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On the Generation of Water Waves in a Flume

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Abstract

The present paper proposes a simple model for the calculation of a time series to control the horizontal motion of the piston of a wavemaker. It is assumed that the initial conditions correspond to water at rest. It is assumed that parameters of kinematics of the fluid (displacement, velocity and acceleration fields) at the initial time are equal to zero. In the first interval the amplitudes grow to an asymptote that corresponds to a regular monochromatic wave. In the second interval the waves decay. Calculated time series were fed into the control system of our wavemaker and the measured horizontal displacements of the piston compared. The control series was supplemented by terms corresponding to the Stokes type solution by addition of terms with multiples of the basic frequency.

1. Introduction

The Institute of Hydroengineering of the Polish Academy of Sciences in Gdańsk has a new wave flume. It is 64 m long, 0.60 m wide and has a depth of 1.40 m. A piston type generator generates the waves. It is possible to create regular monochromatic, irregular and random waves. A time series calculated on the computer may be fed into the control system to obtain the desired displacements of the piston. The displacements must correspond to smooth curves.

The aim of a research program conducted at the Institute is to obtain a computer program based on a rigorous theoretical formulation for irregular waves and reproduces the phenomena observed in the flume. The motion of a piston type generator corresponds to the horizontal component of the orbital motion of long waves and thus long waves are the aim of preliminary studies. In a rigorous theoretical formulation, initial and boundary conditions must be given. The initial conditions correspond to the situation in which at time zero the piston is at rest. The boundary condition at the actual position of the piston corresponds to its position and is thus given in a Lagrange description. Within the standard water wave theory, the fluid is incompressible and thus sudden application of a finite acceleration leads to an acceleration field in the neighbourhood of the piston, resulting immediately in a mass of fluid covibrating with the piston. Thus it is convenient to assume that at the initial time the velocity and acceleration of the piston are zero. Such initial conditions lead to a smooth transition from rest (no velocities and no accelerations in the fluid) to the desired waves.

The theoretical description of long waves is described in detail in an unpublished paper written by P. Wilde (1999) and available in the library of our Institute. The complete experimental data are described in the paper published by the authors P. Wilde, E. Sobierajski and Ł. Sobczak (2000).

The case of monochromatic waves is considered in Chapter Two. The mathematical model of the transient corresponds to the set of differential equations (1). Random functions defined on a similar set, but within the theory of Itô differential equations are studied in the book by P. Wilde and A. Kozakiewicz (1993). The application, of the results presented in this book, leads to the possibility of constructing sample functions for piston motion of random waves.

The motion of the piston must go back to rest in a slow and smooth manner. Thus two regions are defined. In the first interval the amplitudes grow (first subinterval) to an asymptote that corresponds to a regular monochromatic wave (second subinterval). In the second interval the amplitude tends to zero. At the point of transition to the decay interval the values of the displacement, velocity and acceleration have to be continuous. The first interval must be long enough to obtain a part that corresponds to an almost regular wave and the second interval long enough, for the displacements, velocities and accelerations at its end to be negligible.

The measurements in the flume, based on the boundary condition presented in Chapter 2 indicate that along the flume components with multiple frequencies are created. The multiple frequency components take energy from the first one. There is an exchange of energy between the components along the flume. Thus, it is interesting to construct a boundary condition that includes the multiple frequency components in an approximate Stokes type way. Such an approximate boundary condition is presented in Chapter Three. The procedure is similar to the method proposed by Wilde P. & Romańczyk W. (1989) for the construction of a mathematical model used in the analysis of Stokes' type waves. The introduced boundary condition defines the displacements of the piston as a function of time and is thus given in a material description.

The calculated time series of displacements were fed into the control system of the wave generator. The measured displacements of the piston were very close to those given.

2. The Transient to a Regular Monochromatic Wave

2.1. The Mathematical Model of the Proposed Transient

Let us consider the transient motion of the piston of a wave generator described with the help of the solution of the following set of differential equations

$$\begin{aligned} &dA_0/dt + \eta A_0 = 0, \\ &dA_1/dt + \eta A_1 = \eta A_0, \\ &\dots \\ &dA_k/dt + \eta A_k = \eta A_{k-1}, \\ &\dots \\ &dA_n/dt + \eta A_n = \eta A_{n-1}, \end{aligned} \tag{1}$$

where η is a parameter with dimensions 1/s and the set of unknown functions may be represented by the matrix $\mathbf{A}(t) = [A_1(t), A_2(t), \dots, A_k(t), \dots A_n(t)]^T$, where the superscript Tdenotes the transpose. The general solution of the set (1) for initial conditions given at the time $t = 0$ is

$$\mathbf{A}(t) = \Phi(t)\,\mathbf{A}(0), \tag{2}$$

where Φ is the $(n+1) \times (n+1)$ matrix corresponding to the fundamental solution

$$\Phi(\eta t) = \exp(-\tau) \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ \frac{1}{1!}\tau & 1 & 0 & \dots & 0 \\ \frac{1}{2!}\tau^2 & \frac{1}{1!}\tau & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots & 0 \\ \frac{1}{n!}\tau^n & \frac{1}{(n-1)!}\tau^{n-1} & \frac{1}{(n-2)!}\tau^{n-2} & \dots & 1 \end{bmatrix}, \tag{3}$$

where $\tau = \eta t$ is dimensionless time.

By elimination we may express the derivatives of the function $A_n(t)$, as a linear combination of the functions$A_k(t)$, $k = 0, 1 \dots n$, by the relations

$$\begin{aligned} &A_n(t) = A_n(t), \\ &dA_n(t)/dt = \eta(B-1)A_n(t), \\ &d^2A_n(t)/dt^2 = \eta^2(B-1)^2A_n(t), \\ &\dots \\ &d^nA_n(t)/dt^n = \eta^n(B-1)^nA_n(t), \end{aligned} \tag{4}$$

where B is the backshift operator defined by $B^kA_n(t) = A_{n-k}$, $k = 0, 1, \dots, n$. The relations (4) may be written in the form of the matrix relation

$$\mathbf{A}(t) = \mathbf{L_{DA}}\mathbf{A}(t), \tag{5}$$

where the column matrix on the left $\mathbf{A} = \left[A_n^{(0)}, A_n^{(1)}, \dots, A_n^{(k)}, \dots, A_n^{(n)}\right]^T$, the superscript (k) means the k-derivative of the function $A_n(t)$, and $\mathbf{L_{DA}}$ is the $(n+1) \times (n+1)$ matrix of coefficients defined in the relations (4). The most common case is $n = 3$, and in this case the matrix equation (4) is

$$\begin{bmatrix} A_3^{(0)}(t) \\ A_3^{(1)}(t) \\ A_3^{(2)}(t) \\ A_3^{(3)}(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & \eta & -\eta \\ 0 & \eta^2 & -2\eta^2 & \eta^2 \\ \eta^3 & -3\eta^3 & 3\eta^3 & -\eta^3 \end{bmatrix} \begin{bmatrix} A_0(t) \\ A_1(t) \\ A_2(t) \\ A_3(t) \end{bmatrix}. \tag{6}$$

In our case of control of transient motion of the piston of the wave generator, we want the displacement, the velocity and the acceleration to be zero at the initial time. Thus we have to consider at least the case $n = 3$. (If we assume that the initial displacement and velocity is zero then $n = 2$ is sufficient.) In general we can take any value of n.

Let us construct such a function $D_n(t)$ that for t equal to zero the value is one and the values of all derivatives up to the n-derivative are equal to zero. This means

$$\mathbf{Đ_n}(0) = \left[\hat{D}_n^{(0)}(0) = 1, \hat{D}_n^{(1)}(0) = 0, \hat{D}_n^{(2)}(0) = 0, \dots, \hat{D}_n^{(n)}(0) = 0\right]^T.$$

Multiplication of the matrix equation (5) by the inverse of the matrix $\mathbf{L_{DA}}$ leads to the following initial values for the matrix $\mathbf{D}(0)$

$$\mathbf{D}(0) = [D_0(0) = 1, D_1(0) = 1, D_2(0) = 1, \dots, D_n(0) = 1]^T.$$

Thus according to the relation (2) and (3) we have

$$D_n(t) = \left[1 + \frac{1}{1!}\tau + \frac{1}{2!}\tau^2 + \dots + \frac{1}{n!}\tau^n\right] \exp(-\tau).$$

When the time goes to infinity this function tends to zero. A suitable function that deals with our control problem is

$$\widetilde{D}_n(t) = 1 - \left(1 + \sum_{k=1}^{k=n} \frac{1}{n!}\tau^k\right) \exp(-\tau). \tag{7}$$

The first derivative of the function (7) with respect to time is

$$\widetilde{D}_n^{(1)}(\eta t) = \frac{1}{n!}\eta\tau^n \exp(-\tau). \tag{8}$$

Now let us construct a function $\widetilde{A}_n(t)$ that at time $t = 0$ its value and the values of its derivatives up to the $n - 1$ are zero and the n-derivative is equal to one. Following the above outlined procedure the final expression is

$$\widetilde{A}_n(\eta t) = \frac{1}{n!}t^n \exp(-\tau). \tag{9}$$

We now define a complex signal

$$Z_n(t) = x_a\widetilde{A}_n(t) + i\left[\widetilde{D}_n(t) + x_d\widetilde{A}_n(t)\right], \tag{10}$$

where x_a, x_d are real numerical coefficients with dimensions of seconds to the power of $-n$. The square of the absolute value is

$$W_n^2 = \widetilde{D}_n^2(t) + 2x_d\widetilde{D}_n(t)\widetilde{A}_n(t) + \left(x_a^2 + x_d^2\right)\widetilde{A}_n^2(t), \tag{11}$$

and the phase can be calculated from the relation

$$\tan\Psi(t) = \frac{\widetilde{D}_n(t) + x_d\widetilde{A}_n(t)}{x_a\widetilde{A}_n(t)}. \tag{12}$$

The complex signal (10) may be written in the following trigonometric form

$$Z_n(t) = W_n(t)\exp\left[i\Psi(t)\right].. \tag{13}$$

To obtain a signal that has a dominant angular frequency ω one has to multiply the signal (13) by $\exp(-i\omega t)$. It follows that

$$X_n(t) + iY_n(t) = W_n(t)\exp\left\{i\left[\Psi(t) - \omega t\right]\right\}, \tag{14}$$

and the separation into real and imaginary parts leads to the expressions

$$\begin{aligned} X_n(t) &= x_a\widetilde{A}_n(t)\cos\omega t + \left[\widetilde{D}(t) + x_d\widetilde{A}_n(t)\right]\sin\omega t, \\ Y_n(t) &= -x_a\widetilde{A}_n(t)\sin\omega t + \left[\widetilde{D}(t) + x_d\widetilde{A}_n(t)\right]\cos\omega t. \end{aligned}$$

The procedure to calculate the time series of values of real and imaginary parts is completed when a suitable time step Δt is assumed.

2.2. The Properties of the Mathematical Model

The time derivative of the square of the absolute value (11) is

$$\begin{aligned} d\left[W_n^2(t)\right]/dt = {} & 2\widetilde{D}_n(t)\widetilde{D}_n^{(1)}(t) + 2x_d\left[\widetilde{D}_n(t)\widetilde{A}_n^{(1)}(t) + \widetilde{D}_n^{(1)}(t)\widetilde{A}_n(t)\right] + \\ & +2\left(x_a^2 + x_d^2\right)\widetilde{A}_n(t)A_n^{(1)}(t). \end{aligned} \tag{15}$$

Let us commence the discussion on the properties of the phase function by calculation of the time derivative of the function $\sin\Psi$

$$\Psi_n^{(1)}\cos\Psi_n = \frac{x_a^2\widetilde{A}_n^2\left[\widetilde{D}_n^{(1)} + x_d\widetilde{A}_n^{(1)}\right] - x_a^2\widetilde{A}_n\widetilde{A}_n^{(1)}\left[\widetilde{D}_n + x_d\widetilde{A}_n\right]}{W_n^3},$$

where the dependence of functions on time is understood without direct notation. Finally in view of $\cos\Psi_n = x_a\widetilde{A}_n / W_n$ it follows that

$$\Psi_n^{(t)}(t) = \frac{x_a\left[\widetilde{A}_n(t)\widetilde{D}_n^{(1)}(t) - \widetilde{A}_n^{(1)}(t)\widetilde{D}_n(t)\right]}{W_n^2(t)}, \tag{16}$$

The phase of the complex signal with the dominant frequency is given in the relation (14). Let us consider the phase at time $t + \Delta t$. It follows within an approximation to linear terms

$$\Psi_n(t) - \omega t \approx \Psi_n(t_j) - \omega t_j + \left[\Psi_n^{(1)}(t_j) - \omega\right](t - t_j), \tag{17}$$

and thus the time derivative of the phase has the meaning of a local contribution to the angular frequency.

The asymptotic behaviour is clear, as the product of any finite polynomial with the exponential function of a negative argument tends to zero. The behaviour around the initial time ($t = 0$) has to be studied. The expansion of the function $\widetilde{A}_n(t)$ given by the relation (9) in power series is

$$\widetilde{A}_n(t) = \frac{1}{n!}\left[t^n + \sum_{k=1} \frac{(-\eta)^k}{k!} t^{n+k}\right], \tag{18}$$

and that of the function $\widetilde{D}_n(t)$ obtained by expansion in power series its time derivative given by the relation (8) and integration finally is

$$\widetilde{D}_n(t) = \frac{\eta^{n+1}}{n!}\left[\frac{t^{n+1}}{n+1} + \sum_{k=1} \frac{(-\eta)^k}{k!} \frac{t^{n+k+1}}{n+k+1}\right]. \tag{19}$$

It is worthwhile noting that the lowest power term approximations of the functions $\widetilde{A}_n(t)$ and $\widetilde{D}_n(t)$ are

$$\widetilde{A}_n(t) \approx \frac{1}{n!} t^n, \quad \widetilde{D}_n(t) \approx \frac{\eta^{n+1}}{n+1} t^{n+1}.$$

and thus in the limit for the time going to zero $\tan(\Psi)$ given by (12) tends to

$$\Psi_n(0) = \tan^{-1}(x_d/x_a). \tag{20}$$

The limit for the time derivative of the phase is

$$\Psi_n^{(1)}(0) = \frac{x_a}{x_a^2 + x_d^2} \frac{\eta^{n+1}}{n+1}. \tag{21}$$

Now let us calculate the second derivative with respect to time by taking the first derivative of the relation (16). We then substitute the expansions in power series (18) and (19) and go to the limit when times reaches zero. One should be careful, as the denominator has as the lowest power term time to the power 12 and in the numerator the terms with the power 11 cancel and the limit when time approaches zero is

$$\Psi_n^{(2)}(0) = \frac{2}{x_a^2 + x_d^2} \frac{\eta^{n+1}}{n+1}\left[\frac{1}{n+2} - \frac{x_d}{x_a^2 + x_d^2} \frac{\eta^n}{n+1}\right]. \tag{22}$$

In the model there are two wave parameters, the period of motion and the amplitude, and there are three parameters that control the behaviour of the transient, x_a, x_d and η. It is a linear model and thus the amplitude may be assumed equal to one and the final time series multiplied by its actual value. The value of η is crucial for the asymptotic behaviour. (The dimensionless parameter ηT, where T is the period, is more convenient in calculations.) The values of the two last parameters may be chosen arbitrarily.

2.3. The Choice of Parameters and Numerical Examples

There are two possibilities, either the period T of the regular wave or its length L is given. In a linear approximation when one is given the second value this can be calculated from the dispersion relation. The parameter η decides how fast the time series approaches the regular wave. It is reasonable to fix the dimensionless number ηT, (for examples equal to one or two). The behaviour depends upon the choice of the number n that fixes the number of equations in the set (1) equal to $n+1$. In examples the standard value will be $n=3$. Let us discuss the choice of the values of the two parameters x_a, x_d. From the point of view of physics there should be a smooth transition from rest to harmonic motion.

As a first case let us assume that the time derivative of the phase at time equal to zero is equal to ω, $\Psi_n^{(1)}(0)=\omega$ and thus the local angular frequency, according to the relation (17), is equal to zero. As the second condition let us assume that the second derivative of the phase is equal to zero at $t=0$, $\Psi_n^{(2)}(0)=0$. These conditions lead to the following expressions for the coefficients

$$x_a=\frac{\eta}{\omega}\frac{\eta^n}{n+1}\left[1+\frac{\eta^2}{\omega^2}\frac{1}{(n+2)^2}\right]^{-1}, \quad x_d=\frac{\eta}{\omega}\frac{1}{n+2}x_a. \tag{23}$$

In the second case let us assume that $x_a=x_d$ and $\Psi_n^{(2)}(0)=0$. In this case the expressions for the coefficients are

$$x_a=x_d=\frac{1}{2}\frac{n+2}{n+1}\eta^n. \tag{24}$$

The third, and simplest case is

$$x_a=x_d=0. \tag{25}$$

In this case it follows from the relations (11) and (12) that

$$W_n(t)=\widetilde{D}(t), \quad \Psi_n(t)=\pi/2.$$

In the fourth case let us assume that the first coefficient has a suitable value while the second is zero

$$x_a\neq 0, x_d=0. \tag{26}$$

A computer program in MATLAB was written. In the example, let us assume that the wave period T is 2 s, the amplitude of the piston motion is one and the parameter $\eta T = 2$. The values of the last two parameters are assumed to be equal to zeros and n is equal to three. The functions $\widetilde{A}_n(t)$ (9), $\widetilde{D}_n(t)$ and $\widetilde{D}_n(t) = 1 - D_n(t)$ (7) are depicted in Fig. 1. These functions are the basic elements for the construction of the mathematical model. Two functions approach the zero line when the time grows and the third has a horizontal asymptote of value one. The complex signal (10) was calculated for case 3. The absolute value and the phase are depicted in Fig. 2. The asymptote of the absolute value is one and the phase $\pi/2$.

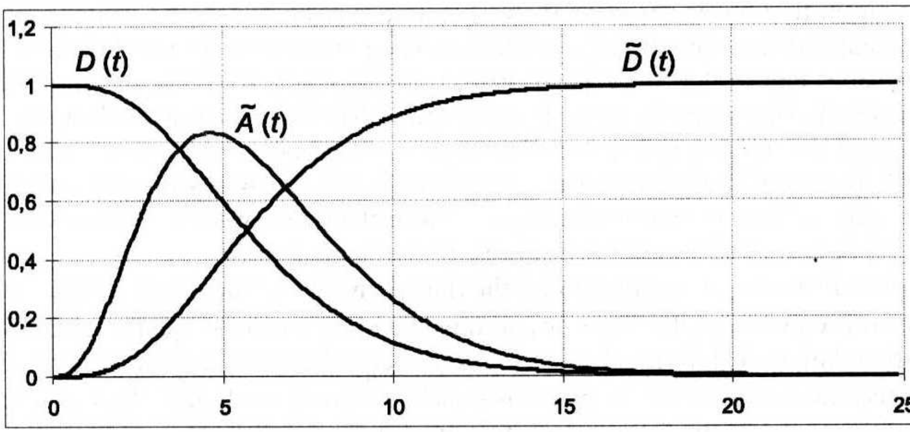

Fig. 1. Basic elements $\widetilde{A}_n(t)$, $D_n(t)$ and $\widetilde{D}_n(t) = 1 - D_n(t)$ for the construction of the mathematical model

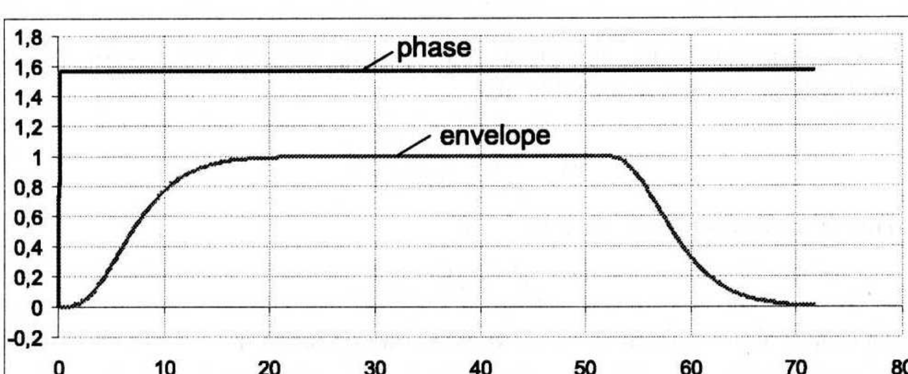

Fig. 2. The absolute value and phase for the case $x_a = 0, x_d = 0$

The calculations were repeated for the parameters of the experiment (8b60r) with parameters: period $T = 2.165$ s, asymptotic wave length $L = 6.466$ m, number of waves in the group 10+5, the parameter $\eta T = 1.5$, the sampling frequency 50 Hz, the parameters x_a, x_d equal to zeros and the asymptotic amplitude $W_1 = 0.143$ m. The complex signal was calculated and its real part $X_n(t)$ was fed into the control system of the generator. The measured displacements of the piston are depicted in Fig. 3 and confronted by the values of the control time series depicted by a dashed line. The differences are very small.

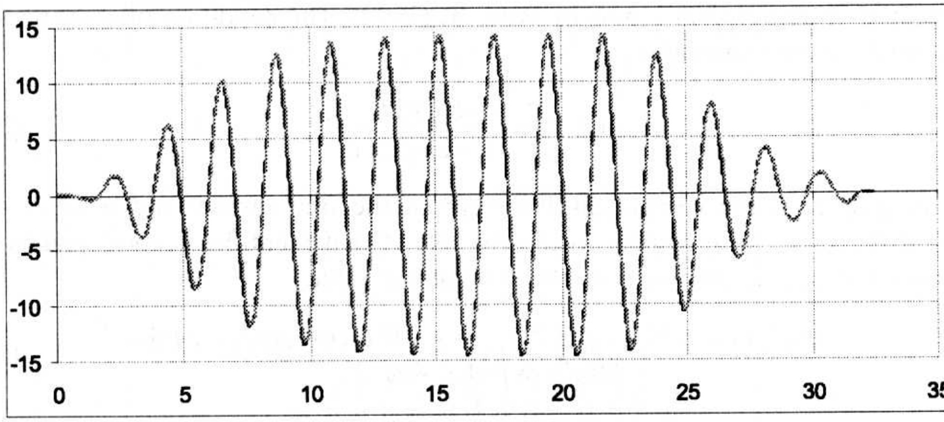

Fig. 3. The motion of the piston corresponding to a Stokes type boundary condition

The curves in Fig. 3 have two intervals. In the first, the values of the envelope grow to the asymptotic value and in its end part are almost exactly equal to it. In the second interval the values decrease asymptotically to zero. At the boundary of these intervals the values of the function and its first and second derivatives have to be equal on both sides. The number n is chosen to be three and thus at time zero the displacement, velocity and acceleration are equal to zeros. The control series for the piston displacements and the measured horizontal displacements have only one component. However, the measured displacements of the free surface and horizontal components of velocities in the fluid have components with multiples of the basic frequency.

3. The Motion of the Piston Corresponding to a Stokes Type Boundary Condition

3.1. The Regular, Monochromatic and Long Wave

Let us assume that the solution for a regular non-linear wave can be approximated by the expression

$$u(Z,t) = -W_1 \sin(kZ - \omega t) - W_2 \sin[2(kZ - \omega t)] - W_3 \sin[3(kZ - \omega t)] \ldots \quad (27)$$

where $u(Z,t)$ is the horizontal displacement, $k = 2\pi/L$ the wave number, Z the distance from the generator at rest and $W_1 > 0$. The motion of the piston corresponds in the material description to $Z = 0$, and thus

$$u(t) = W_1 \sin \omega t + W_2 \sin 2\omega t + W_3 \sin 3\omega t \ldots, \tag{28}$$

For the case of very long, compared to depth water waves, it may be assumed that the vertical material lines remain vertical lines during the motion. Details are presented in the internal report by Wilde P. (1999). The condition of compressibility leads to the expression

$$w(Z,t) = \frac{-u'(Z,t)}{1 + u'(Z,t)} H, \tag{29}$$

where $w(Z,t)$ is the vertical displacement of the free surface and $u'(Z,t)$ the partial derivative with respect to the variable Z that identifies vertical planes by positions at rest. It follows from the relation (27) that

$$\begin{aligned} u'(Z,t) = &-W_1 k \cos(kZ - \omega t) - 2W_2 k \cos[2(kZ - \omega t)] - \\ &-3W_3 k \cos[3(kZ - \omega t)] \ldots, \end{aligned} \tag{30}$$

Substitution into the relation (29) leads to the expression for the free surface in a material description.

$$w(Z,t) = A_1 \cos(kZ - \omega t) + A_2 \cos(kZ - \omega t) + A_3 \cos(kZ - \omega t) \ldots . \tag{31}$$

The values of the coefficients $A_1, A_2, A_3, \ldots$ may be calculated from a set of algebraic equations with the help of the relations (29) when the coefficients W_1, W_2, $W_3 \ldots$ are known. The values of the coefficients W_1, W_2, $W_3 \ldots$ have to be taken from experiments or Stokes types solutions in a material description.

3.2. The Control Time Series

For all the cases of mathematical models in Chapter 2 the complex asymptotic displacements U for an amplitude equal to one are given by the following expression

$$U(t) = i \exp(-i\omega t), \; u_1(t) = real(U) = \sin \omega t. \tag{32a}$$

It follows by raising to the consecutive powers that

$$\begin{aligned} u_2 &= imag\left(U^2\right) = \sin 2\omega t, \\ u_3 &= -real\left(U^3\right) = \sin 3\omega t, \\ u_4 &= -imag\left(U^4\right) = \sin 4\omega t \ldots \end{aligned} \tag{32b}$$

The above-defined functions may be generalised by taking $U(t)$ for any model described in Chapter 2. The graphs of the functions u_1, u_2, u_2 are depicted in

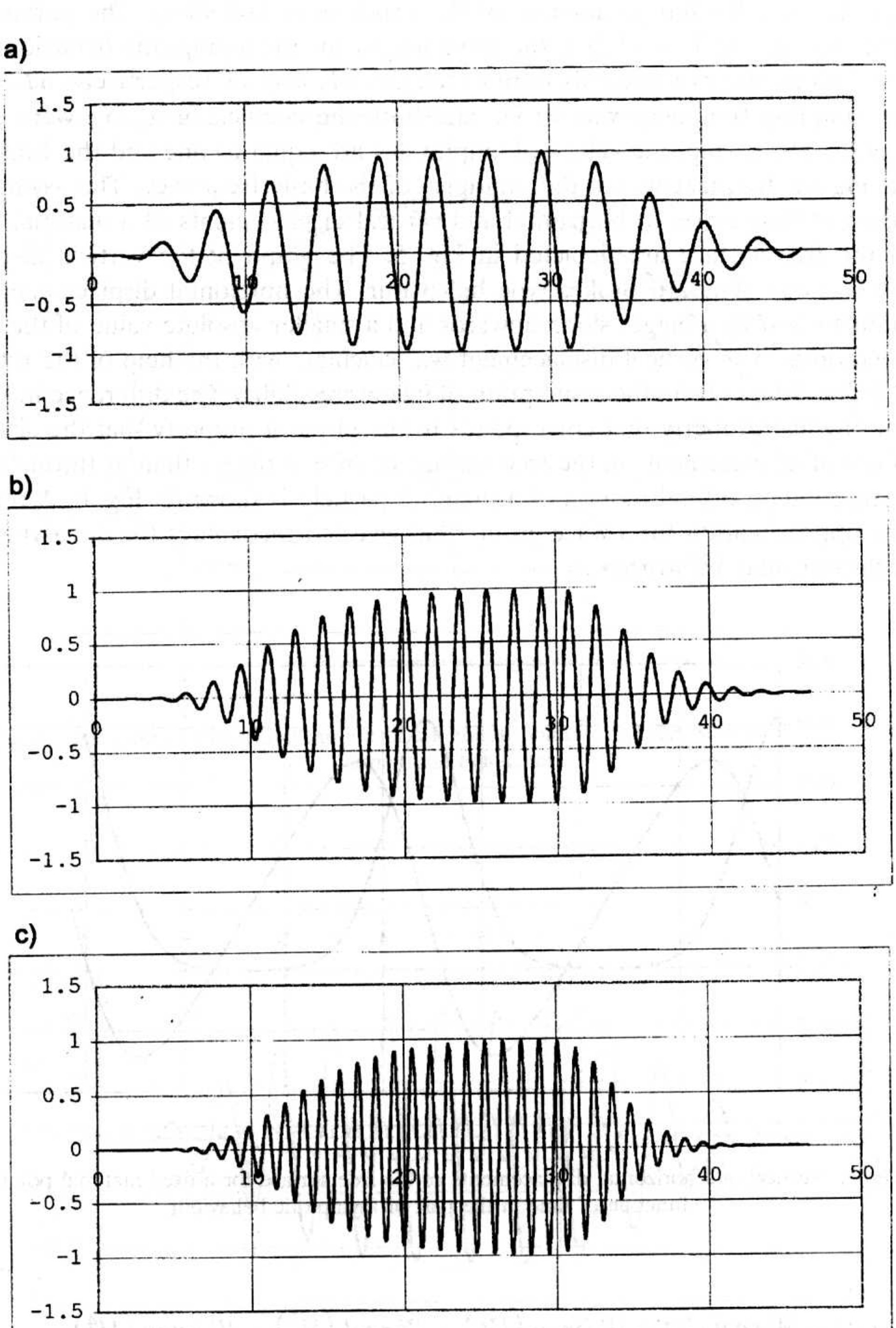

Fig. 4. The component with the amplitude equal to one for the asymptotic part and frequencies equal to: a) basic frequency ω, b) 2ω, and c) 3ω

Figs. 4a, b, c for the parameters of the experiment (14b95er). The parameters were: the period $T = 3.575$ s, the wave length for the asymptotic behaviour was $L = 8.88$ m, the two intervals were of lengths $8T$, and $5T$ respectively, $\eta T = 1.5$, the sampling frequency was 50 Hz and both the coefficients x_a, x_d were equal to zero. The asymptotic values of amplitudes are equal to one and the consecutive angular frequencies are the multiples of the basic frequency. The asymptotic values of time series of horizontal and vertical displacements of a material point on the free surface are depicted in Fig. 5. The graph of the vertical displacements shows standard Stokes type behaviour. The horizontal displacements are characterised by a bigger slope upwards and a smaller absolute value of the slope downwards. The vertical displacement was calculate with the help of the relation (29) that follows from the assumption of incompressibility The difference in slopes is a physical property that corresponds to the physical property that the absolute values of displacement on the free surface at crest is bigger than at through. The corresponding orbital motion of a material particle is shown in Fig. 6. A reasonable approximation for a wave group, that gives correct values for the asymptotic behaviour, may be written as

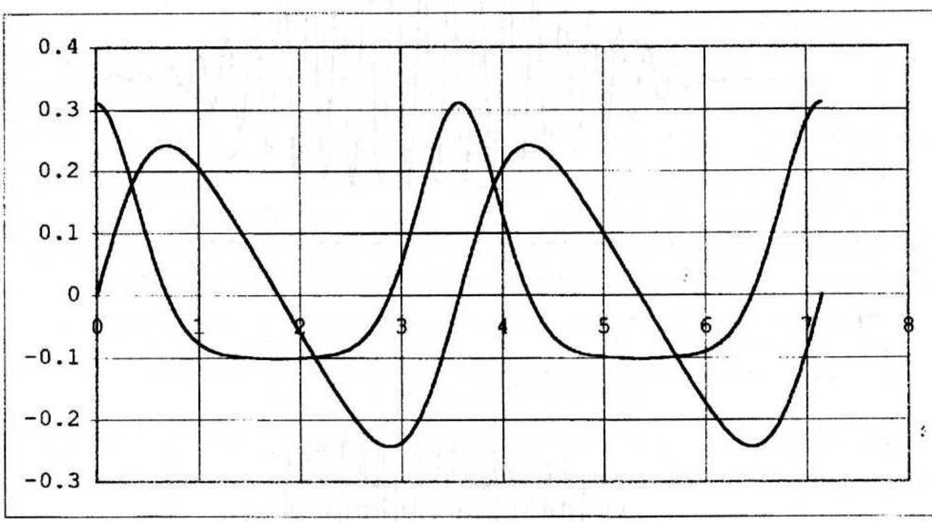

Fig. 5. Vertical and horizontal displacements of the free surface for a fixed material point as a function of time in the part of asymptotic behaviour

$$u(t) = W_1 real(U) + W_2 imag\left(U^2\right) - W_3 real\left(U^3\right) - W_4 imag\left(U^4\right) \ldots \quad (33)$$

Two components of the control time series for $W_1 = 0.231$ m, $W_2 = 0.044$ m, $W_3 = 0$ m, $W_4 = 0$ are depicted in Fig. 7.

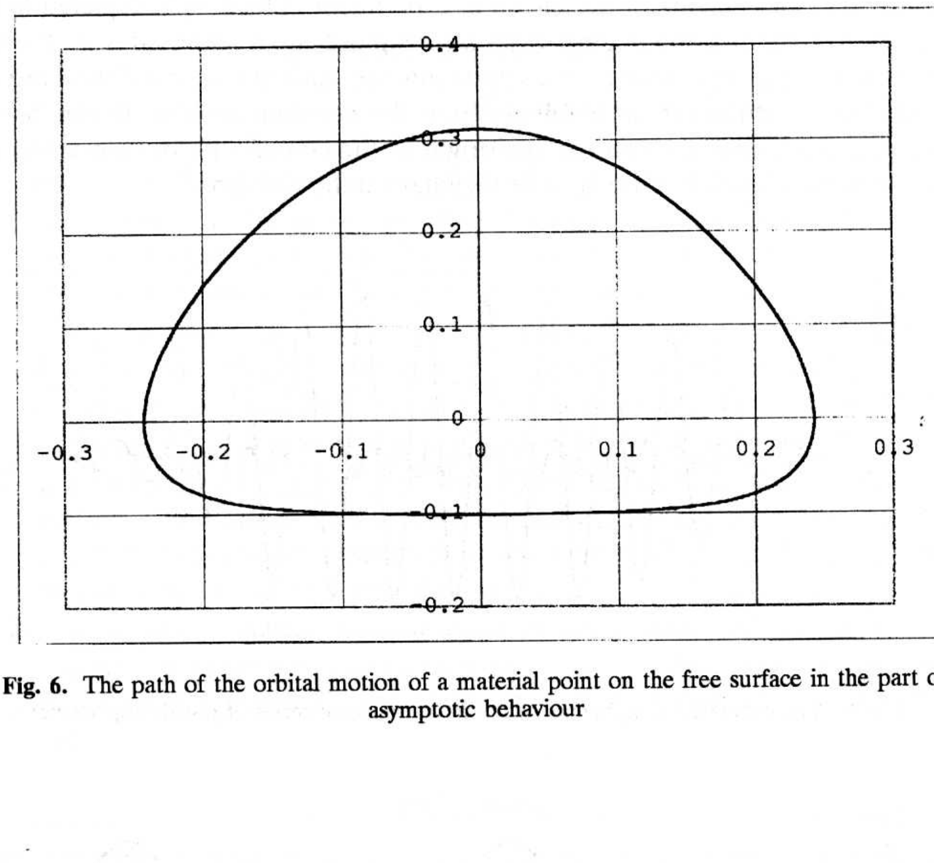

Fig. 6. The path of the orbital motion of a material point on the free surface in the part of asymptotic behaviour

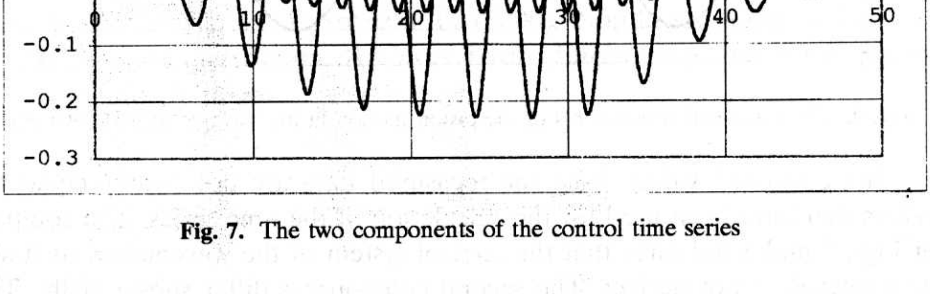

Fig. 7. The two components of the control time series

Experiments were performed with fixed values of W_1, several values of W_2 and $W_3 = 0$. The decomposition into two components by a Kalman filter of the measured displacements of the piston in experiment (14b95er) is depicted in Fig. 8. A part of the measured time series with almost equal amplitudes is shown in Fig. 9. It can be seen (when the components are added) that the downward and upward slopes differ just as it follows from the previous analysis. It may be also seen that it is possible to choose the origin of the co-ordinate in such a way that both amplitudes are positive or that they have different signs.

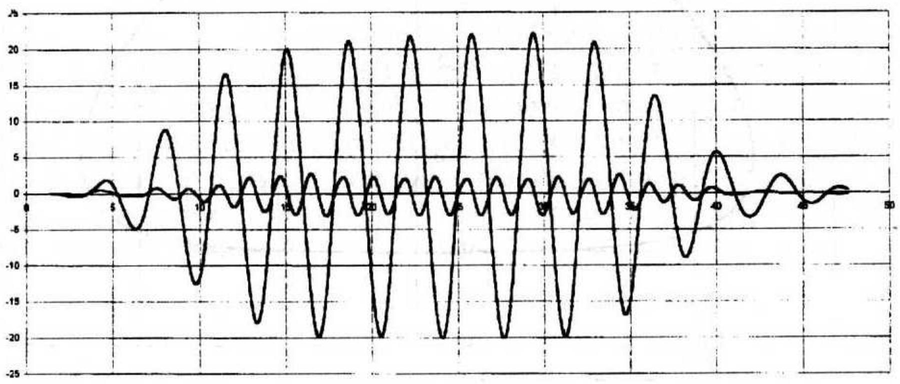

Fig. 8. The estimated components of the measured time series of piston displacements

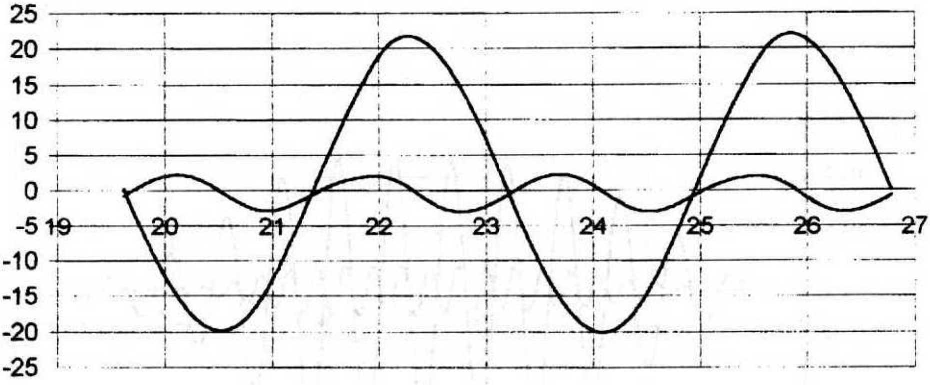

Fig. 9. The estimated components of the piston motion in the asymptotic part (two periods)

The estimated values from the measured data are not exactly equal to the values that have been used for the calculation of the time series. The comparison of Figs. 7 and 8 indicates that the control system of the wavemaker, as it should be expected, is not perfect. The second components differ substantially. Thus it is advisable to consider the measured horizontal displacements of the piston for the boundary conditions in a theoretical analysis.

4. Results and Conclusions

1. The time series, with one dominant frequency, calculated on a computer and given to the control system of the wave maker, results in the motion of the piston. The calculated and measured displacements are very close.
2. As standard, at the initial time the displacements, velocities and accelerations of the piston are zeros. Thus $n = 3$ and at the initial time the velocities and accelerations in the fluid correspond to zero valued vector fields.
3. From the experiments described in the paper (cited in the reference) follows that the velocities and surface elevations might be decomposed into components with multiples of the basic dominant frequency. In the interval of asymptotic behaviour they may be represented by a cosine time series.
4. It follows from the incompressibility condition that for shallow waves the motion of the piston may be decomposed into components with multiples of the basic dominant frequency. In the interval of asymptotic behaviour it may be represented by a sine time series.
5. The calculated control time series with a few components is not exactly reproduced in the measured piston motion. It is better to use the measured piston displacements for the boundary condition, and not the theoretical values.

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