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A Note on Radiation Conditions for Narrow-band Random Waves Generated in Fluid of Constant Depth

J. Kazimierz Szmidt

Institute of Hydro-Engineering of the Polish Academy of Sciences
ul. Kościerska 7, 80-953 Gdańsk

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Abstract

In the paper, the generation of narrow-band random waves in a semi-infinite layer of fluid is considered. The problem is formulated in a discrete space of chosen points by means of the finite difference method. The main goal of the investigations is to construct radiation boundary conditions which enable us to replace the infinite fluid area with a finite domain. The investigations are illustrated with experimental results and numerical examples confirming efficiency of the discrete model developed in the paper.

1. Introduction

In analysis of water waves we usually deal with infinite fluid domains. Frequently the waves are induced by sources of disturbances placed in a finite region of the fluid. In the latter case of waves propagating to infinity the appropriate boundary conditions at infinity are radiation conditions which protect our solution from waves incoming from infinity. In many practical cases involving irregular geometry, it is difficult to construct an analytical solution to the problem and thus we are forced to resort to discrete methods such as the finite difference or the finite element method. With these methods, however, only a finite number of nodal points of an assumed net can be considered. Therefore, these methods are not directly applicable to infinite systems. A standard way of treating the problems is to replace them with problems formulated in finite domains. This can be done by dividing the infinite fluid domain into two parts: a finite region enclosing all sources of disturbances and a regular infinite region. On the boundary between the finite and infinite regions, special boundary conditions (radiation or transmitting) should be specified which allow us to limit our analysis to the finite region and obtain a solution preserving the main features of an unknown solution in the infinite region. Unfortunately, there is no general solution to the latter problem, and thus, for particular cases, only approximate solutions can be obtained. One of the important problems in this field is the problem of radiation conditions for random waves generated in fluid of constant depth. For this case, experimental investigations in a hydraulic flume can be carried out and thus theoretical results may be compared with experimental data. Usually, laboratory experiments provide data describing the generation of water waves by a wavemaker. In the case of random generation, the wavemaker motion is assumed in the form of a stochastic process of known characteristics. In this way the theoretical problem of describing the physical situation is reduced to the examination of the transformation of the generator motion process into one describing the fluid motion. In experiments, a set of sample functions of the processes mentioned is considered.

The literature on the subject is considerable. In order to make the discussion clear and relate the present paper to previous works, some of the earlier methods are quoted below. Lysmer and Kuhlemeyer (1969) proposed a method through which an infinite system may be approximated by a finite system with special viscous boundary conditions assumed on its boundary. They discussed the problem of elastic waves propagating from an excited finite zone outwards, to an exterior infinite region. The boundary conditions assumed on the convex artificial boundary between the finite and infinite regions, ensure that all energy arriving at the boundary is absorbed. In a discrete formulation of the plane problem for an elastic half space, the authors developed a system of dash-pots at the artificial boundary, which absorbs energy of impinging elastic waves. The dash-pots are chosen in such a way that reflection of the waves from the boundary is minimised. For the case of harmonic Love's type waves propagating in an elastic layer Lysmer and Waas (1972) derived a finite element solution expressed in the form of a linear combination of eigenvectors of a stiffness matrix corresponding to nodal points assumed on the layer width. In this way the required boundary conditions on the boundary between finite and infinite zones may be expressed in an explicit form by means of a solution of the eigenvalue problem mentioned. A discussion of absorbing boundary conditions for wave equations may be found in Enquist and Majda (1977). The authors derived a hierarchy of absorbing local boundary conditions for wave equations which approximate theoretical non-local boundary conditions. The obtained local conditions give well-posed mixed initial boundary value problems for the wave equation. The next paper of Enquist and Majda (1979) concentrates on the same problems of radiation boundary conditions for acoustic and elastic wave equations. The problem of approximation of radiation boundary conditions is discussed in Israeli and Orszag (1981). The authors presented a survey of methods for imposition of the boundary conditions in numerical schemes. They found out that a combination of absorbing boundary conditions with damping and wave speed modification, may be a useful tool in constructing solutions to the wave propagation phenomenon. Radiation conditions for the Helmholtz equation in a semi-infinite layer of fluid have been investigated by Szmidt (1983). His approach to solving the problem is similar to that of Lysmer and Waas (1972). The radiation problem is solved with the help of eigenvectors resulting from the finite difference formulation. The final boundary condition is expressed in the form of a linear combination of independent components corresponding to progressive and standing waves, respectively. Exact non-reflecting boundary conditions for a solution of reduced wave equation in an infinite domain were proposed by Keller and Givoli (1989). The authors devised non-local boundary conditions which lead to more accurate results than those obtained by using approximate local conditions.

The problem of reduction of a boundary value problem posed on an infinite domain to one posed on a finite domain emerges not only in analysis of wave equations. There were also other differential equations formulated in infinite domains for which it was required to formulate an associated problem in finite domain. Examples of the latter problems may be found in Lentini and Keller (1980a, b) where the so-called asymptotic boundary conditions were devised.

In most of the papers the radiation conditions correspond to reduced wave equations. In a general case of wave equation, the problem is solved by means of approximate absorbing boundary conditions. The efficiency of applications of the local boundary conditions to discrete formulations of the problems on hand depends inherently on the structure of the conditions. For higher hierarchy of the approximation, the boundary conditions are expressed by equations with higher order derivatives with respect to space and time of solution. The last feature of the conditions may cause some difficulties in their adaptation to discrete formulations.

In the present paper the finite difference solution to the linear problem of random waves propagating in a layer of fluid is considered. We confine our attention to water waves described by narrow-band stochastic processes with prescribed characteristics. It has been found that a combination of local transmitting boundary conditions with discrete integration of the relevant equations in the time domain by means of the WilsonΘ method, gives very accurate results. The main numerical results are compared with results of rigorous analytical solutions to the problem considered and data obtained in experiments.

2. Generation of Water Waves – Formulation in Continuum

Let us consider a semi-infinite layer of fluid shown in Fig. 1. The motion of the fluid is induced by a piston-type wavemaker (rigid vertical wall OA in the figure) placed at the beginning of the layer. It is assumed the fluid is inviscid and incompressible and the velocity field is potential, and thus, a velocity potential $\Phi\,(x, z, t)$ exists which satisfies the Laplace equation

$$\nabla^2\Phi = 0 \tag{1}$$

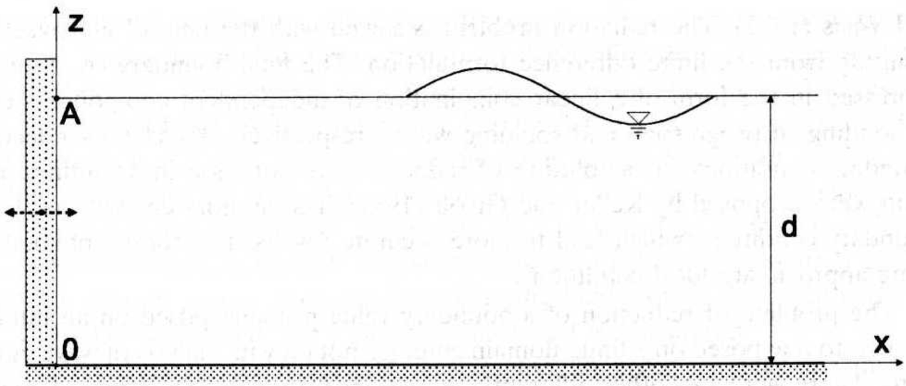

Fig. 1. Semi-infinite layer of fluid

and appropriate boundary conditions:

$$\left.\frac{\partial \Phi}{\partial z}\right|_{z=0} = 0, \quad \left.\frac{\partial \Phi}{\partial x}\right|_{x=0} = v_x = \dot{\tilde{x}}(t), \quad \left.\left(\ddot{\Phi} + g\frac{\partial \Phi}{\partial z}\right)\right|_{z=d} = 0, \tag{2}$$

where d is the water depth and the dots denote derivatives with respect to time.

The first condition means that a perfect reflection occurs on the bottom of the layer. The second condition states that the horizontal component of the fluid velocity at the rigid wall is equal to the wall velocity. The third condition is the linearized boundary condition for the surface waves. In this condition g denotes the gravitational acceleration. For the case of steady-state vibrations the above boundary conditions should be supplemented with the Sommerfeld condition describing the behaviour of the velocity potential at infinity ($x \to \infty$). For a general case of transient fluid motion one has also to specify the initial conditions describing the velocity potential and its time derivative within the fluid domain at $t = 0$. With respect to the boundary conditions at the free surface, its elevation is described by the formula (Stoker 1957)

$$\zeta(x, t) = -\frac{1}{g}\dot{\Phi}\bigg|_{z=d}. \tag{3}$$

In analysis of random generation of water waves we deal with sample functions of a stochastic process describing the wavemaker motion. Analytical solutions for this case may be obtained by means of an impulse response function of the generator-fluid system. In deriving the latter function we need a complex frequency response function for the velocity potential. Thus, let us now consider the case of steady harmonic motion of the wavemaker

$$\tilde{x}(t) = B\exp(i\sigma t), \tag{4}$$

where B is a constant, i is the imaginary unit, σ is the angular frequency and t is the time.

For this case the solution of the Laplace equation assumes the form

$$\begin{aligned}\Phi = &-\frac{4\sigma}{k_0}\frac{\sinh\beta_0}{2\beta_0+\sinh 2\beta_0}\cosh(k_0 z)\exp i(\sigma t - k_0 x)+\\ &-i\sum_j\frac{4\sigma}{k_j}\frac{\sin\beta_j}{2\beta_j+\sin 2\beta_j}\exp(-k_j x)\cos(k_j z)\exp(i\sigma t).\end{aligned} \tag{5}$$

The eigenvalues in the equation satisfy the following dispersion relations

$$\frac{\sigma^2 d}{g} = \beta_0\tanh\beta_0 = -\beta_j\tan\beta_j,\ \beta_0 = k_0 d,\ \beta_j = k_j d, j = 1, 2, \ldots \tag{6}$$

The first term in the solution (5) describes the surface wave propagating in the layer and the second the standing wave which dies out when going to infinity ($x \to \infty$). With respect to the solution obtained, the complex frequency response function for the velocity potential is expressed in the form

$$\begin{aligned}H(\sigma, x, z) = &-\frac{4\sigma}{k_0}\frac{\sinh\beta_0}{2\beta_0+\sinh 2\beta_0}\cosh k_0 z\exp(-ik_0 x)+\\ &-i\sum_j\frac{4\sigma}{k_j}\frac{\sin\beta_j}{2\beta_j+\sin 2\beta_j}\cos k_j z\exp(-k_j x).\end{aligned} \tag{7}$$

For greater values of x, say $x > 2d$, the second term of the equation may be ignored and the function may be assumed as

$$H(\sigma, x, z) \cong -4\sigma\frac{\sinh\beta_0}{2\beta_0+\sinh 2\beta_0}\frac{\cosh k_0 z}{k_0}\exp(-ik_0 x). \tag{8}$$

Knowing the complex frequency response function we can calculate the impulse response function by means of the Fourier transform (Crandall and Mark 1973)

$$h(t, x, z) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} H(\sigma)\exp i\sigma t d\sigma. \tag{9}$$

With the help of the impulse response function, a solution of the problem for arbitrary function describing the wavemaker motion can be obtained by means of convolution of the relevant functions over the time interval $(-\infty, +\infty)$.

In a similar way one can obtain the impulse response function $h_\xi(t) = h(t, x)$ for the free surface elevation. For waves generated by the piston type wavemaker, the function can be expressed as follows (Szmidt 1995)

$$h(t, x) = \frac{1}{\pi}\int_0^\infty\frac{\tanh sd}{s}\cos(sx - rt)ds + \frac{1}{\pi}\int_0^\infty\frac{\tanh sd}{s}\cos(sx + rt)ds, \tag{10}$$

where

$$r^2 = gs \tanh sd. \tag{11}$$

For a sufficiently large value of x (say $x > 2d$) and $t > 1s$, the second integral in Eq. (10) may be neglected and the impulse response function assumed in the form

$$h(t,x) \cong \frac{1}{\pi}\int_0^\infty \frac{\tanh sd}{s} \cos(sx - rt)ds. \tag{12}$$

Knowing the generator velocity $v_g(t) = \dot{\hat{x}}(t)$, where $\hat{x}(t)$ is the generator displacement, we can calculate the free surface elevation at $x =$ const. by means of the following convolution integral

$$\xi(x = \text{const.}, t) = \int_0^t v_g(t-\tau)h(\tau, x = \text{const.})d\tau. \tag{13}$$

3. Random Generation of Water Waves in a Hydraulic Flume

In order to verify results of discrete models describing the random generation of water waves in fluid of constant depth, experimental investigations in a hydraulic flume were performed. In experiments, the waves were generated by a programmable piston type wavemaker. The experimental set-up is shown in Fig. 2. The motion of the generator was assumed in the form of sample functions of stochastic processes of known characteristics. With respect to the equation (4), where B is a constant amplitude of the generator motion, let us consider the following process describing the generator motion

$$X(t) = A_n(t)\cos\sigma_1 t + D_n(t)\sin\sigma_1 t, \tag{14}$$

where σ_1 is the dominant frequency, $A_n(t)$ and $D_n(t)$ two independent stationary normal processes without dominant frequencies and n indicates the degree of differentiability of the processes. The functions $A_n(t)$ and $D_n(t)$ are statistically independent, and assumed to have zero means and the same correlation functions (Wilde and Kozakiewicz 1993). For the discussed problem of water motion starting from rest, the generator motion is assumed to be described by a three times differentiable stochastic process. In order to create such a process of known characteristics we here attached some important results given in Wilde and Kozakiewicz (1993). The starting point is the set of stochastic differential equations of processes without dominant frequencies

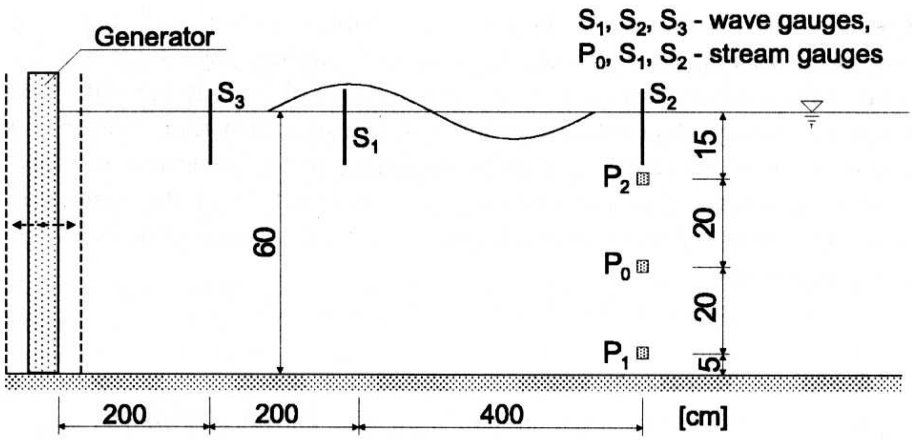

Fig. 2. Experimental set-up

$$
\begin{aligned}
&dA_0(t) + \eta A_0(t)dt = \alpha dB(t),\\
&\frac{dA_1}{dt} + \eta A_1 = \eta A_0,\\
&\frac{dA_2}{dt} + \eta A_2 = \eta A_1,\\
&\frac{dA_3}{dt} + \eta A_3 = \eta A_2.
\end{aligned}
\tag{15}
$$

where $A_r(t)$ is an r-times differentiable stochastic process ($r = 0, 1, 2, 3$), $B(t)$ is the Brownian motion process of unit variance, η is a memory parameter and α – another parameter of the set.

For our purposes it is convenient to introduce the discrete time steps with constant Δt. The solution of the first equation in (15) may be written in the following form

$$
\begin{aligned}
&A_0(0) = \sqrt{P}U(0),\\
&A_0(r+1) = \exp(-\eta\Delta t)A_0(r) + \sqrt{P[1-\exp(-2\eta\Delta t)]}U(r+1).
\end{aligned}
\tag{16}
$$

where $t_r = r\Delta t$, $P = \alpha^2/2\eta$ and $U(r)$ is an independent random variable with normal distribution $N(0,1)$.

In calculating sample functions of the process at the point $t_r = (r+1)\Delta t$, $A_0(r)$ in the second equation of (16) is a known number as the sample value of the process at the previous instant of time. The solutions to the remainder equations of (15) are described by the formula

$$
A_k(r+1) = \exp(-\eta\Delta t)A_k(r) + \int_{r\Delta t}^{(r+1)\Delta t} \eta\exp[-\eta(t-\tau)]A_{k-1}(\tau)d\tau. \tag{17}
$$

Knowing the distribution of the random variables $A_k(0)$ ($k = 0, 1, 2, 3$) at the starting point we may calculate the sequence of numbers representing a sample function of the process $A_k(t)$ in the considered range of time. In laboratory experiments, we frequently deal with the problem of the generator-fluid motion starting from rest. In order to ensure a smooth beginning of the generator motion, it is reasonable to assume that both velocity and acceleration of the generator are equal to zero at the starting point. In such a case the initial conditions for the process $A_3(t)$ reads

$$A_3(0) = 0, \dot{A}_3(0) = 0, \ddot{A}_3(0) = 0. \tag{18}$$

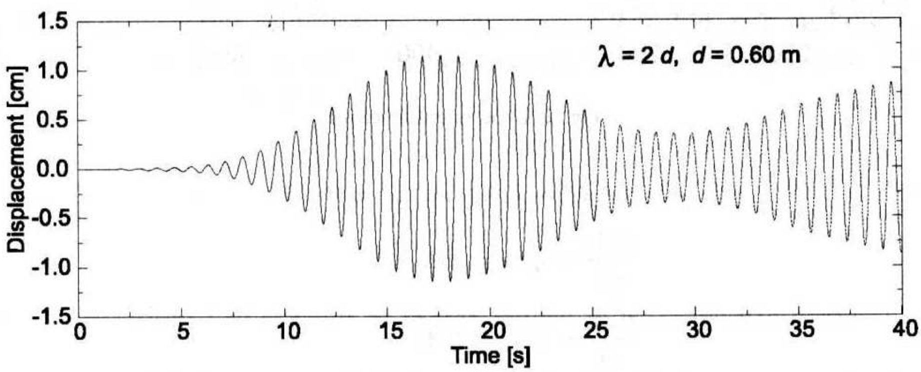

Fig. 3. Sample function of the generator motion process

Similar conditions are valid for the process $D_3(t)$. With respect to the assumed initial conditions of the generation, the Gaussian process of the generator motion is not a stationary process, but it is an asymptotically stationary Gaussian process (for details see Wilde and Kozakiewicz 1993). A sample function of the process (14) describing the generator motion is shown in Fig. 3. From the plot it can be seen that within the first range of time (a small elapse of time from the starting point), the amplitude of the generator motion grows very slowly, and thus, a relatively long time is needed to reach an assumed, mean level of the generation amplitude. Therefore, in preparing the input data for the wavemaker steering system, it has been decided to shorten the first range of the motion by means of a deterministic description of the motion within this range of time. A random generation is switched on at a chosen moment of time when the amplitude is close to its mean value. Such a sample function describing the wavemaker motion may be obtained by means of the computer program '*FALOROB*' available in our institute. The input data for this program consists of the sampling frequency, the assumed length of water wave corresponding to the dominant frequency of the wave spectrum, and defines the length of time record divided into three stages. Within the first stage of the generator motion the amplitude of the motion grows in time up to a certain value. Then, we have a basic range of time during which the amplitude changes slowly. The third, and last stage describes the diminution of the generation. A sample function obtained with the help of the computer program is expressed in the form of sequence of numbers corresponding to the assumed sampling frequency. Having the sample function, in the second step, the discrete record is multiplied by an amplification factor, and then it is used as the input data for the generator steering system. In experiments, both the generator motion and the surface waves were measured by means of the displacement and the wave gauges, respectively. Some of the results obtained in experiments are shown in Fig. 4 where sample functions of the generator motion and associated elevation of the free surface measured at the distance x_ξ from the generator are depicted. One can show that for the discussed case of narrow-band process of the generator motion the surface elevation may be expressed in the form of a linear combination of the generator motion process and its time derivative both shifted in the time scale.

4. Discrete Formulation and Radiation Conditions

With respect to the discussion and results given in the preceding sections, let us consider now the finite difference solution of the aforementioned problem of random generation of water waves in a semi-infinite layer of fluid. The assumed finite difference mesh is shown in Fig. 5. The artificial boundary for the discrete description is assumed at the distance x_b from the vertical rigid plate of the wavemaker (from wall OA in the figure). Let a and b be the spacing of vertical and horizontal lines of the assumed net, respectively. In the discrete formulation by means of the finite difference method (FDM) the representation of the Laplace equation assumes the form of a system of algebraic equations written for all nodal points of an assumed net including the boundary points. Each of the equations includes unknown values of the potential at five nodal points. For a typical point (i, j) within the fluid, where i denotes the horizontal coordinate and j denotes the vertical coordinate, the finite difference equation for the Laplace equation assumes the form

$$-\varepsilon\Phi_{i-1,j} - \Phi_{i,j-1} + 2(1+\varepsilon)\Phi_{i,j} - \Phi_{i,j+1} - \varepsilon\Phi_{i+1,j} = 0, \tag{19}$$

where $\varepsilon = (b/a)^2$.

In order to write the equations at boundary points ($x = 0, x = x_b, z = 0$ and $z = d$) we have to extend the finite difference net in such a way that together with each of the boundary points, a neighbouring nodal point is placed at the outward normal to the boundary at the considered point. The unknown values of the potential function at these external points are expressed by means of the boundary conditions, in terms of the values of the function at points belonging to the fluid area. In this way, the resultant system of the difference equations corresponds to the nodal points of the fluid domain.

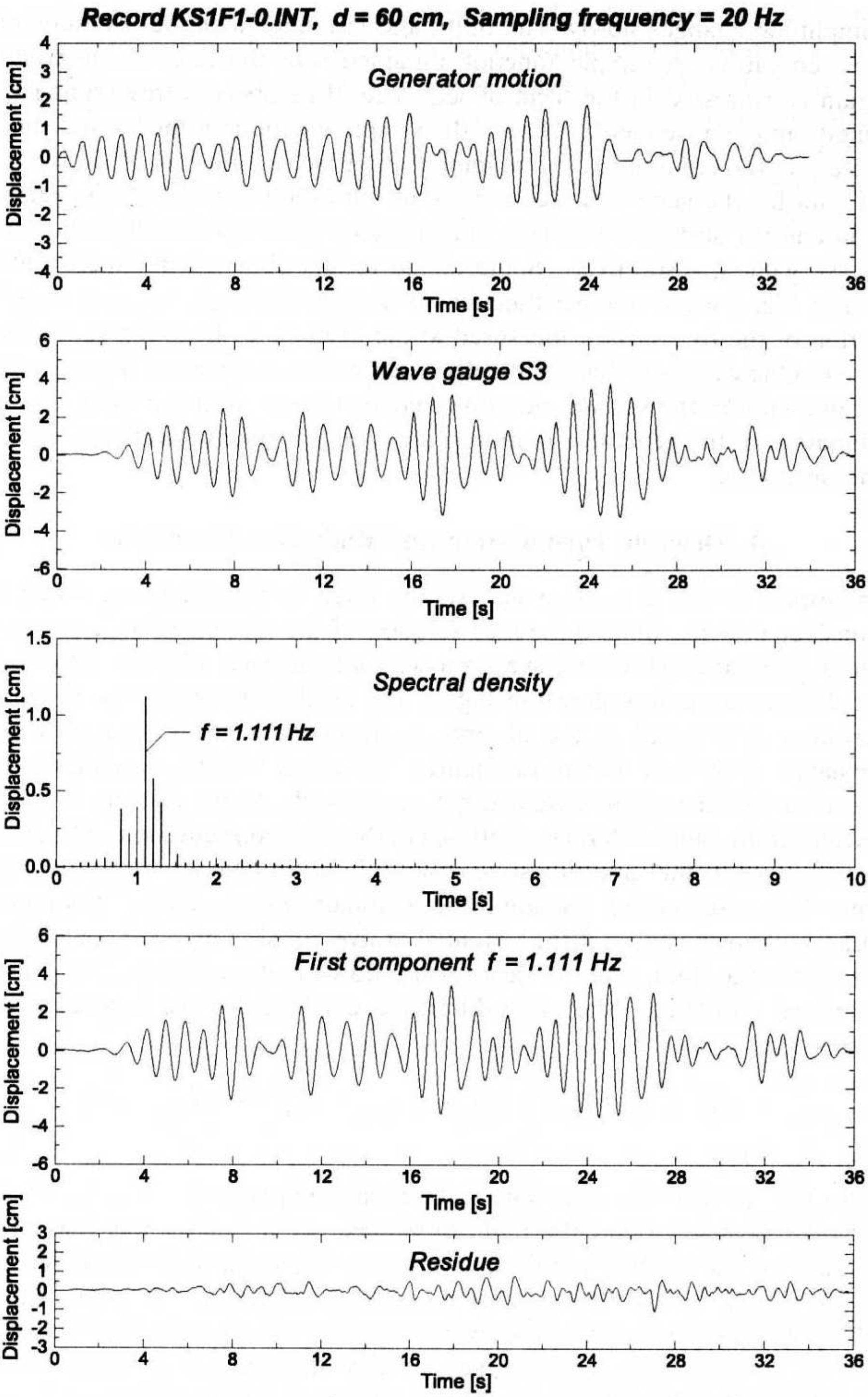

Fig. 4. Sample function of the generator motion and free surface elevation

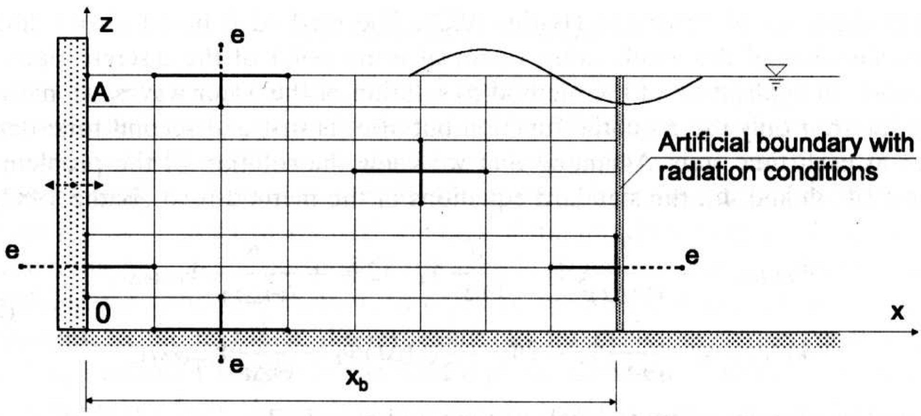

Fig. 5. Finite difference spacing of nodal points

Let e, b, and i denote the subscripts of the external, boundary and internal points, respectively. According to the boundary condition at the wave maker, the following relation holds

$$\Phi_e = \Phi_i - 2a v_x, \tag{20}$$

where v_x is the velocity of the wall OA (Fig. 1).
The boundary condition at the bottom of the layer leads to the result

$$\Phi_e = \Phi_i. \tag{21}$$

The linear boundary condition for the surface waves gives

$$\Phi_e = \Phi_i - \frac{2b}{g}\ddot{\Phi}_b. \tag{22}$$

For the discussed case of water waves with narrow-band spectral density, the radiation condition at the artificial boundary is assumed in the following form

$$\frac{\partial \Phi}{\partial x} + \frac{k}{\sigma}\frac{\partial \Phi}{\partial t} \cong 0, \tag{23}$$

where k is the wave number corresponding to the dominant frequency σ.
From the last equation if follows that

$$\Phi_e = \Phi_i - \frac{2ak}{\sigma}\dot{\Phi}_b. \tag{24}$$

It is seen that the boundary condition for the artificial boundary and the free surface contains the derivatives with respect to time of the potential function. Therefore, in equations for the velocity potential corresponding to a certain instant of time there are also time derivatives of the potential at some boundary points. In order to transform the equations to standard algebraic equations written at a common moment of time we use the WilsonΘ method well established in the dynamics of structures (Bathe 1982). The method is based on the linear approximation of the acceleration vector at every point of the discrete time. In our case, in application of the method to solution of the water waves, we have to calculate not only the potential function but also its first and second time derivative at each time step. Assuming that we know the solution of the problem at time t (Φ, $\dot{\Phi}$ and $\ddot{\Phi}$), the standard equations of the method read (Bathe 1982)

$$\begin{aligned}\ddot{\Phi}_{t+\Theta\Delta t} &= -\frac{6}{(\Theta\Delta t)^2}\Phi_t - \frac{6}{\Theta\Delta t}\dot{\Phi}_t - 2\ddot{\Phi}_t + \frac{6}{(\Theta\Delta t)^2}\Phi_{t+\Theta\Delta t},\\ \dot{\Phi}_{t+\Theta\Delta t} &= -\frac{3}{\Theta\Delta t}\Phi_t - 2\dot{\Phi}_t - \frac{1}{2}(\Theta\Delta t)\ddot{\Phi}_t + \frac{3}{\Theta\Delta t}\Phi_{t+\Theta\Delta t},\end{aligned} \tag{25}$$

where the subscripts denote levels of time and $\Theta = 1.47$.
From substitution of Eq. (25) into Eq. (22) it follows

$$\Phi_e = \Phi_i - \frac{12b}{g(\Theta\Delta t)^2}\Phi_b + F_1, \tag{26}$$

where

$$F_1 = \frac{2b}{g}\left[\frac{6}{(\Theta\Delta t)^2}\Phi_t + \frac{6}{\Theta\Delta t}\dot{\Phi}_t + 2\ddot{\Phi}_t\right]_b. \tag{27}$$

In a similar way, Eq. (24) is reduced to the form

$$\Phi_e = \Phi_i - \frac{6ak}{(\Theta\Delta t)\sigma}\Phi_b + F_2, \tag{28}$$

where

$$F_2 = \frac{2ak}{\sigma}\left(\frac{3}{\Theta\Delta t}\Phi_t + 2\dot{\Phi}_t + \frac{1}{2}\Theta\Delta t\,\ddot{\Phi}_t\right)_b. \tag{29}$$

Due to the last relations, the finite difference equations of the problem written for $t + \Theta\Delta t$ do not contain time derivatives of the potential function. Having the solution at $t + \Theta\Delta t$ it is a simple task to calculate the values of $\ddot{\Phi}$, $\dot{\Phi}$ and Φ at the time $t + \Delta t$. Simple manipulations give

$$\begin{aligned}\ddot{\Phi}_{t+\Delta t} &= \ddot{\Phi}_t + \frac{\ddot{\Phi}_{t+\Theta\Delta t} - \ddot{\Phi}_t}{\Theta\Delta t}\Delta t,\\ \dot{\Phi}_{t+\Delta t} &= \dot{\Phi}_t + \ddot{\Phi}_t\Delta t + \frac{1}{2}\frac{\ddot{\Phi}_{t+\Theta\Delta t} - \ddot{\Phi}_t}{\Theta\Delta t}(\Delta t)^2,\\ \Phi_{t+\Delta t} &= \Phi_t + \dot{\Phi}_t\Delta t + \frac{1}{2}\ddot{\Phi}_t(\Delta t)^2 + \frac{1}{6}\frac{\ddot{\Phi}_{t+\Theta\Delta t} - \ddot{\Phi}_t}{\Theta\Delta t}(\Delta t)^3.\end{aligned} \tag{30}$$

The method of discrete solution of the problem of initial generation of the waves presented above has been applied to test problems of assumed generation of the wave, and to the input data used in laboratory experiments.

5. Numerical Examples

In order to illustrate the discussion and examine the accuracy of the discrete solutions, we attach here some numerical examples. Results of numerical calculations are compared with results of analytical solutions in continuum and data obtained in experiments performed in a hydraulic flume. The analytical solution may be considered as an exact theoretical solution of the problem mentioned. Numerical calculations have been performed for sample functions of a stochastic process describing the generator motion. In order to examine accuracy of the radiation conditions, in the first step, discrete solutions for the sample function shown in Fig. 3 and different lengths of the fluid domain (different distances x_b of the artificial boundary from the generator in Fig. 2) have been carried out. The results obtained in calculations are shown in Fig. 6 where the free surface elevations are depicted. From comparison of these results it may be seen that the approximate radiation boundary conditions lead to proper solution of the problem on hand. In the next step, analytical and discrete solutions for the input data used in laboratory experiments are created. The results obtained in calculations are depicted in Fig. 7. The upper graph in the figure shows the free surface elevation measured by the wave gauge $S3$. Then, in the two subsequent plots, the analytical solutions based on the impulse response functions (10) (solution I) and (12) (solution II) are presented, respectively. The next plot in the figure shows the difference between these two analytical solutions, and the subsequent one represents the finite difference solution. In order to compare the results obtained in numerical calculations, in the last graph zoomed sections of the relevant plots are depicted. From the plots in Fig. 7 it is seen that the radiation conditions developed above lead to solutions of acceptable accuracy. Obviously, in experiments, we deal with a physical situation where the fluid viscosity as well as non-linearity of the problem influences final results. Therefore, with growing amplitude of the generator motion one may expect greater discrepancy between the theoretical solution, based on linear approximation, and results of laboratory experiments.

6. Concluding Remarks

Theoretical and experimental analysis of the problem of generating random water waves described in the preceding sections has revealed some important features of the phenomenon. The results obtained lead to the following conclusions

- For a moderately small amplitude of the generator motion the local radiation condition in the FD formulation may be derived with the help of a solution for a monochromatic wave.
- The WilsonΘ method has proved to be an efficient tool in performing integration of the system equations in the time domain.

- FDM with proper radiation conditions is a convenient tool in analysing water wave phenomenon in fluid of finite depth.

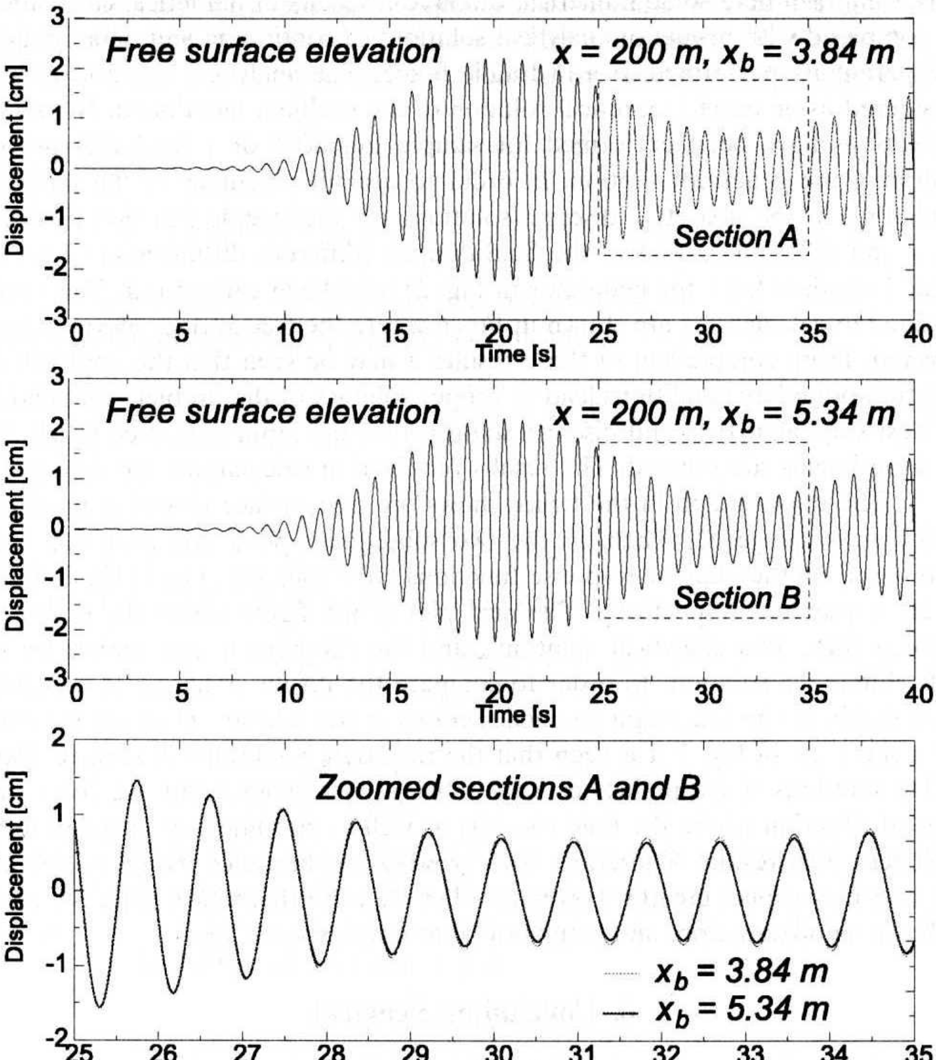

Fig. 6. Free surface elevations for different fluid domains

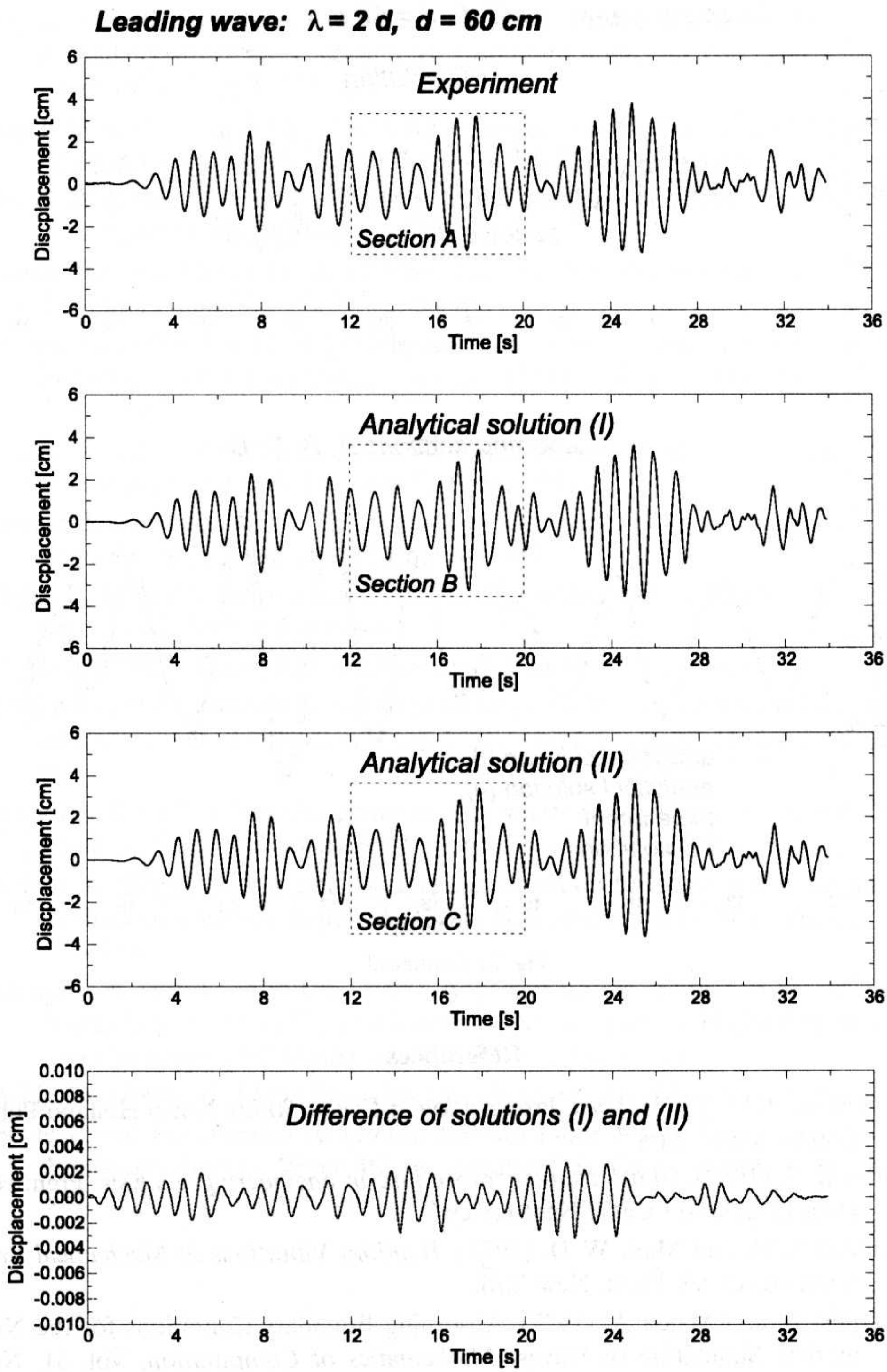

Fig. 7a. Free surface elevation at $x_\xi = 2$ m

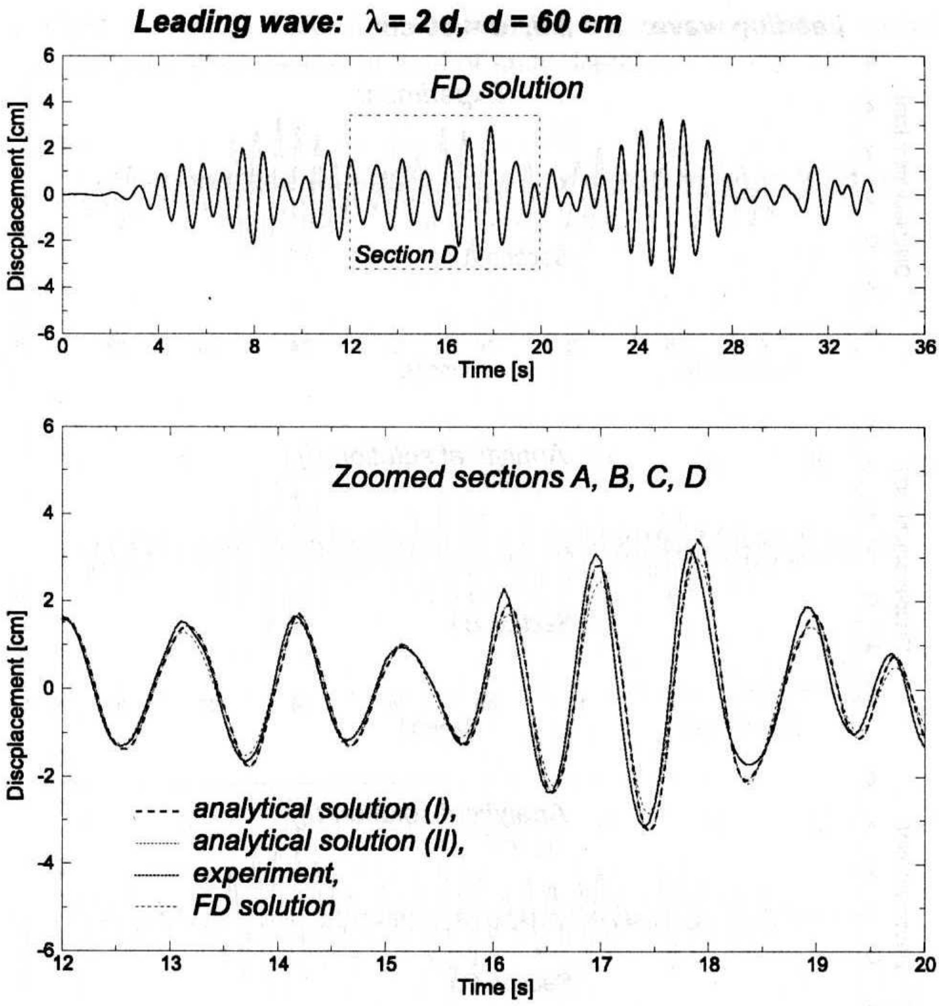

Fig. 7b. Continued

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