineering*, 8, 347–365.

Svendsen I. A., Buhr-Hansen J. (1986), The interaction of waves and currents over longshore bar, *Proc. 20th Int. Conf. Coastal Eng.*, Taipei.

Svendsen I. A., Buhr-Hansen J. (1987), Cross-shore currents in surf zone modelling, *Coastal Engineering*, 12, 23–42.

Svendsen I. A., Buhr-Hansen J. (1988), Cross-Shore Currents in Surf-Zone Modelling, *Coastal Engineering*, 12, 23–42.

Svendsen I. A., Lorenz R. S. (1989), Three dimensional velocity profiles in combined undertow and longshore currents, *Coastal Engineering*, 13, 1, 55–80.

Svendsen I. A., Putrevu U. (1990), Nearshore circulation with 3D profiles, *Proc. 22nd Coastal Engineering Conference ICCE*, Chap. 18, 242–254.

Szmytkiewicz M., Skaja M. (1993), Model prądów wzdłużbrzegowych dla rewowego profilu dna oraz wielokrotnego załamania fali, *Rozprawy Hydrotechniczne*, Gdańsk.

Szmytkiewicz M. (1994), *Modele prądów pochodzenia falowego w strefie brzegowej morza*, Raport Instytutu Budownictwa Wodnego w Gdańsku.

Szmytkiewicz M. (1997), *Pomiary falowania i prądów w wielorewowej strefie brzegowej w ramach ekspedycji „Lubiatowo 96"*, Praca wewnętrzna IBW PAN, Gdańsk.

Szmytkiewicz M. (2002), *Prądy pochodzenia falowego w morskiej strefie brzegowej*, Wydawnictwo IBW PAN, Gdańsk, p. 178.

Thornton E. (1970), Variation of longshore current across the surf zone, *Proceedings of the 12th Conference on Coastal Engineering*, Vol. I.

Thornton E., Guza R. T. (1983), Transformation of wave height distribution, *J. Geophys. Res.*, Vol. 88, C10.

Thornton E. B., Guza R. T. (1986), Transformation surf zone longshore currents and random waves, Field Data and Models, *J. Phys. Oceanogr.*, Vol. 16, No. 7, 1165–1178.

Van Dongeren A. R., Sancho F. E., Svendsen I. A., Putrevu U. (1994), SHORECIRC: A quasi 3-D nearshore model, *Proc 24th Coastal Engineering Conference ICCE*, 2740–2754.

Van Rijn L. C. (1993), *Principles of sediment transport in rivers, estuaries and coastal seas*, Aqua Publications, the Netherlands.

Visser P. J. (1984), Uniform Longshore Current Measurements and Calculations, *Proc. Coastal Engineering Conference ASCE*, 2192–2207.

De Vriend H. J., Stive M. J. F. (1987), Quasi-3D modelling of nearshore currents, *Coastal Engineering*, 11, 565–601.

De Vriend H. J., Kitou N. (1990), Incorporation of wave effects in a 3D hydrostatic mean current model, *Proc. 22nd Coastal Engineering Conference, ASCE*, 1005–1018.

Yamashita T., Tschiya Y., Suriamihardja D. A. (1990), Vertically 2-D nearshore circulation model, *Proc. 22nd Coastal Eng. Conf., ASCE*, 150–163.