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Local Scour in Non-uniform Bed Material Below a Horizontal Solid Apron

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Abstract

A two-dimensional mathematical model of local scour in non-uniform non-cohesive soil below a horizontal solid apron is presented. The model parameters were identified on the basis of own experimental laboratory data. Verification of the model, accomplished using other laboratory data, showed fairly good conformity, however, additional verification under field conditions is needed.

List of Symbols

b – width of flume, [L]
D – grain diameter (size), [L]
D_i – representative grain size of sediment class i, [L]
D_p – grain size for which $p\%$ are finer, [L]
$e = D_{95}/D_5$ – coefficient of grain non-uniformity, [–]
g – acceleration due to gravity, [L T^{-2}]
h – initial flow depth, [L]
$H = h + z$ – water depth in scour hole, [L]
n – sediment porosity, [–]
p_i – fraction, by mass, of size D_i in initial bed material, [–]
p_{ai} – fraction, by mass, of size D_i in the armour coat, [–]
q_i – probability that grain of size D_i remains as part of stable armour coat, [–]
q_{sv} – volumetric rate of bed load transport per unit width, [L^2 T^{-1}]
Q – flow discharge, [L^3 T^{-1}]
t_p – time of prognosis, [T]

t – time, [T]
T – thickness of mixed layer, [L]
T_a – thickness of armour coat, [L]
u – mean velocity, $[\mathrm{L\,T^{-1}}]$
$\widetilde{u} = u + u'$ – instantaneous flow velocity in x-direction, $[\mathrm{L\,T^{-1}}]$
$u\prime$ – velocity fluctuation, $[\mathrm{L\,T^{-1}}]$
u_{no} – non-scouring velocity, $[\mathrm{L\,T^{-1}}]$
u_{ni} – incipient velocity at which grain i-th class starts moving, $[\mathrm{L\,T^{-1}}]$
χ – horizontal distance from the end of apron, [L]
z – scour depth, [L]
α – angle of free-jet discharge, [–]
γ_s – specific weight of soil, $[\mathrm{M\,L^{-2}T^{-2}}]$
$\varepsilon = \sigma_u/u$ – mean turbulence intensity at section of the apron end, [–]
ρ_s – density of soil, $[\mathrm{M\,L^{-3}}]$
σ_g – geometric standard deviation of grain size, [–]
σ_u – standard deviation of velocity in x direction. $[\mathrm{L\,T^{-1}}]$

Subscript z means value at the section of scour hole of depth z.

1. Introduction

Local scour occurs commonly in erodible soils due to the action of mean or fluctuating velocities created by various obstacles to flow e.g. constrictions, bridge piers, channel bends etc. Another cause of local scour is non-equilibrium in sediment transport due to deficiency of bed or suspended load in a flow. Such a phenomenon is observed below reservoirs from which clear water is released at high velocities. Downstream of stilling basins the flow has often continues to have so high an energy that it can pick up soil particles from an earth-channel bed and erode its bottom and banks.

Mathematical description of the erosion process is rather sophisticated due to the random nature of turbulence and soil characteristics. In cohesive soils one can observe detaching of aggregates, in non-cohesive ones – separated particles in the form of bed or suspended load. In non-uniform loose soils a selective washing of bed material occurs, which leads to the creation of a protective coat of coarser grains on the eroded surface. This layer usually increases resistance of the bed material against further erosion.

A reliable prognosis of the scour development below stilling basins and other forms of bed protection would allow for optimal design of these members. Such a procedure should enable estimation of the risk of undermining and collapse of the protective members and the protected structures themselves. The paper presents a relatively simple mathematical model of two-dimensional local clear-water scour in non-uniform bed materials below a horizontal solid apron. The model was identified and verified on the basis of laboratory test data.

2. Mathematical Description of Scouring

The presented model of local scour describes the temporal two-dimensional development of the scour hole in non-cohesive, non-uniform bed material. It enables prediction of the maximum depth of scour hole and composition of surface soil layer at the deepest point of the hole.

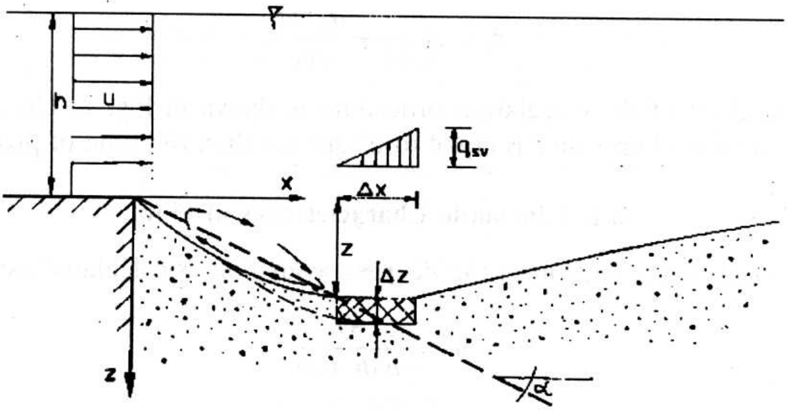

Fig. 1. Definition sketch of local scour below the horizontal apron

Generally, changes in depth z of a two-dimensional (x, z see Fig. 1) scour hole in time can be described by the continuity equation written in the form:

$$\frac{\partial z}{\partial t} = \frac{1}{\gamma_s}\frac{\partial q_s}{\partial x} \tag{1}$$

where γ_s – specific weight of soil, N/m^3; q_s – bed load transport rate per unit width, N/sm.

For the sake of simplicity the following assumptions are made:

a) clear water scour, i.e. no sediment inflow,

b) the protective solid apron is horizontal and followed by an initially horizontal erodible bed,

c) the bed consists of non-cohesive, non-uniform soil,

d) the flow on the apron is subcritical ($Fr < 1$),

e) distributions of mean velocities over water depths h and $h+z$ are uniform, i.e. plots of the velocities in vertical sections at depths h and $h+z$ are rectangular.

It is also assumed, that after short time interval Δt the bed load transport rate increases along distance Δx by $\Delta q_{sv} = q_{sv} - 0 = q_{sv}$ (Fig. 1). These assumptions enabled calculation of an increase in scour depth Δz using the following form of Eq. (1):

$$\Delta z = \frac{q_{sv}\Delta t}{\Delta x} \tag{2}$$

where: q_{sv} – volumetric bed load transport per unit width, m^3/sm.

The maximum depth of scour hole after time t can thus be calculated from

$$z_t = z_{t-\Delta t} + \frac{q_{sv}\Delta t}{\Delta x}. \tag{3}$$

A flow-chart of the calculation procedure is shown in Fig. 2. The procedure stops when time of erosion t is equal to or greater than the time of prognosis t_p.

2.1. Kinematic Characteristics of Flow

Mean velocity at the section of the deepest scour hole is calculated as:

$$u_z = \frac{Q}{b\,(h+z)} \tag{4}$$

where:

u_z – mean flow velocity over a vertical section, m/s,
Q – flow discharge, m^3/s,
b – width of flume, m,
h – initial flow depth, m,
z – scour depth, m.

To estimate grain stability it is necessary to know the bottom velocity. The mean bottom velocity is given by Eq. (4). The variance of fluctuating bottom velocity depends mostly on approach flow depth and velocity, as well as distance from the apron end.

Using experimental data for two-dimensional free turbulent jets (Abramovich 1984) the velocity variance may be roughly estimated by:

$$\sigma_{u_z}^2 \approx 0.04\frac{hu^2}{x} \tag{5}$$

where:

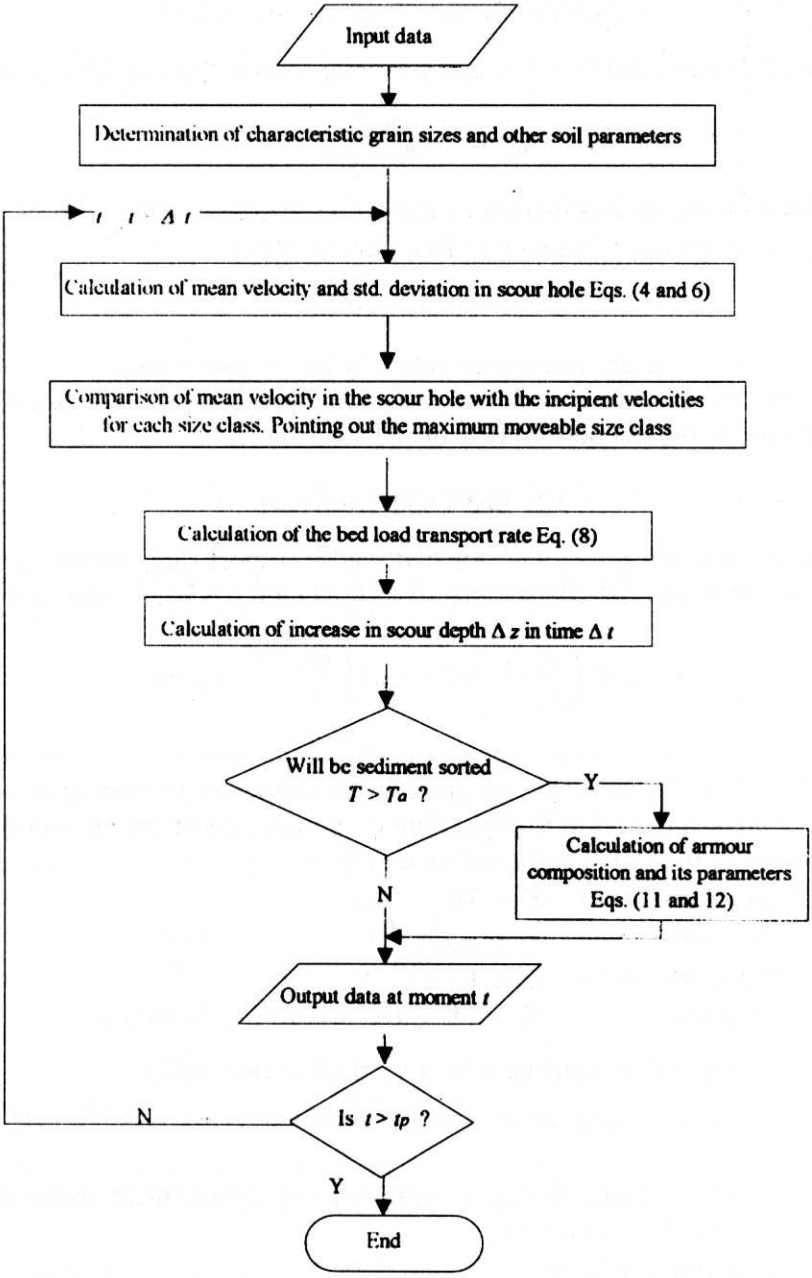

Fig. 2. Flow-chart of calculation procedure

h – half-thickness of jet equal to water depth on the solid apron,
$u = Q/bh$ – mean velocity at the apron end,
x – horizontal distance from the apron end.

Introducing relationship $z = x$ tg a (see Fig. 1) into the Eq. (5) one obtains:

$$\sigma_{u_z}^2 \approx 0.04\frac{hu^2 tg\alpha}{z}. \tag{6}$$

The angle α can be interpreted as that of jet discharge and it can be approximated by an empirical relationship (Błażejewski 1989):

$$\alpha = 0.39\varepsilon + 0.13 \tag{7}$$

where $\varepsilon = \sigma_u/u$ – average turbulence intensity at the apron end.

Knowing the value of α one can also evaluate the horizontal distance from the apron end to the deepest section of the scour hole.

2.2. Bed Load Transport

Shamov's formula (Shamov 1954) for bed load transport rate, recommended for non-uniform bed material coarser than 0.15-0.20 mm, has been used in the form:

$$q_s = K\left(\frac{u_z}{u_{on}}\right)^3 (u_z - u_{on})\left(\frac{D_w}{H}\right)^{1/4}, \text{kg/ms} \tag{8}$$

where K – coefficient of proportionality, kg/m^2, dependent on grain diameter D_K calculated as a weighted mean for grains with diameters greater than 1–2 mm. There are four formulae for K depending on percentage of the coarser ($> 1 - 2$ mm) fractions in the initial soil (bed material):

40 – 70%	$K = 3D_K^{2/3}$,
70 – 80%	$K = 2.5D_K^{2/3}$,
10 – 20% or 80 – 90%	$K = 1.5D_K^{2/3}$,
otherwise	$K = 0.95\sqrt{D_K}$, where D_K in meters.

u_z – mean flow velocity over a vertical section, m/s,
u_{on} – non-scouring mean velocity calculated as $u_{on} = 3.7D_w^{1/3}H^{1/6}$, m/s,
D_w – characteristic diameter expressed as weighted mean for all fractions in bed material, m,
H – water depth, m.

Shamov's formula (8) is one of the few which bound the bed-load transport rate with flow velocity instead of shear stress and hydraulic gradient which are difficult to determine under scour-hole conditions.

In the case of high non-uniformity of bed material a critical grain diameter should be calculated by the relationship (Shamov 1954):

$$D_{cr} = \left(\frac{u_z}{4,4H^{1/6}}\right)^3 \cong \frac{u_z^3}{85\sqrt{H}}. \tag{9}$$

Grains larger than D_{cr} are excluded from the sediment transport. The next step in Shamov's procedure is the calculation of percentage of the remaining (mobile) fractions, taking their sum as 100%, followed by determination of characteristic diameters D_K and D_w.

Volumetric sediment transport is expressed by

$$q_{sv} = \frac{q_s}{\rho_s(1-n)} \tag{10}$$

where ρ_s – sediment density, n – sediment porosity.

2.3. Sorting Bed Material during Scouring

Assessment of composition of the armouring layer is based on the methodology proposed by Gessler (1970), as well as Hsu and Holly (1992). The percentage of the i-th fraction in the armour coat is estimated by

$$p_{ai} = \frac{p_i q_i}{\sum\limits_i p_i q_i} \tag{11}$$

where p_i – percentage of the i-th fraction in bed (initial) material, q_i – probability that grain of size i remains as part of the stable armour coat.

Probability q_i in our model is equal to the probability that an instantaneous bottom velocity is smaller than non-scouring velocity for the i-th fraction ($\widetilde{u}_z < u_{ni}$). By assuming that the instantaneous velocity $\widetilde{u}_z = u_z + u_z'$ is normally distributed, it holds that

$$q_i = P\left(\widetilde{u}_z < u_{ni}\right) = \frac{1}{\sigma_m\sqrt{2\pi}} \int\limits_{-\infty}^{u_{ni}/u_z} \exp\left(-\frac{(m-1)^2}{2\sigma_m^2}\right) dm \tag{12}$$

where $m = \widetilde{u}_z/u_z$ – dimensionless variable ($\overline{m} = 1$) $\sigma_m = \sigma_{u_z}/u_z$ – the standard deviation of the $\widetilde{u}_z/u_z$ distribution, u_z and σ_{u_z} can be obtained from Eq. (4) and Eq. (6).

The incipient velocity for each fraction can be calculated by Shamov's formula (Shamov 1954):

$$u_{ni} = 4.4 D_i^{1/3} H^{1/6}, \text{m/s} \tag{13}$$

where D_i and H are in meters.

An armour coat of thickness T_a equal to one large grain diameter, e.g. D_{90} (Błażejewski and Zawadzki 1990), may occur after washing a soil layer of thickness T in which the content of fractions creating armour coat is p_a:

$$T = \frac{T_a}{p_a}. \tag{14}$$

The washed layer of thickness T may consist of two thinner layers: the surface one with partially sorted and the deeper one with unsorted (initial) material. Their contribution in volume of the washed layer is then:

$$p_{Ti} = \frac{T_a}{T} p_{ai} + \frac{T - T_a}{T} p_i. \tag{15}$$

The deeper the scour hole, the lower the mean flow velocity u_z and higher the probability of remaining grains in the scour hole. The content of coarser fractions increases and the thickness of washed layer T, calculated by Eq. (14) decreases. If a surface layer consists of a majority of stable grains then thickness T may prove to be smaller than the assumed armour coat thickness T_a. This means the end of sorting, but not of scouring. Due to variations in flow direction and turbulence at the bottom, further washing out of particular grains is still possible. The bottom layer may occur at dynamic equilibrium – the washed out grains are replaced by others.

2.4. Identification of Model Parameters

Some model parameters (e.g. sediment density, texture, porosity) were measured directly in the laboratory, others (increments Δx and Δt in Eq. (2)) were determined on the basis of experimental data using the least square fitting procedure.

Experiments were conducted in a glass-sided rectangular flume 50 cm wide located in the hydraulic laboratory of the Department of Hydraulic Engineering - Agricultural University of Poznań (Zawadzki 1998). Inside the flume a concrete horizontal solid apron 330 cm long was built, followed by an erodible reach of 300 cm. All experiments were conducted at flow rate $Q = 0.055$ m^3/s and two water depths at the end of the apron $h = 17.0$ cm and 20.0 cm. Five different mixtures of soils indicated by symbols **A**, **B**, **C**, **D** and **E** were tested. Grain diameters ranged from 0.5 mm to 10.0 mm. Table 1 depicts characteristics of the soil mixtures. In the last column there is a geometric standard deviation of grain diameters given by

$$\sigma_g = \sqrt{D_{84.1}/D_{15.9}}. \tag{16}$$

As a criterion of non-uniformity $\sigma_g > 1.35$ was taken after Breusers and Raudkivi (1991). Before experiments the soil mixtures were placed in the flume and compacted in 10 cm layers using a rammer.

Table 1. Basic parameters of investigated soils (Zawadzki 1998)

Soil	D_5 [mm]	D_{50} [mm]	D_{95} [mm]	D_{100} [mm]	σ_g [–]
A	0.50	0.90	1.25	1.25	1.44
B	0.50	0.95	3,00	4.00	1.66
C	0.50	0.95	4.00	6.30	1.92
D	0.50	0.95	6.30	10.00	2.50
E	0.50	1.00	8.50	10.00	2.95

Longitudinal profiles of scour hole along a distance 180 cm downstream of the apron end were measured after stopping flow at t = 15, 30, 60, 120, 240, 360 and 480 minutes from the beginning of every test. A detailed description of the test parameters is given in Table 2.

Table 2. Basic parameters of tests conducted by Zawadzki (1998)

Test	Soil	Depth h, [m]	Mean velocity u, [m/s]	Standard deviation σ_u, [m/s]	Test duration [h]	Number of replicates
A	A	0.20	0.55	0.05	8	3
B	B	0.20	0.55	0.05	8	3
C	C	0.20	0.55	0.05	8 – 10	3
D	D	0.20	0.55	0.05	8 – 60	3
E	E	0.20	0.55	0.05	8	3
F	E	0.17	0.60	0.07	8 – 60	8

Table 3. Texture of soils investigated by Popova (1977)

Soil	Percentage of fraction between diameters D [mm]			
	0.1–1.0	1.0–2.0	2.0–4.0	4.0–7.5
A_P	80	20	0	0
B_P	75	25	0	0
C_P	70	15	15	0
D_P	80	0	0	20

Every scour hole began just below the solid apron boasting a mild slope and downward concave shape as shown in Fig. 1. Similar changes in grain composition were observed in all tests; the hole bottom at the deepest section was covered by the largest grains. Elsewhere, the percentage of the largest grains was lower.

It was assumed that the increment Δx was proportional to the mean grain diameter D_{50}. It turned out (Fig. 3) that a better fitting was achieved using in

Table 4. Basic parameters of tests conducted by Popova (1977)

Test No.	Water depth h [cm]	Mean velocity u [cm/s]	Turbulence intensity σ_u,	Test duration t [h]	Soil
1	9.6	32,7	0.70	130	$\mathbf{A}_P$
2	9.6	32.6	0.70	100	$\mathbf{B}_P$
3	9.6	32.6	0.70	240	$\mathbf{C}_P$
5	9.6	55.5	0.12	200	$\mathbf{C}_P$
9	9.6	32.6	0.70	50	$\mathbf{D}_P$
12	21.2	86.5	0.12	60	$\mathbf{D}_P$

Table 5. Texture of soils investigated by Bondarczuk (1986)

Percentage of grains of diameters $D < D_{\%}$ [%]	Grain size diameters $D_{\%}$ [mm] in soils			
	$\mathbf{A}_B$	$\mathbf{B}_B$	$\mathbf{C}_B$	$\mathbf{D}_B$
0	0.25	0.25	0.25	0.25
10	0.55	0.55	0.37	0.31
50	0.90	0.90	0.90	0.90
90	1.50	2.50	5.30	8.00
100	2.00	3.60	10.00	20.00

addition some grain non-uniformity characteristics:

$$\Delta x = 10 e D_{50} \text{ when } \sigma_g \geq 2 \tag{17a}$$

where $e = D_{95}/D_5$ – coefficient of grain non-uniformity [–], or

$$\Delta x = 50 D_{50}/\sigma_g \text{ when } \sigma_g < 2. \tag{17b}$$

The limit $\sigma_g = 2$ was proposed earlier by Gessler (1970). The author argued that soils with $\sigma_g < 2$ did not create any armour coat, and single coarse grains increased turbulence behind them and accelerated erosion of finer grains. The differentiated values of Δx may also be caused by the mode of grain detachment: a uniform bed material is eroded from relatively small areas whereas the non-uniform – from larger areas (clusters) due to interaction between grains. The best fitting was obtained using of Eq. (17) was obtained for tests **A** and **C** and the worst – for test **E**.

3. Model Validation

To verify the above proposed model, results of tests conducted by Popova (1977), Bondarczuk (1986) and own tests lasting over 8 hours (see Tab. 2) were adopted. The choice was confined to well documented data sets.

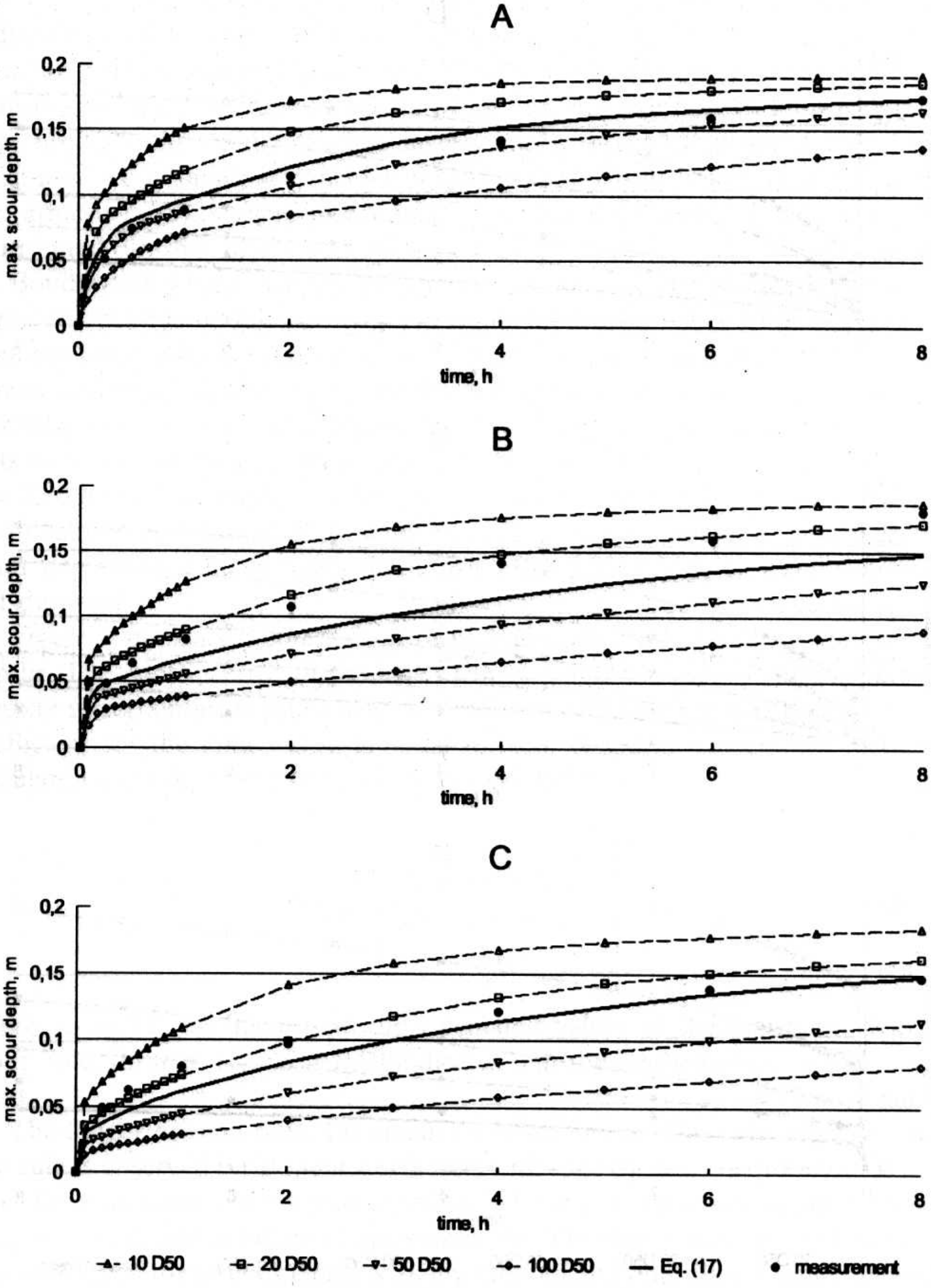

Fig. 3a. Model identification; comparison of calculated and measured scour depths for different values of Δx (Zawadzki 1998)

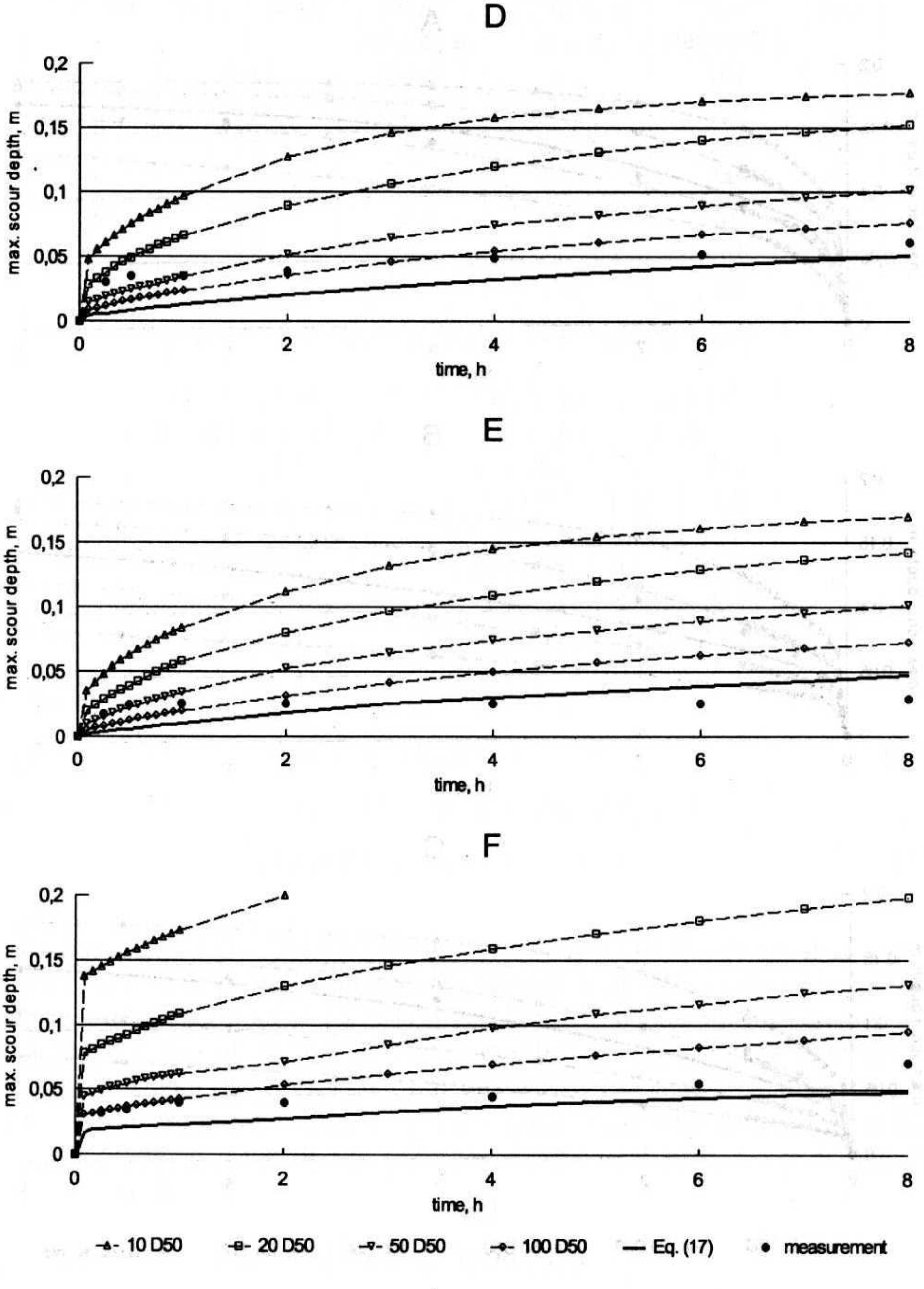

Fig. 3b. Model identification; comparison of calculated and measured scour depths for different values of Δx (Zawadzki 1998)

Popova (1977) carried out her investigations in a hydraulic flume 0.5 m wide and 15.0 m long. Clear water ran freely or under a movable gate creating a hydraulic jump. The Froude number $Fr = u/\sqrt{gh}$ ranged between 0.11 and 0.57. Four mixtures of sand and gravel (see Tab. 3) were investigated. Basic parameters of tests conducted by Popova are shown in Tab. 4. Only runs of known duration were used for the validation. The investigations showed that about 75% of the maximum scour depth was achieved during first 10–20 hours of test duration. The armour coat consisted of a single layer of coarser grains. The largest grains contributed 30–75% of the surface-layer mass.

Bondarczuk (1986) conducted his experiments in a glass-walled hydraulic flume 16 cm wide and 4.5 m long. An erodible stretch, located downstream of a concrete horizontal apron, was filled with soil mixtures described in Tab. 5. Soil density was equal to 2650 kg/m^3. At the end of the erodible stretch a vertical thin retaining wall was installed, followed by a soil slope at the angle of repose. All tests were carried out at constant water depth $h = 25.0$ cm and mean flow velocity $u = 63.0$ cm/s. The turbulence level at the apron end was low $(\sigma_u/u < 0.09)$and mean velocity profile was well established. Tests lasted 6 hours. Measurements of scour depths were made along flume axis after 0.25; 0.5; 1; 2; 4 and 6 hours of test duration.

Figure 4 presents a comparison of measured and calculated maximum scour depths. The relative error of predicted values ranges between + 97% and –55%. Despite uncertainties in input data conformity is reasonably good. The correlation coefficient for the data shown is equal to 0.50. A special correlation coefficient calculated as (Ozga-Zielińska and Brzeziński 1994):

$$RS = \left[\frac{\sum\limits_{i=1}^{n} \left(2z_{mi}z_{ci} - z_{ci}^2\right)}{\sum\limits_{i=1}^{n} z_{mi}^2} \right]^{\frac{1}{2}} \tag{19}$$

where z_m and z_c are measured and calculated values of the maximum depth of scour hole, for $n = 17$, $RS = 0.89$ indicates that the mathematical model is fairly good.

The longer erosion time, the smaller relative errors. The only departure from that rule is observed for Bondarczuk's tests with soils C_B and D_B, however, in these cases the maximum scour depths equalled 3–3.5 cm, i.e. they were comparable with $D_{\max} = 2.0$ cm and accuracy of measurements. The above, mentioned tendency is explainable, as the phenomenon is highly random and only after a longer period is an equalisation of effects provided. A relatively high scattering of results obtained from replicates is evidence of somewhat chaotic development of the scour hole.

The model predictive power concerning armour coat composition is also fairly good, it will, however, be presented and discussed in a separate paper.

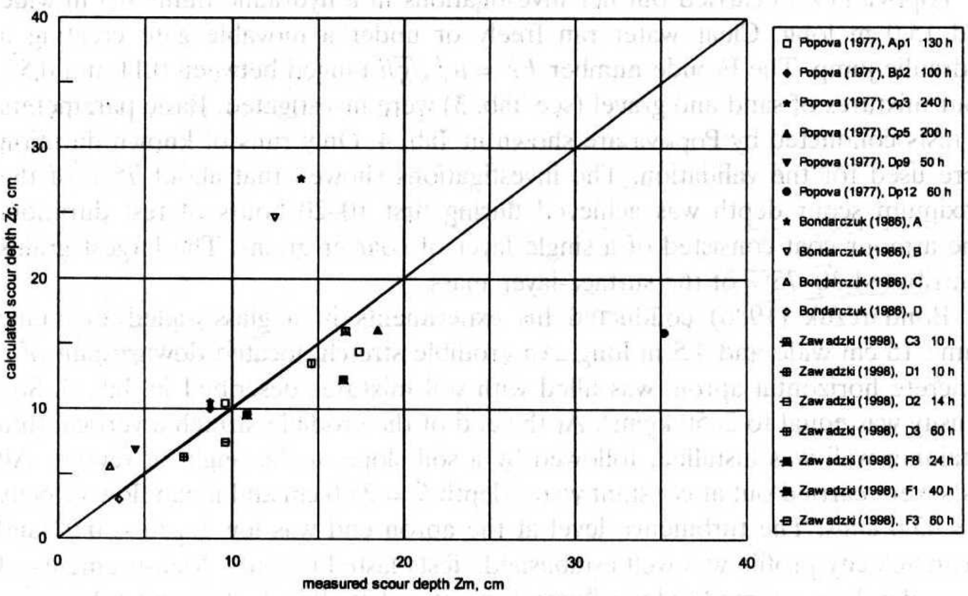

Fig. 4. Comparison of calculated and measured scour depths

4. Conclusions

Creation of armour coat and decrease in bottom velocities when scouring a non-uniform bed material are basic factors which slow down sediment transport rate in time to zero upon a stabilisation phase.

Discrepancy between the maximum scour hole depth calculated by the mathematical model presented and experimental data ranged from + 97% till –55%. This discrepancy can be explained by a certain oversimplification of the phenomenon in the mathematical model as well as uncertainties in the empirical data. It seems that due to turbulence and soil texture variability (including grain shapes) some chaotic effects might occur. However, sensitivity analysis using the mathematical model did not show such great variability in output data as obtained in experiments.

Further validation using laboratory and field data is needed.

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