

Flow Characteristics of Homogeneous Mixture in Laminar Flow Zone

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Abstract

Knowledge of the rheological characteristics of a mixture enables the defining of hydraulic losses in horizontal pipelines on the basis of dimensionless criterion type λ (Re_{gen}), where λ is the pipe resistance dimensionless coefficient from the Darcy-Weisbach formula and Re_{gen} is the generalized Reynolds number for the adopted rheological model.

In order to describe the rheological character of plastic and viscous mixtures, the 3-parameter generalized Herschel-Bulkley model was applied. The complete generalized Reynolds number Re_H for the Herschel-Bulkley model was formulated.

The estimation of relative error Re_H value determination for the laminar flow was conducted for two cases: when the yield point τ_0 is taken into account and when it is not considered.

Symbols

- D – pipeline diameter [m],
- g – gravity constant [$m \cdot s^{-2}$],
- G – true deformation rate [s^{-1}],
- G_p – apparent deformation rate [s^{-1}],
- I_m – decrease of pressure level in the mixture flow in the pipeline,
- K – rigidity coefficient for the Vočadlo model [Pa^{1/n_s}],
- k – rigidity coefficient for the Herschel-Bulkley model [$Pa \cdot s^n$],
- k_R – coarseness of pipeline [m],
- L – length of pipeline [m],
- Q – flow intensity [$m^3 \cdot s^{-1}$],
- Re – Reynolds number,
- Re_{crit} – critical Reynolds number,
- Re_{gen} – generalized Reynolds number,

v	-	mean stream velocity in pipeline [$\text{m}\cdot\text{s}^{-1}$],
ΔP	-	pressure losses [Pa],
ε	-	relative pipeline coarseness,
λ	-	pipeline resistance coefficient,
η	-	dynamic viscosity coefficient [Pa·s],
η_{pl}	-	plastic viscosity [Pa·s],
τ	-	shear stresses [Pa],
τ_0	-	flow threshold [Pa],
τ_R	-	shear stresses on face of rotary viscometer [Pa],
τ_w	-	shear stresses on face of pipeline [Pa],
ρ	-	density [$\text{kg}\cdot\text{m}^{-3}$].

1. Introduction

Homogeneous mixtures composed of solid particles and fluid, boast different conditions of solid particle transport, depending on physical properties of the solid and fluid components. Therefore, taking into account these factors, we can divide mixtures as follows (Parzonka, Omelański 1970):

1. Proper suspended matter, in which fine solid particles (colloidal) are transported both when they move and rest due to Brown movements. They can have both Newtonian and non-Newtonian properties.
2. Structural (one-phase) hydro-mixtures, containing mainly dust particles and a number of colloidal particles. They function like viscous and homogeneous substances. At high concentrations of the solid component, these mixtures function like non-Newtonian fluid, and at low concentrations – like the Newtonian. Solid particles are transported in the mixture during movement at a velocity sufficient to keep them in suspended form.
3. Fine-dispersion hydro-mixtures, including mainly fine granular particles (e.g. fine sand). As in structural mixtures, the solid particles are transported in a jet only during movement, at velocities appropriately higher than those for structural mixtures. Thus, both these groups of mixtures can be called dynamic suspended matter. Fine-dispersion hydro-mixtures though, function like two-phase fluids as a rule, with increased density as compared with the fluid component. The properties of the non-Newtonian fluid occur at medium and high concentrations and low flow velocities.
4. Coarse-dispersion hydro-mixtures and non-homogenous dispersion systems. They contain coarse and medium grains (e.g. gravel, sand, etc.). These are typical two-phase mixtures, whose particular components do not mix, and the fluid medium does not change viscosity even at high concentrations of the solid component.

Therefore, both proper suspended matter (in tests as well as during movement) and dynamic suspended matter, i.e. structural and fine-dispersion mixtures (above specified flow velocity securing uniform dispersion of solid particles) can be treated as homogeneous mixtures. The regime of solid particle settling in the kinds of mixtures is laminar mentioned, consistent with Stokes' law.

The flow characteristics of homogeneous mixtures should be conducted on the basis of experimental tests, since the diversified character of solid particles and fluid is the reason why the rheological behaviour of mixtures cannot be described in a theoretical way, e.g. on the basis of the knowledge of physical properties of both components. The mixture flow curves must be measured by visco-meter, and the basic physical properties of the solid component, and its concentration in particular, need to be determined.

The identification of rheological properties of homogeneous mixtures is based on rheo-stable system models, i.e. with the assumption of shear stresses being independent of shearing time. Such an approach is justified by the absence of thixotropy, within the mixture concentration assigned for hydro-transport, or the homogenization effects, produced by pumps, centrifugal separators or mixers.

2. Rheological Models for Viscous-Plastic Mixtures

The appearance of the flow threshold τ_0 at constant plastic viscosity in a mixture requires the use of the Bingham model for a plastic body

$$\tau = \tau_0 + \eta_{pl} \cdot G. \quad (1)$$

Multi-parameter models (Czaban 1987, Eckstädt 1984, Kembłowski 1975, Kempniński 1986, Parzonka 1967) are indispensable in approximation of the rheological properties of a mixture with a clear flow limit τ_0 and alternating plastic viscosity. The generalized three-parameter models are most widely applied:

- Herschel-Bulkley (Czaban 1987, Eckstädt 1984)

$$\begin{aligned} \tau &= \tau_0 + k \cdot G^n & \text{for } \tau > \tau_0 \\ G &= 0 & \text{for } \tau < \tau_0 \end{aligned} \quad (2)$$

- Vočadlo (Parzonka 1967, Voadlo 1973, Kempniński 2000 a)

$$\begin{aligned} \tau &= \left(\tau_0^{1/n} + KG \right)^n & \text{for } \tau > \tau_0 \\ G &= 0 & \text{for } \tau < \tau_0 \end{aligned} \quad (3)$$

The most versatile three-parameter model is the generalized Vočadlo, containing simpler two- and single-parameter models. This was proved by the results of research studies conducted by (among others) Vočadlo and Parzonka (1967), Vočadlo and Charles (1973).

This model was used for the approximation of flow curves by, among other researchers, Sozański (1988) in post coagulatory sediments, Czaban (1987) for kaolin and sewage sediment, and Kempniński (1986, 1999) for cattle and swine manure.

The Herschel-Bulkley model was applied for the approximation of flow curves by Eckstädt in 1984 for manure, by Czaban in 1987 for water mixture of kaolin, chalk and ash.

2.1. Defining True Flow Curve

Nowadays the selection of the mathematical representation of the rheological model and setting the rheological parameters are conducted using the methods of statistical analysis of true mixture flow curves. The transformation of a pseudo-curve, obtained directly from the measurements with the use of rotary viscometers, into the true flow curve is possible using the equation provided by Krieger, Elrod (1953) and Maron (1954), and Švec (1983).

$$G = G_p \left\{ K_1 + K_2 (m - 1) + K_3 \left[(m - 1)^2 + 0.4343 \frac{dm}{d^2 \log \tau_1} \right] \right\} \quad (4)$$

where:

$$K_1 = \frac{1 - \frac{1}{\alpha}}{2} \left(1 + \frac{1}{\ln S} + \frac{\ln S}{3} \right) \quad (5)$$

$$K_2 = \frac{1 - \frac{1}{\alpha}}{2} \left(1 + \frac{2}{3} \ln S \right) \quad (6)$$

$$K_3 = \frac{1 - \frac{1}{\alpha}}{6} \ln S \quad (7)$$

with: $\alpha = \left(\frac{R_2}{R_1} \right)^2$ and $S = \frac{R_2}{R_1} = \alpha^{\frac{1}{2}}$,

and R_1 is the radius of internal cylinder, R_2 is the radius of external cylinder of the rotational viscosimeter.

$$m = \frac{d \log G_p}{d \log \tau_R} \quad (8)$$

Parameter m is a tangent slope to the given point of curve $\log G_p = f(\log \tau_R)$. Equation (4) defining the true velocity of deformation on the internal cylinder surface can be applied for values $S < 1, 2$.

This method enables determination of the true flow curve of the examined fluid without adopting any rheological model whatsoever, assuming complete shearing in the slot of the viscometer and the absence of slide on the face of the spinning cylinder.

Similar treatment is adopted for the approximation of pseudo-curves of flow, obtained from the measurements conducted with the use of pipe and capillary

viscometers, in order to specify true deformation rates and set the rheological parameters of the model.

Rabinowicz, Mooney (Kembłowski 1973, Wilkinson 1963) provided the method of calculation of true deformation rate gradients for the laminar flow of rheologically stable fluids in pipe and capillary viscometers, assuming lack of slide on the face of the pipeline.

$$G = \frac{dv}{dr} = 3 \frac{8Q}{\pi D^3} + \frac{D\Delta P}{4L} \frac{d(8Q/\pi D^3)}{d(D\Delta P/4L)} \quad (9)$$

The equation suggested by Metzner and Reed (Wilkinson 1963) is more convenient in practice.

$$G = \frac{3n' + 1}{4n'} \cdot \frac{8V}{D} = \frac{3n' + 1}{4n'} \cdot G_p \quad (10)$$

where:

$$\dot{n}' = \frac{d \log \tau_w}{d \log G_p} \quad (11)$$

2.2. Setting Rheological Parameters

Given the true flow curve $\tau = f(G)$ and using the statistical analysis it is possible to select the optimal rheological model specifying the rheological behavior of the mixture and set the rheological parameters. All the calculations are performed by electronic computational method.

The application of statistical methods and computer calculation techniques enables quick and detailed determination of the rheological parameter values and facilitates suitable selection of an optimal rheological model.

The evaluation of statistical characteristics and after the physical assessment of the rheological parameter values. Sozański (1988), when analysing parameter τ_0 final determination of the rheological model should take place after for the post-coagulation deposit, specified by statistical methods for Bingham model, Herschel-Bulkley model and Vočadlo one, stated that it reached various values, considerably different from one another, depending on the analytical form of the accepted rheological model. This statement is unacceptable as regards the physical aspect, no matter how precisely the deposit rheological properties are described by these models.

It confirms the correctness of tasks concerning the experimental determination of the flow threshold τ_0 , particularly when Herschel-Bulkley rheological model is used for the approximation of flow curves.

An additional criterion of usability of the determined rheological parameters of the analyzed model should be the evaluation of correlation between the values of these parameters and the concentration of mixture solid particles.

3. Determination of Hydraulic Losses in Vertical Pipelines on the Basis of the Dimensionless Criterion of $\lambda - Re_{gen}$ Type

Knowledge of rheological features enables generalization of research results by means of the use of the dimensionless criterion λRe_{gen} , where λ is the dimensionless pipe resistance coefficient from the classical Darcy-Weisbach formula (12), and Re_{gen} is the generalized Reynold number for the rheological model considered.

$$\Delta p = \lambda \frac{L}{D} \cdot \frac{v^2}{2} \rho \quad (12)$$

The application of the three-parameter generalized rheological model is definitely worth while. According to the researcher, Herschel-Bulkley (2) model and Vočadlo model (3) (Kempinski 2000 b) is the optimal model for homogenous mixtures. In the dimensioning of the main transport installation, conducting of introductory pipe examinations becomes important, enabling appropriate determination of $\lambda(Re_{gen})$ dependence for the entire flow range.

3.1. Determination of Pipe Resistance Coefficient λ in Laminar Flow Zone

The linear resistance coefficient λ for Newton's model in the laminar flow equals

$$\lambda = \frac{64}{Re} \quad (13)$$

where Reynolds number Re is determined with the use of the following formula:

$$Re = \frac{vD\rho}{\eta} \quad (14)$$

For the rheological models of Bingham (1), Herschel-Bulkley (2) and Vočadlo (3) there are, sometimes incomplete, formulas for the generalized Reynolds number in the flow in a pipeline with diameter D , obtained with the assumption that in the laminar flow zone

$$\lambda = \frac{64}{Re_{gen}} \quad (15)$$

i.e. analogical to the dependence for the Newtonian fluids (13), resulting from Poiseuille's formula.

In the author's opinion, the appropriate description of the generalized Reynolds number Re_{gen} is crucial due to the fact that it constitutes the flow criterion and also enables to model the flow.

Bingham model is applied to the flow description of substances characterized by the flow threshold τ_0 and plastic viscosity η_{pl} . The fluid flow described by the Bingham model is characterized by the Reynolds number in the following formula:

$$Re_B = \frac{vD\rho}{\eta_{pl}} \quad (16)$$

This formula, however, does not account for the influence of the flow threshold τ_0 , which is crucial in the laminar flow zone. As the researcher proved, the application of this formula in determining the loss coefficient λ results in serious errors (Kempiński 2000 a).

Buckingham-Reiner (Parzonka 1977) provided the complete generalized Reynolds number for the Bingham model which includes the flow threshold τ_0 .

$$Re_B = \frac{vD\rho}{\eta_{pl}} \left[1 - \frac{4}{3} \left(\frac{\tau_0}{\tau_w} \right) + \frac{1}{3} \left(\frac{\tau_0}{\tau_w} \right)^4 \right] \quad (17)$$

Having introduced the simplification suggested by Reiner and Fiłatow for $\tau_0/\tau_w < 0.5$ they arrived at the simplified formula

$$Re_B = \frac{vD\rho}{\eta_{pl}} \left[1 - \frac{4}{3} \left(\frac{\tau_0}{\tau_w} \right) \right] \quad (18)$$

Reiner and Fiłatow (Sozański 1988) provided the simplified Reynolds number formula for the Bingham model, corresponding to formula (18). But their formula was more practical as it did not require the knowledge of shear stresses τ_w on the face of the pipeline.

$$Re_B = \frac{vD\rho}{\eta_{pl} + \frac{\tau_0 D}{6v}} \quad (19)$$

Czaban (1986) proved that this simplification is important when $v \geq \frac{\tau_0}{\eta_{pl}} \cdot \frac{D}{8}$. It constitutes approximately 1/2 of the laminar flow range for the flow of homogeneous mixtures.

The flow of plastic and viscous fluid defined by the Vočadlo model (3) is characterized by the generalized Reynolds number which Kempieński (2000 b) gives in formula (20):

$$Re_v = \frac{8v^{(2-n)}D^n\rho}{(2K)^n} \left[\frac{n}{3n+1} - \frac{1}{3} \left(\frac{\tau_0}{\tau_w} \right)^{1/n} + \frac{1}{3(3n+1)} \left(\frac{\tau_0}{\tau_w} \right)^{3+1/n} \right]^n \quad (20)$$

According to the analysis conducted by Reiner and Fiłatow (Sozański 1988) for Bingham fluids, for $\tau_0/\tau_w < 0.5$ it is possible to omit the last component of the formula in the brackets.

$$Re_v = \frac{8v^{(2-n)}D^n\rho}{(2K)^n} \left[\frac{n}{3n+1} - \frac{1}{3} \left(\frac{\tau_0}{\tau_w} \right)^{1/n} \right]^n \quad (21)$$

The assumption that the influence of the flow threshold τ_0 disappears in the turbulent flow zone in the flow of plastic and viscous substances, as confirmed by

numerous researchers (Vočadlo, Charles 1973), simplifies the formula above to the following form

$$Re_v = \frac{8v^{(2-n)}D^n\rho}{K^n\left(6 + \frac{2}{n}\right)^n} \quad (22)$$

Formula (22) is simplified and does not include the flow threshold τ_0 . It should not be used in calculating the linear loss coefficient λ in the laminar flow zone.

The Herschel-Bulkley model (2) is more and more frequently applied also in the description of the fluid flow with clear flow limit and alternating plastic viscosity.

According to Czaban (1987) the intensity of laminar flow in R radius pipeline described by Herschel-Bulkley model is:

$$Q = \pi R^3 \left(\frac{\tau_w}{k}\right)^{\frac{1}{n}} \frac{n}{n+1} \left(1 - \frac{\tau_0}{\tau_w}\right)^{\frac{n+1}{n}} \times \left\{1 - \frac{2n}{3n+1} \left(1 - \frac{\tau_0}{\tau_w}\right) \left[1 + \frac{n}{2n+1} \cdot \frac{\tau_0}{\tau_w}\right]\right\} \quad (23)$$

After replacing the flow intensity with the product of the pipeline cross-section face and the mean rate, the dependence of the shear stress on the face of the pipeline is obtained.

$$\tau_w = \frac{(2v)^nk}{D^n \left| \frac{n}{n+1} \left(1 - \frac{\tau_0}{\tau_w}\right)^{\frac{n+1}{n}} \left\{1 - \frac{2n}{3n+1} \left(1 - \frac{\tau_0}{\tau_w}\right) \left[1 + \frac{n}{2n+1} \cdot \frac{\tau_0}{\tau_w}\right]\right\} \right|^n} \quad (24)$$

Using formulas (12) and (15), the following formula is obtained. This characterizes the generalized Reynolds number:

$$Re_v = \frac{8\rho v^2}{\tau_w} \quad (25)$$

By placing τ_w from formula (24), the researcher obtained the complete Reynolds' number formula for Herschel-Bulkley model, after author

$$Re_H = \frac{8v^{(2-n)}D^n\rho}{2^nk} \left| \frac{n}{n+1} \left(1 - \frac{\tau_0}{\tau_w}\right)^{\frac{n+1}{n}} \times \left\{1 - \frac{2n}{3n+1} \left(1 - \frac{\tau_0}{\tau_w}\right) \left[1 + \frac{n}{2n+1} \cdot \frac{\tau_0}{\tau_w}\right]\right\} \right|^n \quad (26)$$

The assumption that the influence of the flow threshold τ_0 disappears in the turbulent flow zone in the flow of plastic and viscous substances, simplifies the formula above to the following form

$$Re_H = \frac{8v^{(2-n)}D^n\rho}{k \cdot \left(6 + \frac{2}{n}\right)^n} \quad (27)$$

Formula (27) is the simplified one, which doesn't consider the flow threshold τ_0 and is equivalent to Reynolds' number formula for the De Waele-Ostwald fluids. This formula (27) should not be used for the calculation of coefficient linear losses λ in the laminar flow zone.

Setting the Reynolds' number Re_H according to formulas (26) requires knowledge of the shear stresses on the face of the pipeline: $\tau_w = D\Delta P/4L$.

Knowing the model rheological parameters measured with rotative viscosimeter and using formula (24), we can determine iteratively the value of shearing stress on pipeline wall τ_w .

The suggested method allows for correct determination of pressure losses in the laminar flow zone on the basis of the rheological parameters of the mixture obtained with the use of rotary viscometer.

Since the presented Reynolds number for Herschel-Bulkley model (26) is actually the generalized Reynolds number Re_H , changing into Reynolds numbers for the simpler two- and single-parameter models as the fluid structure is simplified, thus, the presented method has also general applications. Inserting $n = 1$, $k = \eta_{pl}$ respectively to formula (3.13), the Reynolds number for the Bingham model is obtained according to (17), as provided by Buckingham-Reiner, when $\tau_0 = 0$, $n = 1$, $k = \eta$ the Reynolds' number for Newton's model (14).

The pipeline hydrotransport of highly concentrated mixtures, close to thixotropic concentration $c_{s,tix}$, takes place mainly in the laminar flow zone. Excluding the flow threshold τ_0 when setting pressure losses may result in serious calculation errors and, consequently, in the hydrotransport installation failure.

The researcher conducted the analysis of the relative error δ committed when calculating the Reynolds' number Re_H with the use of simplified formula (27), which did not include the flow threshold τ_0 in the laminar flow zone, in relation to the complete, generalized Reynolds number specified according to formula (26).

$$\delta = \left(\frac{Re_{H(27)} - Re_{H(26)}}{Re_{H(26)}} \right) \cdot 100\% \quad (28)$$

$$\delta = \left[1 - \frac{\left[\left(1 - \frac{\tau_0}{\tau_w} \right)^{\frac{n+1}{n}} \left\{ 1 - \frac{2n}{3n+1} \left(1 - \frac{\tau_0}{\tau_w} \right) \left[1 + \frac{n}{2n+1} \cdot \frac{\tau_0}{\tau_w} \right] \right\} \right]^n}{\left(1 - \frac{2n}{3n+1} \right)^n} \right] \cdot 100\% \quad (29)$$

The relative error δ was calculated with the use of the alternating parameter n and the (τ_0/τ_w) ratio. Figure 1 depicts the graphic representation of the alternation of $\delta = f(\tau_0/\tau_w, n)$ dependence.

For the practical applications of pipeline hydraulic transport, according to the analysis done by Reiner and Filatow (Sozański 1988), the value of $\tau_0/\tau_w < 0.5$. The

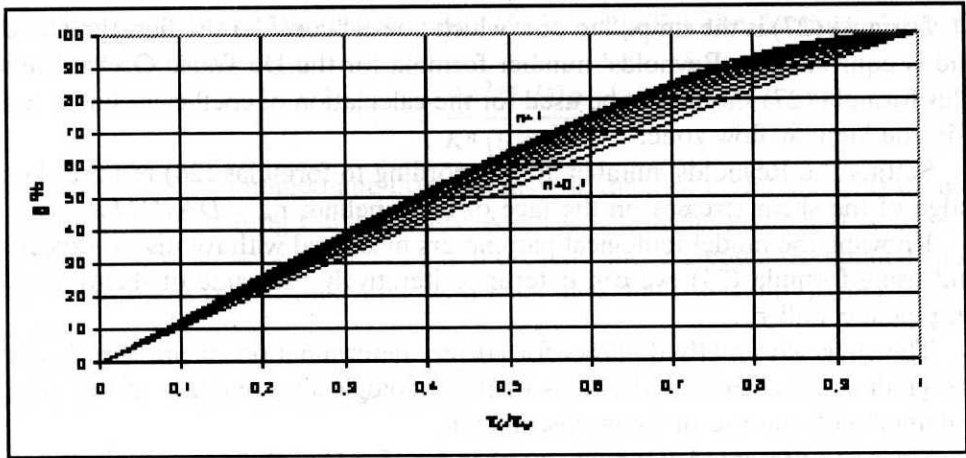


Fig. 1. Relative error $\delta = f(\tau_0/\tau_w, n)Re_H$ determination

author has established for manure flow in the laminar flow zone in pipelines of diameter $D = 25.6; 40.6; 51.6$ mm a changeability of τ_0/τ_w ranged between $0.1 \div 0.3$. Therefore application of the simplified formula of calculating the Reynolds number (27) can cause relative error of determination of the Reynolds number of the order of $10 \div 40\%$.

3.2. Critical Reynolds' Number Re_{crit} During Pipeline Flow

Ryan and Johnson (1959) provided the theoretical basis for specifying the critical Reynolds' number for a non-Newtonian fluid on the basis of the alternation analysis of the function specifying the stability number Z_R .

Using the Ryan and Johnson method, Czaban (1987) specified the formulas for the critical Reynolds number for Herschel-Bulkey and Bingham models.

The formulas enabling the determination of Re_{cr} for Herschel-Bulkey and Bingham models require additionally, knowledge of the critical flow rate v_{cr} , corresponding to the transfer from the laminar into turbulent flow. This hinders the use of the provided formulas for practical applications without conducting laboratory pipe tests.

Conducting laboratory pipe tests is expensive and requires the engagement of a specialized research team. We cannot always afford it.

The author presented the iterant method of determining the critical Reynolds number $Re_{B,cr}$ on the basis of viscometric analysis. To achieve this, the researcher compared the generalized Reynolds number for the Bingham model with the critical Reynolds number of this model in the boundary conditions (transfer from the laminar into turbulent flow) (Kempínski 2000 b).

This method is also proper for the three-parameter Herschel-Bulkey model when comparing the generalized Reynolds number Re_H with the critical Reynolds number $Re_{H,crit}$ according to (Czaban 1987).

4. Conclusion

Viscometric tests of non-Newtonian fluids with the use of rotary or pipe viscometers allow for obtaining the flow pseudo-curves of $\tau(G_p)$. This constitutes the basis for determining true flow curves with the use of Krieger, Elrod and Maron method for pipe and capillary viscometers.

This procedure allows for elaborating experimental tests in the form of true flow curves without prior adoption of any rheological model.

The application of mathematical methods of statistical analysis of true flow curves enables selection of a proper rheological model and setting the rheological parameters.

Knowledge of the rheological properties enables generalization the investigation results applying the $\lambda(Re_{gen})$ dimensionless criterion.

In this research, the formula for the generalized Reynolds number for the three-parameter Herschel-Bulkley model was first provided, which enables precise determination of the pipes' resistance coefficient λ in the laminar flow zone for this model.

Since the adopted model is generalized and includes simple two- and one-parameter models, the suggested determination method of λ refers to mixtures with various behaviour characteristics (viscous, pseudo-plastic, plastic, plastic-viscous).

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