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Experimental Investigations of Elastic Anisotropy of Sands

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Abstract

Some results of experimental investigations of sands, performed in the triaxial compression apparatus with additional measurement of lateral strains are presented. These results suggest that the reversible (elastic) response of specimens is anisotropic. A simple experimental programme enables determining the elastic constants for the configuration analysed. Extensive discussion of results obtained is also presented.

1. Introduction

Despite rapid development of soil mechanics, some fundamental questions regarding the mechanical behaviour of granular media have not yet been answered. One of these basic problems deals with the reversible (elastic) response of these materials, and there still exist various opinions regarding this matter, see discussion presented in Sawicki and Świdziński (1998). More specific questions concern such problems as: whether the elastic response of granular materials is linear or non-linear, is it isotropic or anisotropic, etc.? It seems that most classical theories describing the behaviour of granular materials are based on the presumption as to linear and isotropic reversible response of these media. A brief review of the methods of determination of isotropic elastic moduli of particulate materials is presented in Świdziński (2000a), and extensive review of the field and laboratory tests is provided by Tatsuoka and Shibuya (1992).

Analysis of some recent experimental data suggests that sands display some anisotropic elastic properties (Tatsuoka 2000). Therefore, the plastic strains, which can be obtained as the difference between total and elastic strains, differ from those obtained under the assumption concerning isotropic elastic response. This difference may influence various theories of soil plasticity, which means that the problem considered is of great importance.

The aim of this paper is to present some results of experimental investigations of sands, performed in a triaxial compression apparatus with additional measurement of lateral strains. These results suggest that the reversible (elastic) response of sand is generally anisotropic. A simple experimental programme proposed, enables the determination of elastic constants for the configuration analysed. It is shown that these constants are different for loose and dense sands. Extensive discussion of the results obtained is also presented.

2. Experimental Technique and Programme

The triaxial compression tests were carried out in a computer-controlled hydraulic triaxial testing system from GDS Instruments Ltd., see Menzies (1988), Świdziński (2000b). The system has been additionally equipped with special gauges, adopting the Hall effect to measure both radial and axial local deformations of the specimen. The measurement accuracy of the gauges assured by the manufacturer is 1 μm (10^{-6} m) which for the specimen's height of 80 mm is equivalent to strain of 10^{-5}. Nevertheless, practical resolution of the gauges, for a sample with an average diameter of 38 mm and height of 80 mm, corresponds to a change of radial and axial strains of 10^{-4}.

It should be mentioned that the vertical strains were also measured externally in terms of volume change of water in the lower chamber of the triaxial apparatus responsible for the vertical movement of the pedestal, as well as in terms of volume change of pore water in the case of fully saturated drained samples.

All tests were performed on "Lubiatowo" sand which consists of medium subrounded grains. The characteristics of this sand are: mean grain diameter D_{50} = 0.25 mm, minimum void ratio $e_{\min}$ = 0.49, maximum void ratio $e_{\max}$ = 0.74, coefficient of uniformity C_u = 1.5, specific gravity G = 2.65. The samples were approximately 80 mm high with an average diameter of 38 mm. All specimens were prepared by air-dry pluviation from different heights, which varied depending on the required density. Special attention was paid to prepare loose and very loose specimens. In this case the funnel with sand was raised very slowly keeping its open end just above the specimen surface. In turn, dense and very dense samples were compacted in thin layers in terms of light tamping. After reconstituting the sample to its full height, the upper surface was carefully levelled in terms of wet pore stone. In the case of drained saturated samples, the saturation process was followed by carbon dioxide flushing which assured full saturation of the samples.

After having prepared a sample, a negative pore pressure of 15 kPa was applied in order to simplify the installation of local strain gauges. These were glued to the rubber membrane confining the samples, always at the same locations. The radial gauge was positioned precisely half-way up the sample height. In turn, the bases of the axial strains' gauges were installed on the sample symmetrically with respect to radial gauge. The distance between bases was 50 mm. This means that both 15 mm ends of the sample were excluded from the measurement of axial deformation together with bedding errors substantially influencing the measurement (see Tatsuoka and Shibuya 1991, Jardin at al. 1984). The specimen with all gauges installed is shown in Fig. 1.

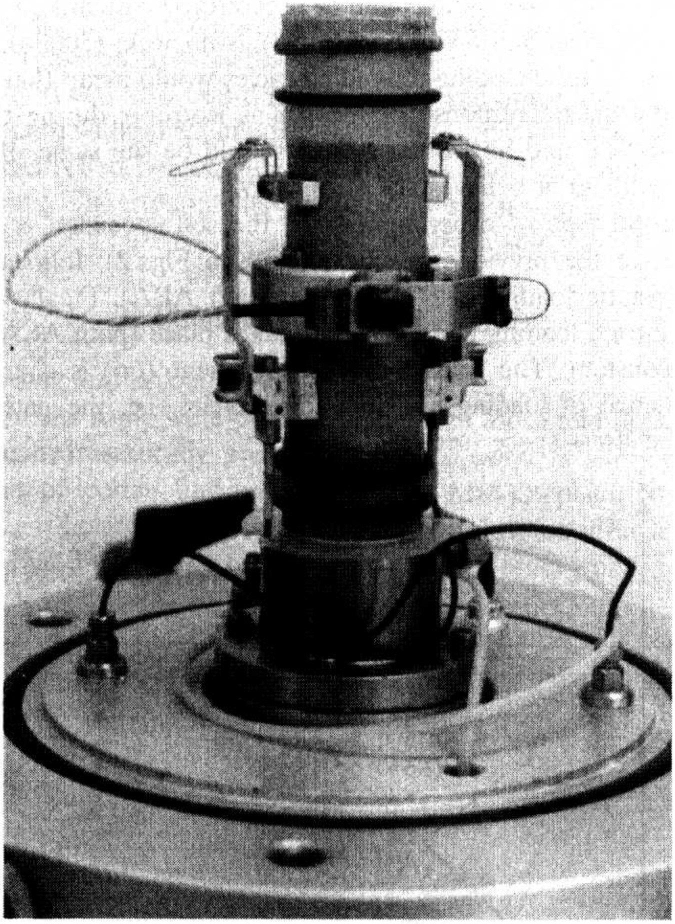

Fig. 1. Specimen of sand with gauges for local measurement of axial and radial strains

After assembling the gauges, the triaxial cell was installed and the cell pressure continuously applied with simultaneous reduction of negative pore pressure, maintaining a constant effective pressure of 15 kPa.

The experiments were performed for three types of specific loading paths. In the first (pp) the hydrostatic loading and unloading was applied. The specimen was initially pre-loaded, to a hydrostatic pressure level of p_0 which was treated as a reference stress. Then, a few cycles of hydrostatic loading and unloading were applied. The aim of such experiments is to detect whether the unloading stress-strain curves may be considered as elastic, and whether the soil response is isotropic or anisotropic.

Some comments as to the above criteria are necessary. Previous experimental investigations by the Authors have shown that large parts of unloading stress-strain curves have almost the same shape, and can therefore be identified with the elastic response – see Sawicki and Świdziński (1998), Świdziński (2000b). Similar behaviour during hydrostatic loading-unloading cycles would mean that the unloading is elastic. If the elastic response of the soil is isotropic during the hydrostatic loading, the vertical and horizontal strains should be the same. If they are not, means that the material behaves anisotropically.

In the second type of experiment (pq), the soil specimen is hydrostatically pre-loaded, as in the previous case (path OA in Fig. 2), followed by a single cycle of hydrostatic loading and unloading (path ABA). Finally, a single cycle of purely deviatoric loading and unloading takes place (path ACA), keeping the mean stress constant. The third type of experiment (qp) is characterised by a different sequence of loading and unloading cycles, i.e. the path OACABA is followed in the stress space (see Fig. 2).

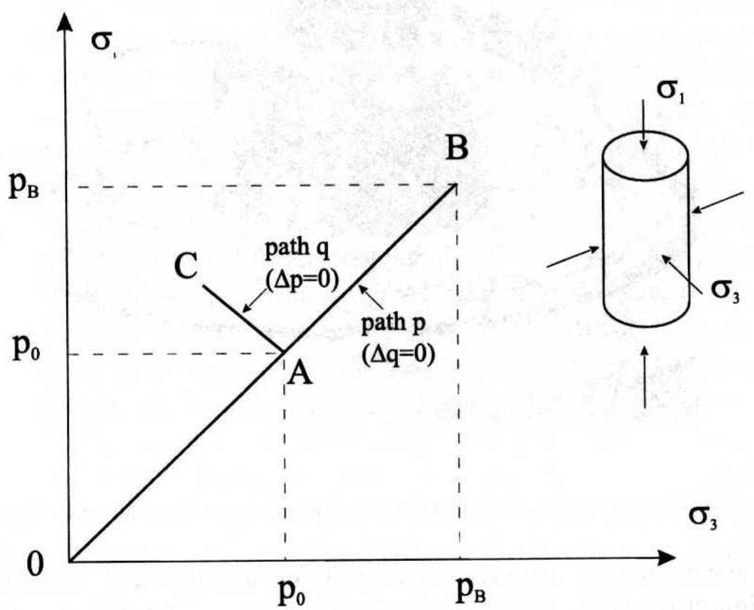

Fig. 2. Stress paths

The aim of the above experiments is to detect whether unloading paths BA and CA may be treated as elastic and eventually to determine the elastic constants from the data obtained. If the soil response is elastic during the unloading, the sequence of loading-unloading cycles (pq or qp) should not significantly influence the results.

In order to assure permanent contact between the piston and the specimen top cap, a very small stress deviator of the order of 1 to 2 kPa was kept during each test. It was necessary for the external measurement of axial deformation as well as to enable smooth transition from hydrostatic to purely deviatoric loading, especially in its early stage. It was also useful in controlling any other turning points of the assumed stress path without influencing the final results. All the stress paths were automatically controlled by computer programme and the results stored in the computer memory. The experiments were carried out at similar stress rate (stress controlled mode).

3. Some Experimental Results

Hydrostatic loading and unloading (pp)

Of 8 experiments some were performed on both loose and dense sands. The specimens were hydrostatically preconsolidated to point A ($\sigma_1 = \sigma_3 = p_0$) and then three cycles of loading and unloading (path ABA) applied. Note that σ_1 = vertical stress, σ_3 = horizontal stress, (Fig. 2). A typical result of those experiments is presented in Fig. 3 where the respective strain path is shown.

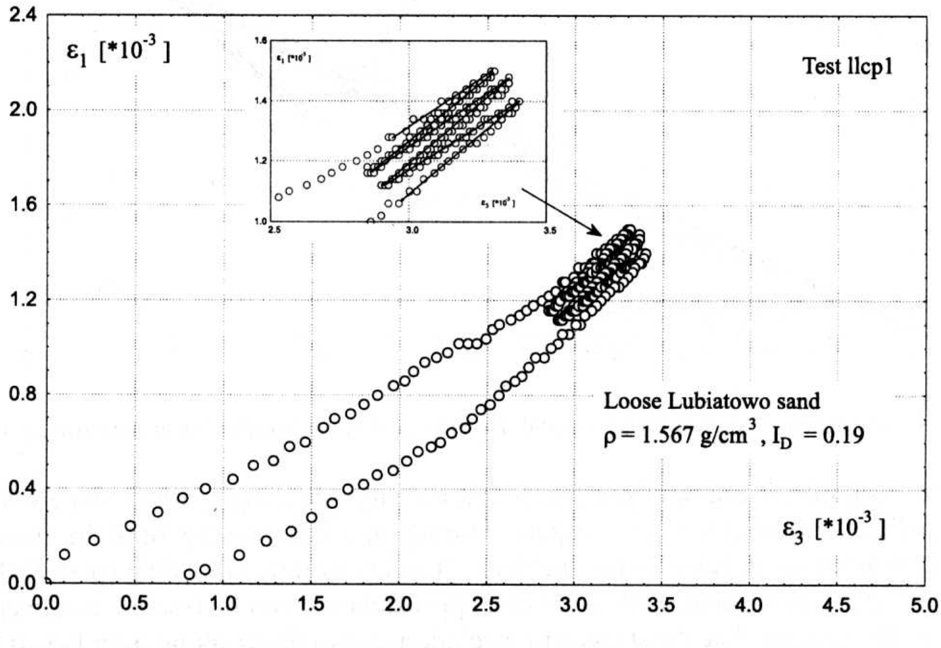

Fig. 3. Hydrostatic loading and unloading. Specimen's response in the strain space

The unloading strain paths (BA) are parallel to each other which supports the assumption as to elastic unloading. Moreover, the unloading paths BA can be approximated by linear sectors. Therefore, the first unloading sector is representative for all three unloading paths.

Path pq

During the pq test, the path OABACA in the stress space was followed. In this special case it was: $p_0 = 2 \times 10^5$ N/m^2, $p_B = 6 \times 10^5$ N/m^2, $\sigma_1^C = 3 \times 10^5$ N/m^2, $\sigma_3^C = 1.5 \times 10^5$ N/m^2, see Fig. 2. The soil mechanics sign convention is used in this paper, where the plus sign denotes compression. Fig. 4 shows a typical strain path corresponding to this stress path for loose "Lubiatowo" sand (ρ = 1.545 g/cm^3, I_D = 0.10). Note that this path can be approximated by linear sectors. It certainly does not mean that the sand's behaviour is linear which follows from the analysis of Figs. 5 and 6 supplementing Fig. 4.

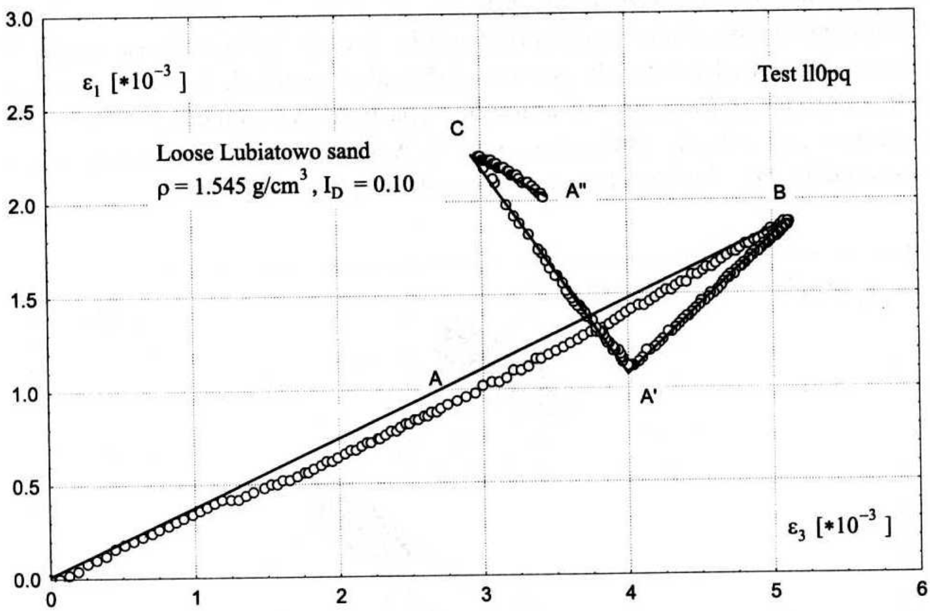

Fig. 4. Strain path in pq test for loose sand. Experimental record against linear approximation

For example, it follows from Fig. 5 that the virgin loading (path OAB) gives a parabolic relationship in the ε_3, p space. During this virgin loading, both the elastic and plastic strains develop in the specimen. The hydrostatic unloading (path (BA) is only slightly non-linear, and in the first approximation can be treated as a linear sector. This means that the stress-strain relation is also linear along path BA. It is not possible from Fig. 5 to detect the behaviour during the deviatoric unloading

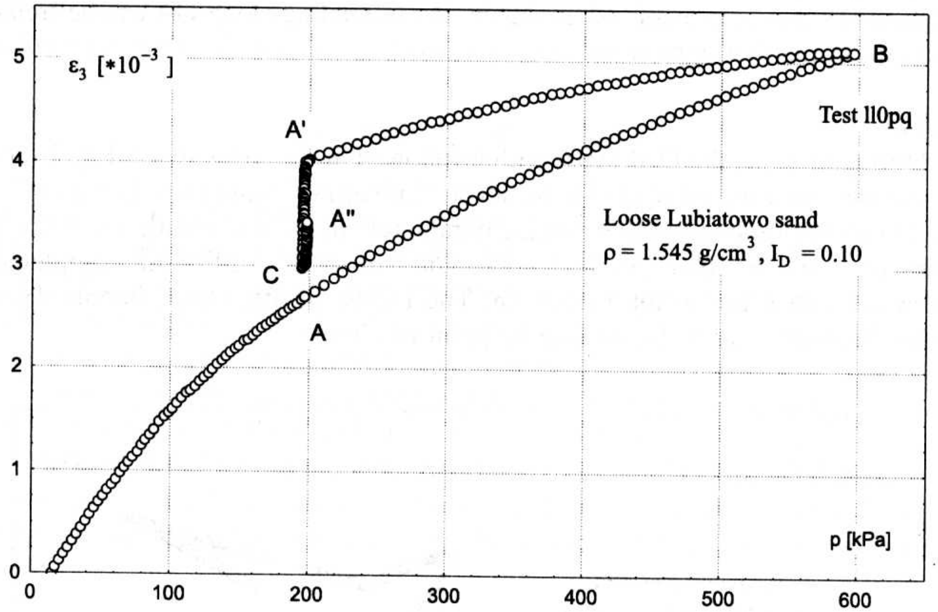

Fig. 5. Horizontal strain against mean stress. pq test for loose sand. Experimental record

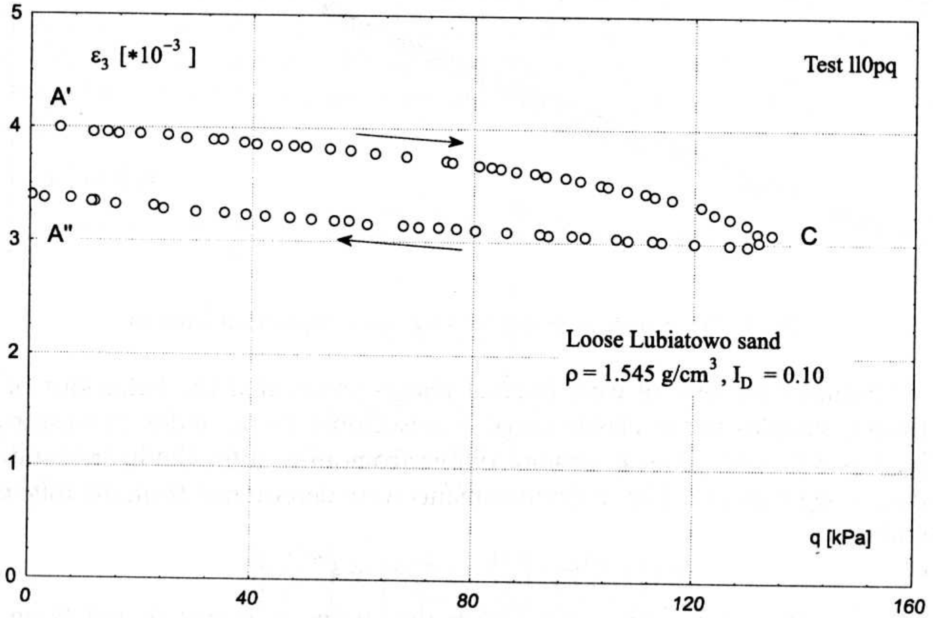

Fig. 6. Horizontal strain against stress deviator. pq test for loose sand. Experimental record

(path CA), as p = const along this path. It can be done from the analysis of Fig. 6 which shows the ε_3, q relationship. Again, the unloading along CA can be treated as elastic in the first approximation.

Path qp

During qp tests, the OACABA path is followed in the stress space. Fig. 7 shows the experimental record of strains for loose "Lubiatowo" sand ($\rho = 1.55$ g/cm^3, $I_D = 0.13$) corresponding to the following parameters of the stress path: $p_0 = 3 \times 10^5$ N/m^2, $p_B = 6 \times 10^5$ N/m^2, $\sigma_1^c = 4 \times 10^5$ N/m^2, $\sigma_3^c = 2.5 \times 10^5$ N/m^2, supplemented by respective linear approximation. The results of other experiments display similar behaviour from the qualitative point of view.

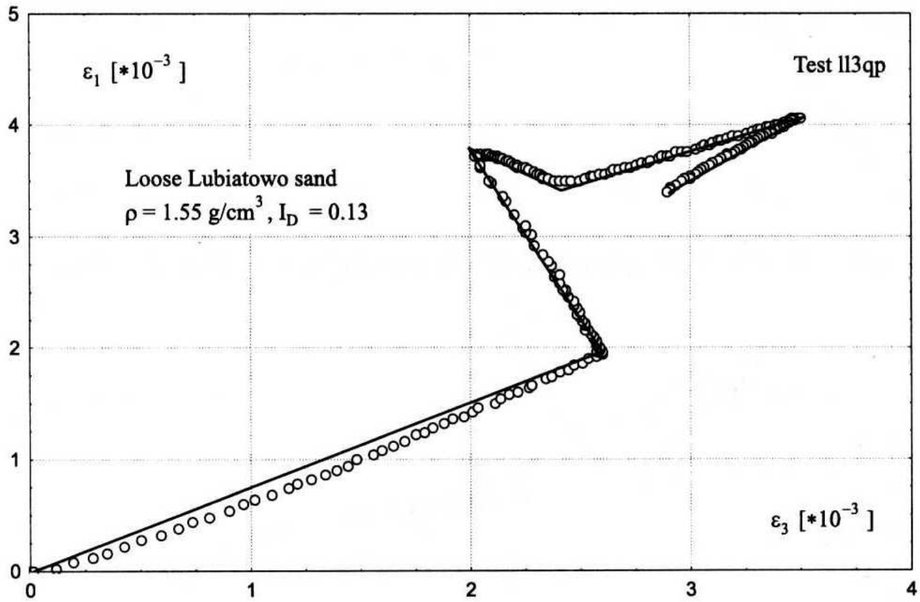

Fig. 7. Strain path in qp test for loose sand. Experimental record

Preliminary analysis of experimental results shows that the behaviour of investigated samples in the elastic range is anisotropic, as the index of anisotropy, defined as $IA = \Delta\varepsilon_1/\Delta\varepsilon_3$, in general differs from unity, after the hydrostatic unloading along path BA. The strain increments were determined from the following formulae:

$$\Delta\varepsilon_1 = \varepsilon_1^A - \varepsilon_1^B , \qquad \Delta\varepsilon_3 = \varepsilon_3^A - \varepsilon_3^B , \tag{1}$$

where respective superscripts distinguish the strains at points A and B on the unloading path BA. According to the assumption adopted in this paper, the strain increments (1) are elastic.

The average anisotropy index calculated from eight chosen experiments, performed on both loose and dense sands, is $IA = 0.75$. The average deviation from the average index of anisotropy, defined as:

$$D(IA) = \frac{1}{n}\sum_{i=1}^{n} |IA - (IA)_i|, \tag{2}$$

was found to be 0.16. Here, $n = 8$ denotes the number of experiments, and the subscript "i" distinguishes the particular experiment. It is realised that the number of experiments chosen for the present analysis is too small to perform any serious statistics, but despite this, the above numbers give some idea as to the character of sands deformation during hydrostatic loading.

A similar analysis, performed separately for loose and dense sands, gives the following numbers: $IA = 0.933$ and $D(IA) = 0.184$ for loose sand, $IA = 0.644$ and $D(IA) = 0.123$ for dense. It follows from these numbers that the anisotropy is more pronounced in the case of dense sand. It may result from the fact that during preparation of dense and very dense samples they were compacted in layers.

4. Determination of Elastic Moduli

The elastic moduli will be determined from experimental data represented in Figs. 2 and 4 or 2 and 7, and particularly from their unloading parts BA and CA. Recall that the stress deviator is zero along path BA, and the mean stress p = const along path CA. It is therefore possible to study the influence of the mean stress and stress deviator on sand's deformation separately. It is convenient to introduce the following classical invariants:

$$p = \frac{1}{3}(\sigma_1 + 2\sigma_3), \tag{3}$$

$$q = \sigma_1 - \sigma_3, \tag{4}$$

$$\varepsilon_v = \varepsilon_1 + 2\varepsilon_3, \tag{5}$$

$$\varepsilon_q = \frac{2}{3}(\varepsilon_1 - \varepsilon_3). \tag{6}$$

It is also convenient to introduce the following general form of the elastic strain-stress relationship:

$$\begin{Bmatrix} \varepsilon_v^e \\ \varepsilon_q^e \end{Bmatrix} = \begin{bmatrix} M_{vv} & M_{vq} \\ M_{qv} & M_{qq} \end{bmatrix} \begin{Bmatrix} p \\ q \end{Bmatrix}, \tag{7}$$

where M_{ij} play a role of elastic compliances. The first subscript indicates to which strain a particular coefficient is attributed, i.e. $i = v$ means that to volumetric strain and $i = q$ to the deviatoric. The second subscript shows which stress corresponds to a coefficient: $j = v$ denotes the mean stress and $j = q$ the stress deviator.

Note that in the case of isotropic elasticity, there is $M_{vv} = 1/K$ and $M_{qq} = 1/3G$, where K = bulk modulus and G = shear modulus. There is also $M_{vq} = M_{qv} = 0$ as there is no coupling between the volumetric and deviatoric effects. A constitutive equation of the same form as (7) is suggested in literature, see Atkinson (1993), but without any reference to the behaviour of actual soils.

During the hydrostatic unloading along path BA only the increment of mean stress Δp appears, and $\Delta q = 0$. Therefore, it follows from Eq. (7):

$$M_{vv} = \frac{\Delta\varepsilon_v^e}{\Delta p}, \qquad M_{qv} = \frac{\Delta\varepsilon_q^e}{\Delta p}. \tag{8}$$

The respective increments should be calculated as differences between the values of respective quantity at points A and B.

During the deviatoric unloading along path CA there is $\Delta p = 0$, and from Eq.(7) we have:

$$M_{vq} = \frac{\Delta\varepsilon_v^e}{\Delta q}, \qquad M_{qq} = \frac{\Delta\varepsilon_q^e}{\Delta q}. \tag{9}$$

Respective coefficients have been determined separately for loose specimens (average $I_D = 0.14$) and dense ones ($I_D = 0.74$). Corresponding matrices of average elastic compliances are the following:

$$M_{loose} = \begin{bmatrix} 0.642 & -0.344 \\ -0.015 & 0.246 \end{bmatrix} \times 10^{-8}\ \mathrm{m^2/N}, \tag{10}$$

$$M_{dense} = \begin{bmatrix} 0.479 & -0.088 \\ -0.043 & 0.190 \end{bmatrix} \times 10^{-8}\ \mathrm{m^2/N}. \tag{11}$$

The average deviations can also be presented in the matrix form:

$$D\left(M_{loose}\right) = \begin{bmatrix} 0.058 & 0.061 \\ 0.026 & 0.039 \end{bmatrix} \times 10^{-8}\ \mathrm{m^2/N}, \tag{12}$$

$$D\left(M_{dense}\right) = \begin{bmatrix} 0.050 & 0.053 \\ 0.015 & 0.058 \end{bmatrix} \times 10^{-8}\ \mathrm{m^2/N}. \tag{13}$$

It would be more readable to present matrices (12) and (13) in a normalized form, which can be done by dividing each number from expressions (12) and (13) by the corresponding absolute values of the coefficients from matrices (10) and (11); and multiplying the results by 100%. For example: $(0.058/0.642) \times 100\% = 9\%$, etc. Respective matrices are the following:

$$D_1\ (\mathrm{loose}) = \begin{bmatrix} 9 & 18 \\ 173 & 16 \end{bmatrix} \%, \tag{14}$$

$$D_1\,(\text{dense}) = \begin{bmatrix} 10.4 & 60.0 \\ 35.0 & 30.5 \end{bmatrix} \%. \tag{15}$$

5. Discussion

a) The experimental results presented in Section 3 show that the investigated sand behaves anisotropically under the hydrostatic compression. The average index of anisotropy is 0.75 with the average deviation of 0.16. In the case of loose sand, respective numbers are 0.933 and 0.184 which means that the behaviour is nearly isotropic. The anisotropy is more pronounced in dense sand since respective numbers are the following: 0.644 and 0.123. The above conclusion has been drawn from purely empirical considerations, without any theoretical presumption. The normalized average deviations (for example $(0.123/0.644) \times 100\%$ are about 20% which seems to be a reasonable result. It should be remembered that it is not possible to prepare two identical samples from the same set of grains, and therefore some scatter in experimental data is unavoidable.

b) The anisotropy of dense sand may have been induced by a sample preparation method due to tamping the samples in several layers. Such a method may lead to transverse isotropy with privilege direction coinciding with the vertical (σ_3 and ε_3 directions). Transversely isotropic elastic materials are defined by five independent constants, see Wood (1990). The elastic stress-strain relation for such a material reduces to the form (7), in the case analysed, where $M_{vq} = M_{qv}$. Inspection of relations (10) and (11) shows that the ratio M_{vq}/M_{qv} equals 23 in the case of loose sand, and is 2 for dense sand. It would mean that the sample preparation method may have led to some kind of transverse isotropy indeed.

c) The elastic compliances have been determined from the assumption that the unloading is elastic. This assumption is supported by experimental results, but after all it is a kind of hypothesis. Experimental data show that the elastic response can be described by a general relationship (7) which contains four independent coefficients. These coefficients may be considered as constants for the applied stresses (maximum stress was 6×10^5 N/m^2), at least in the first approximation. The shape of the matrix of elastic compliances is typical for anisotropic response.

d) Eq. (7) shows that there is a coupling of volumetric and deviatoric elastic effects. This means that the volumetric strain is affected by both the mean stress p and the stress deviator q. Similarly, the deviatoric strain ε_q is also affected by p and q.

e) Inspection of matrices (10) and (11) shows that loose sand is more susceptible to volumetric changes than dense sand (the value of M_{vv} for loose sand is greater than that corresponding to the dense one). Note that the coefficients M_{vv} have been determined precisely in both cases, since the normalized average deviation is 9 and 10.4 % respectively, see Eqs. (14) and (15). Also, the loose sand is more susceptible to deviatoric changes than the dense one, as the value of M_{qq} is also greater in the case of loose sand. The coefficients M_{qq} display a bigger scatter from their averages, which is 16 and 30.5% respectively. Recall that in the case of isotropic elastic response ($M_{vq} = M_{qv} = 0$) we have $M_{vv} = 1/K$ and $M_{qq} = 1/3G$. The above results also mean that the elastic stiffness of dense sand is greater than that characterising the loose sand, which is intuitively the correct result.

f) Very interesting are the coupling effects, represented by the coefficients M_{vq} and M_{qv}. First, it should be noted that their values are negative for both loose and dense sands, in contrast to the positive values of M_{vv} and M_{qq}. For example, consider the volumetric changes. When $q = 0$ and p increases we have positive volumetric change (compression gives compaction) which is consistent with physical intuition. The stress deviator gives the opposite effect which can be identified with some kind of elastic dilation. Decreasing values of q contribute to sand compaction, whilst decreasing p gives volumetric expansion. This effect is more pronounced in loose sands. The coefficients M_{vq} responsible for this behaviour have been determined more accurately for loose sands than for dense ones, as the values of normalized average deviations are 18 and 60% respectively, see Eqs. (14) and (15).

g) The coefficient M_{qv} represents the influence of mean stress p on deviatoric deformation. In the case of loose sand, this effect is very small. Moreover, the normalized average deviation is 173% in this case (!) which suggests that this coefficient has been determined with very low accuracy. The above observations suggest that there are probably some experimental errors, and the coefficient $M_{qv} \cong 0$ for loose sands. This coefficient has been determined more accurately for dense sands because the average normalized deviation is 35% in this case. Recall that the respective deviation for M_{vq} is 60%. Because of these deviations and the fact that M_{vq} and M_{qv} do not differ essentially, it may be assumed that $M_{vq} \cong M_{qv}$ for dense sands. This means that the matrix of elastic compliances is symmetric, as that for the transversely isotropic material. This is consistent with the previous conclusion that the method of preparation of dense samples may induce this kind of anisotropy.

h) The relation (7) can be represented in the following alternative form:

$$\left\{ \begin{array}{c} \varepsilon_1^e \\ \varepsilon_3^e \end{array} \right\} = \left[\begin{array}{cc} S_{11} & S_{13} \\ S_{31} & S_{33} \end{array} \right] \left\{ \begin{array}{c} \sigma_1 \\ \sigma_3 \end{array} \right\}, \tag{16}$$

where:

$$S_{11} = \frac{1}{3}\left(\frac{1}{3}M_{vv} + M_{vq} + M_{qv} + 3M_{qq}\right),$$
$$S_{13} = \frac{1}{3}\left(\frac{2}{3}M_{vv} - M_{vq} + 2M_{qv} - 3M_{qq}\right),$$
$$S_{31} = \frac{1}{3}\left(\frac{1}{3}M_{vv} + M_{vq} - \frac{1}{2}M_{qv} - \frac{3}{2}M_{qq}\right),$$
$$S_{33} = \frac{1}{3}\left(\frac{2}{3}M_{vv} - M_{vq} - M_{qv} + \frac{3}{2}M_{qq}\right).$$

6. Conclusions

The main results can be summarised as follows:

1. An original experimental method of determining elastic compliances of sand, in the triaxial compression tests, has been proposed. This method is based on the assumption as to the elastic behaviour of sand during unloading.
2. Experimental results show that the elastic response of sand is generally anisotropic. The behaviour of loose sand may be treated as nearly isotropic in the first approximation. The elastic response of dense sand may be treated as transversely isotropic in the first approximation, with the privilege direction coinciding with the vertical. This kind of anisotropy has probably been induced by the method of sample preparation.
3. The volumetric elastic behaviour of loose sand is affected by both mean and deviatoric stresses. The deviatoric deformation of loose sand is caused in practice by the stress deviator. The influence of mean stress on this deformation can be neglected.
4. The elastic compliances have been determined with differing accuracy, measured by average deviations. More experimental data is needed to perform more precise statistical analysis of quantitative results. From the qualitative point of view, the results obtained are acceptable.

Acknowledgement

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