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The Numerical Solution for the Problem of Heat and Mass Flow in the Soil Heated with Warm Air

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Abstract

In this paper a difference method of solving the system of differential equations is presented for differential equations, describing the distribution of temperature and water content in the greenhouse substrate heated with a system of heating pipes. The algorithm of solving the proposed method (explicite-implicite difference scheme) is presented. In addition, the effects of temperature and water content changes obtained from the solution of proposed the model as well as the model where the thermal diffusion of mass was included were compared.

Notation

a – coefficient of heat diffusion,
c – water vapour content in the heating air,
c_m – water vapour content in the soil,
c_{lm} – water vapour content in the air directly containing soil particles,
c_l – water vapour content in the ambient air,
$\mathbf{n}$ – normal to the pipe surface,
u_g – water content of the soil,
C – volumetric thermal capacity,
D – coefficient of moisture diffusion of the soil water,
D_T – thermal moisture diffusivity as vapour and liquid,
D_ϑ – isothermal moisture diffusivity as vapour and liquid,
K – conductivity of the soil water,
L – latent heat of evaporation of water,
$S(u_g)$ – water consumption of by plants per unit,
T – temperature of the heated air,

T_g – temperature of the soil,
T_1 – ambient temperature,
α, α_l – heat transfer coefficient in natural and forced convection,
α_m, α_{lm} – mass transfer coefficient in natural and forced convection,
λ – thermal conductivity of the soil,
κ_w – specific heat of the soil,
τ – time,
χ_1, χ_2, η_1, η_2, s_1, s_2 – technical date of the repeatable element of the soil heating system.

1. Introduction

There are many factors influencing the crop yield of plants grown under the monitored conditions in greenhouses; the optimum temperature of greenhouse beds is, for example, a requisite. Heating by warm air is one of the technical solutions to carry out the target function (raising the soil temperature). Warm air is fed into a soil through perforated pipelines buried in it (Bernier at all. 1988; Boulard et al. 1989; Kurpaska, Ślipek 1994; Mavroyanopoulos, Kyristis 1986). The heated and undersaturated (with water vapours) air flows through soil pores and capillaries causing the space-timely increase of soil temperature. This proceeds until a thermodynamic quasi-equilibrium is obtained. Another physical phenomenon which does not depend on temperature changes and which occurs during the heating process is continuous development of a drying front drifting towards the bed layers lying at a distance from the symmetry axis of the perforated pipelines.

All the factors influencing the physical phenomena generated while heating the soil by warm air can be divided into the following categories:

- physical parameters of heating air (temperature – T, water content in air – c) and ambient air (temperature – T_1 and water content – c_1), and (in consequence) the heat and mass transfer coefficients on the boundary surface: the upper layer of the soil (the environment) and the heating pipeline (surrounding soils) α_1, α_{1m}, α, α_m,
- technical data of the heating pipe installation (the diameter, the depth of heating pipelines location – d, h),
- soil material features (thermophysical properties and water conductivity of soils – a, κ_w, λ, D, $K = f(u_g)$).

Physical processes (heat and mass transfer) generated during the flow of undersaturated air through soil pores and capillaries constituted the objective for mathematical modelling, however, in those formulated models, the heat and the mass transfer were separately considered (Boulard, Balie 1986; Hanks et al. 1971; Van Keulen, Hillel 1974; Brandt et al. 1971; Van der Ploeg 1974; Walker 1982).

Ahmed et al. (1983), Bruger (1984), Kindelan (1980), Merbaun et al. (1983), Parker (1981), Pile et al. (1978) and Puri (1987) analyzed the combined heat and mass transfer in soils heated by a system different from the one as described in the present paper.

To describe the simultaneous heat and mass transfer, the authors employed a model evolved by Philip and de Vries (1957) and based on the basic hypothesis assumed by them that the combined thermodiffusive mass transfer occurs in form of fluid and vapour. Due to lack of complete characteristics of the convective heat and mass transfer coefficients (thermal and isothermal coefficients of liquid and vapour diffusivity) in the relevant literature it is rather difficult to adequately and fully utilize the cited model.

The analyzed horticultular substrate (usually utilized in a greenhouse production) consisted of the following ingredients: peat, tree bark and pearlite. Thus, it is impossible to determine the convective transfer coefficient with a theoretical method, and an experimental identification seems to be a very uphill task requiring a specialized equipment (Jury and Miller 1974).

Because of these impediments, an optional model (based on equations of heat and mass convective transfer, generally employed in drying technology) is introduced in the present paper; it describes physical processes occurring while heating the soil beds. The convective heat and mass transfer (in natural and forced convection) was determined from the correlation equations implemented in chemical engineering.

The analysis was performed in a two-dimensional repeatable element of the heating system under analysis (Fig. 1). This means that the entire studied area of heating installation can be covered with sub-areas conforming to border insulation terms. The perforated heating tube was placed in the symmetry axis line of repeatable element. Thus, there is no gradient of water content nor of temperature (Neumann's conditions) contained in the symmetry axis between the heating pipelines. Heat and mass transfer within the analyzed bed was described and verified for the repeatable element which was constructed as a soil duct (canal) with insulated (by polystyrene and PE foil) bottom and side parts. A diagrammatic drawing of the test stand is shown on Fig. 1.

The description of processes occurring in the heated substratum shall be carried out relying on the concepts proposed in the theory presented in Luikov (1975). According to this theory, the local change of water content (du_g) in the heated substratum consists of two effects: the transport of liquid phase (du_{gL}) and water steam developed in effect of phase transformation of soil water (du_{gV}). Thus the following formula applies:

$$du_g = du_{gL} + du_{gV}. \tag{1}$$

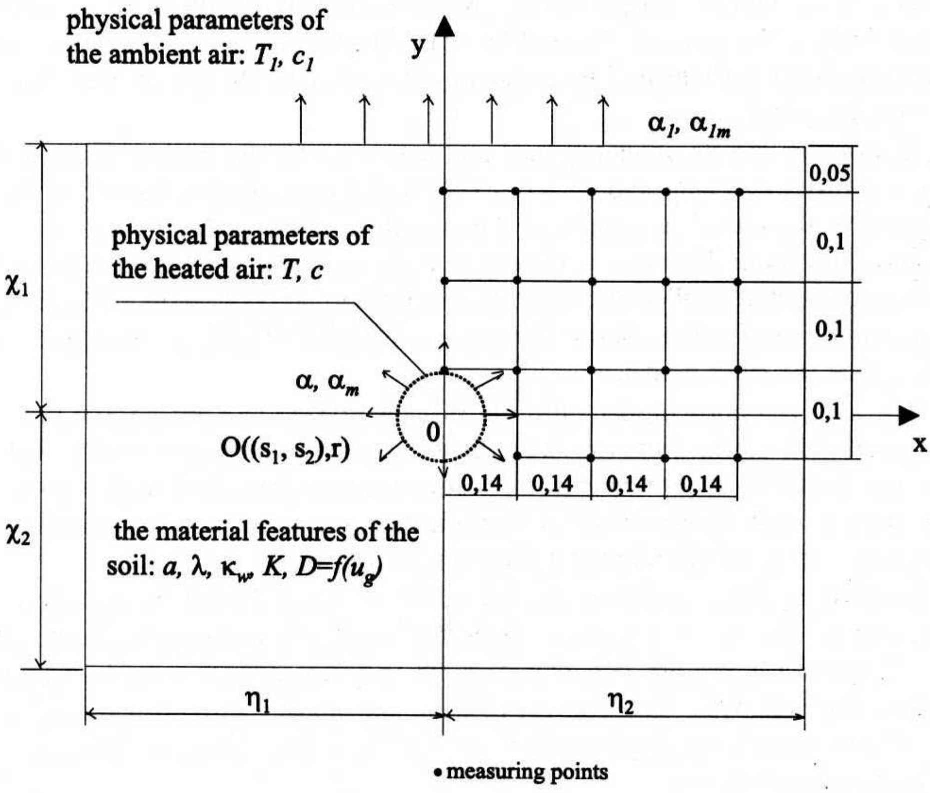

Fig. 1. Analyzed repeatable element of the soil heating system

Considering the conversion of water into water steam (ε) in the form $\varepsilon = du_{gV}/du_g$ while $0 \leq \varepsilon \leq 1$, equation (1) can be formulated as follows:

$$du_g = du_{gL} + \varepsilon du_g. \tag{2}$$

Applying the law of mass conservation in the body (of ρ_s density) and finite volume V limited by the surface A, the non-stationary field of water content inside it can be defined by the following equation:

$$\rho_s = \frac{du_g}{d\tau} = -div\ qu_g + \varepsilon\rho_s\frac{du_g}{d\tau}. \tag{3}$$

Considering the relation defining the stream of mass, with respect to the liquid phase (qu_g), in the form: $qu_g = -D_L\rho_s(\nabla u_g + \delta\nabla T_g)$, having applied proper transformations and adopted the assumption that the value of thermal gradient coefficient of water transport (δ) in the course of heating remain quasi-constant, the following is received:

a) for liquid phase:

$$\frac{\partial u_g}{\partial \tau} = div\left(D_L \nabla u_g + D_L \delta \nabla T_g\right) + \varepsilon \frac{du_g}{d\tau}; \tag{3a}$$

b) for water steam:

$$\frac{\partial u_g}{\partial \tau} = div\left(D_V \nabla u_g + D_V \delta \nabla T_g\right), \tag{3b}$$

where: D_L, D_V are respectively the coefficient of mass diffusion in the form of liquid (D_L) and steam (D_V).

Having adopted the assumption about insignificantly low impact of temperature gradient in the studied space V upon the water transport process, and combining equations 3a and 3b (according to equation 1), the classic Fick law formulation is received, which - for the iso-thermal process of water transport in the body of capillary, colloidal and porous structure, adopts the following form:

$$\frac{\partial u_g}{\partial \tau} = D \nabla^2 u_g. \tag{3c}$$

In the case of substratum (where D is coefficient of moisture diffusion of the soil water) both in the form of liquid and water steam is the function of water content in substratum) the following is obtained:

$$\frac{\partial u_g}{\partial \tau} = \frac{\partial}{\partial x}\left[D(u_g)\frac{\partial u_g}{\partial x}\right] + \frac{\partial}{\partial y}\left[D(u_g)\frac{\partial u_g}{\partial y}\right] - S(u_g). \tag{4}$$

This equation, when supplemented with source element, defining the consumption of water by the plant, was verified by Watson (acc. Kaniewska and Kowalik, 1979); the authors obtained satisfactory consistency between the solution and measurements.

The differential equation defining the temperature field in studied body volume V, results of the assumption that the stream of heat used in heating the water-steam developed by phase transformation of soil water, is insignificantly small in comparison to the heat required to warm up the liquid phase, while the temperature of liquid and steam in every point of the body is equal to its temperature. With these assumptions, the change of enthalpy in the finite volume V is equal to the sum of the divergence of the heat stream vector and the power of internal stream of heat source (q_v). The equation defining temperature field must consider both the heat stream contained within the material and the stream developed through the existence of internal heat source generated by water appearing in the process of phase transformation. Similarly to the above formulated equation of water movement, the equation defining temperature field in heated body can be presented as follows:

$$\frac{\partial (c_w \rho T_g)}{\partial \tau} = \nabla(\lambda \nabla T_g) \pm q_v \varepsilon \rho_s \frac{\partial u_g}{\partial \tau}. \tag{5}$$

Formally, the (+) sign standing before the second element of equation (5) appears in the case when heat is generated in the process of steam condensation, while (−) sign appears in the case of receiving heat from the body in the course of evaporation.

Considering the relation defining the heat source, and having introduced the concept of substratum heat capacity L, the specific heat of the soil κ_w, after the assumption about iso- thermal character of λ coefficient is adopted, and in result of transformations written in equation (5), the following is obtained:

$$\frac{\partial T_g}{\partial \tau} = a\nabla^2 T_g + \varepsilon \frac{L}{\kappa_w} m \frac{\partial u_g}{\partial \tau}. \tag{6}$$

Thus, the description of non-stationary fields of water content and temperature in studied process relies on equations 4 and 6. Equation 6 is often used in the process of drying the agricultural produce (Pabis 1983).
Initial conditions:

$$T_g\ (x, y, 0) = T_{g0} = \text{const.} \tag{7}$$

$$u_g\ (x, y, 0) = u_{go} = \text{const.} \tag{8}$$

The real heat and mass transfer, having a convective character from heated air to the soil bed particles, was included in the boundary conditions in form of:

$$\lambda \frac{\partial T_g}{\partial y} = \alpha_1 (T_g - T_1) \quad \text{and} \quad K \frac{\partial u_g}{\partial y} = \alpha_{1m}(c_{1m} - c_1) \quad \text{at}: \ y = \chi_1; \tag{9}$$

$$\frac{\partial T_g}{\partial x} = 0 \quad \text{and} \quad \frac{\partial u_g}{\partial x} = 0 \quad \text{at}: \ x = -\eta_1 \ \text{and} \ x = \eta_2; \tag{10}$$

$$\frac{\partial T_g}{\partial y} = 0 \quad \text{and} \quad \frac{\partial u_g}{\partial y} = 0 \quad \text{at}: \ y = -\chi_2; \tag{11}$$

$$-\lambda \frac{\partial T_g}{\partial \mathbf{n}} = \alpha (T - T_g) \quad \text{and} \quad -K \frac{\partial u_g}{\partial \mathbf{n}} = \alpha_{1m}(c_m - c) \quad \text{at}: \ O((0,0), r); \tag{12}$$

where:
$O((s_1, s_2), r) := \{(x, y) : x^2 + y^2 = r^2\}$,

T_g – temperature of the soil, °C,
u_g – water content of the soil $\text{kgkg}^{-1}_{\text{dry soil}}$,
L – latent heat of evaporation of water, Jkg^{-1},
$S(u_g)$ – water consumption of by plants per unit. In our case, $S(u_g) = 0$,
$\mathbf{n}$ – normal to the pipe surface, m,
$\chi_1, \chi_2, \eta_1, \eta_2, s_1, s_2$ – technical date of the repeatable element of the soil heating system (see Fig. 1), m,
a – coefficient of heat diffusion, m^2s^{-1},

λ	– thermal conductivity of the soil, $Wm^{-1}K^{-1}$,
κ_w	– specific heat of the soil, $Jkg^{-1}K^{-1}$,
D	– coefficient of moisture diffusion of the soil water, m^2s^{-1},
K	– conductivity of the soil water, ms^{-1},
α, α_1	– heat transfer coefficient in natural and forced convection, $Wm^{-2}K^{-1}$,
α_m, α_{1m}	– mass transfer coefficient in natural and forced convection, ms^{-1},
c_m, c_{1m}	– water vapour content in the soil and air directly contacting soil particles, kgm^{-3},
c, c_1	– water vapour content in heating air (c) and ambient air (c_1), kgm^{-3}.

The quantities c_m, c_{1m}, depending on soil water potential, were calculated from the Kelvin's relation (Hanks and Ashcroft, 1980).

The water vapour content in heated air (c, c_1) was calculated from a standard psychrometric relation. The coefficients of heat transfer by natural and forced convection were computed for a mean temperature (arithmetic mean of soil and flowing air). More details referring to assumed simplifying presumptions – among other things – are contained in the work by Kurpaska and Ślipek (1996). A diagram of complementary interdependencies among the presented systems is shown on Fig. 2.

The model ensued involves physical parameters of heated and ambient air, technical data of the installation, as well as thermophysical and water properties of horticultural beds, thus, it can be employed to study the heat and mass transfer in horticultural beds by use of computer simulation. However, the suggested mathematical model of a soil heating system displays a strong conjugation between the equations of heat and mass diffusion. Moreover, soil material features and coefficients of convective heat and mass transfer depend on soil humidity. As a consequence, the analyzed system becomes a non-linear system. Also, nonlinearity of the investigated issue derives from the fact that thermo and hydrophysical characteristics are a function of soil bed humidity changing within the process proceeding. The conclusion is that an efficient numerical algorithm should be formed for the purpose to pinpoint the values of temperature and water content in soil. This is precisely the main objective of the study.

2. Presentation of the Differential Problem

Let us express the system of the equations ($4, 6 \div 12$) in a compact form in order to facilitate investigation of the developed numerical method.

Let $\mathbf{R} \supset \overline{H} := [0, a_1] \times [0, a_2] \setminus F((s_1, s_2), r)$, where $a_i > 0$, $s_i\,(i = 1, 2)$, $r > 0$ given real numbers, while $F((s_1, s_2), r) := \left\{(x_1, x_2) : x_1^2 + x_2^2 < r^2\right\} \subset [0, a_1] \times [0, a_2]$. Let $T > 0$ be a given number.

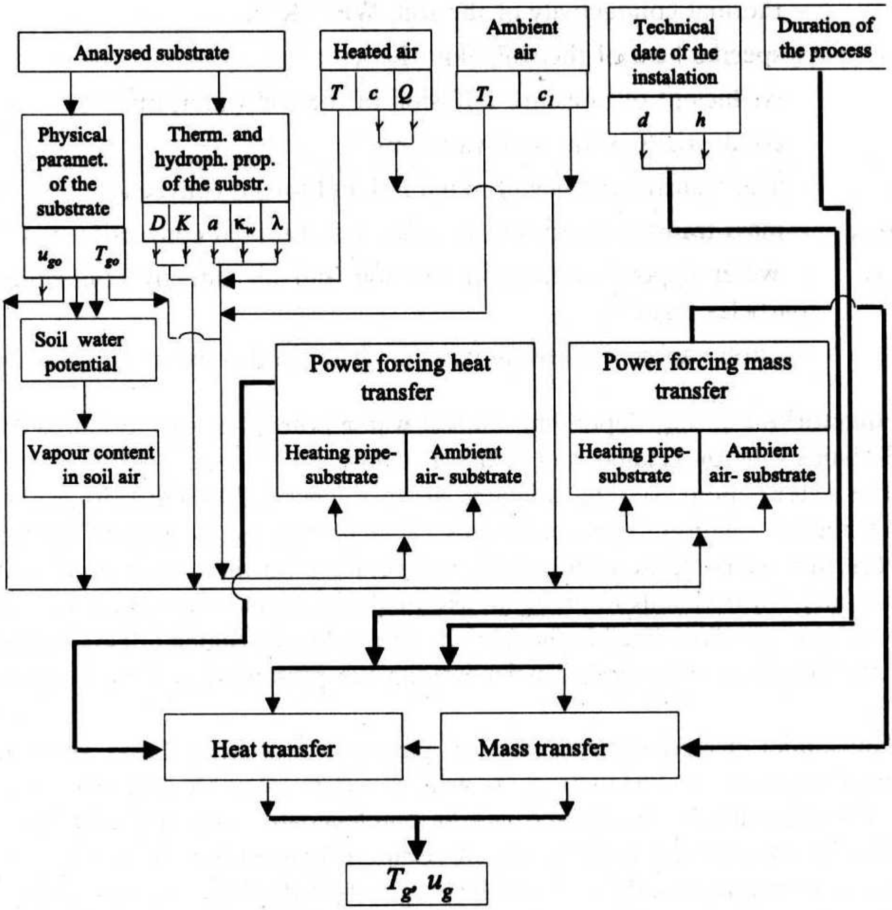

Fig. 2. Diagram of complementary interdependencies among presented systems Fig. 1

We consider a system of differential equations:

$$\begin{cases} \dfrac{\partial u_1}{\partial \tau}(\tau, x) = f_1\left(u_2(\tau, x)\dfrac{\partial u_2}{\partial \tau}(\tau, x), (\Delta u_1)(\tau, x)\right)(\tau \in (0, T], x \in H), \\ \dfrac{\partial u_2}{\partial \tau}(\tau, x) = f_2\left(u_2(\tau, x), (\Delta u_2)(\tau, x)\right)(\tau \in (0, T], x \in H), \end{cases} \tag{13}$$

with initial conditions:

$$u_1(0, x) = u_{i0}(x) \quad (i = 1, 2, \;\; x \in H), \tag{14}$$

and boundary conditions in a form:

$$\begin{cases} \dfrac{\partial u_1}{\partial \mathbf{n}}(\tau, x) = 0 \\ \left(i = 1, 2,\ \tau \in (0, 1],\ x \in \{0\} \times [0, a_2) \cup [0, a_1] \times \{0\} \cup \{a_1\} \times [0, a_2]\right), \\ \dfrac{\partial u_1}{\partial \mathbf{n}}(\tau, x) = g_1\left(u_1(\tau, x), u_2(\tau, x)\right) \\ \left(\tau \in (0, 1],\ x \in [0, a_1] \times \{a_2\} \cup O\left((s_1, s_2), r\right)\right), \end{cases} \tag{15}$$

where $x = (x_1, x_2)$, $(\Delta u_1)(\tau, x) := \frac{\partial^2 u_1}{\partial x_1^2}(\tau, x) + \frac{\partial^2 u_i}{\partial x_2^2}$ $(i = 1, 2)$. It is essential to note, that on the boundary of the rectangle $[0, a_1] \times [0, a_1]$ the normal outward derivative of the boundary of the domain is equal to the partial derivative of the corresponding variable.

Digitizing and difference method.

Let $N_i \in \mathbf{N}$ $(i = 1, 2)$ be given natural numbers. We define $k_i := a_i \dot{N}_i^{-1}$ $(i = 1, 2)$.

A set of points

$$S := (A_1 \cap A_2) \cup \left((A_1 \cup A_2) \cap O\left((s_1, s_2), r\right)\right), \tag{16}$$

where

$$\begin{cases} A_1 := \left\{0, \left(j - \frac{1}{2}\right) k_1\ (j = 1, \ldots, N), a_1\right\} \times \mathbf{R}, \\ A_2 := \mathbf{R} \times \left\{0, \left(j - \frac{1}{2}\right) k_2\ (j = 1, \ldots, N), a_2\right\} \end{cases} \tag{17}$$

we call a difference net.

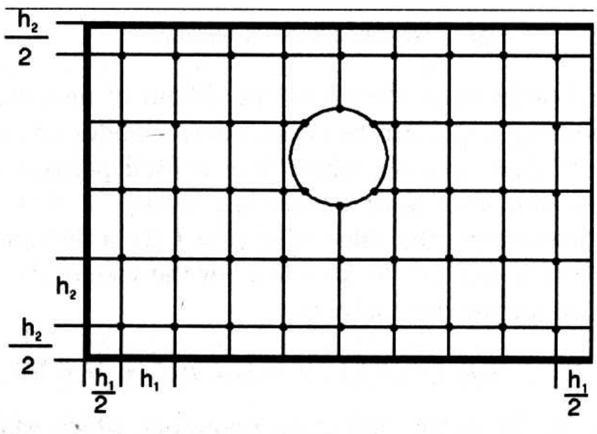

Fig. 3. Difference net

Let assume, that the points of the net S we will denote x^M, where M will be considered an index. Therefore we can write:

$$\mathsf{S} = \left\{x^M : M \in \mathsf{M}\right\}, \tag{18}$$

where M is a appropriate set of indices. We divide the nodes of the net S into interior and border nodes (Fig. 1):

$$\mathsf{S}^0 := \left\{x^M : x^M \in H^0\right\}, \partial\mathsf{S} := \left\{x^M : x^M \in \partial\ H\right\}. \tag{19}$$

This sets correspond to appropriate sets of indices, which we denote, respectively, M^0 and $\partial\mathsf{M}$. Let see, that directly from the definition a relationships

$$\mathsf{M} = \mathsf{M}^0 \cup \partial\mathsf{M},\ \ \mathsf{M}^0 \cap \partial\mathsf{M} = \emptyset \tag{20}$$

arises.

If $M \in \mathsf{M}^0$, then nodes x^M correspond exactly with four neighbouring node points, which we denote: $x^{-1(M)}$, $x^{1(M)}$, $x^{-2(M)}$, $x^{2(M)}$ and the distance between them and x^M is equal to h_{-1}, h_1, h_{-2}, h_2 – Fig. 4.

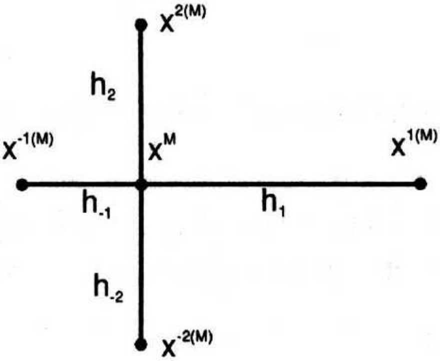

Fig. 4. Neighbourhood of nodal point x^M

If x^M is not a border node, then it is a neighbour of one, two or three nodes, while the neighbouring nodes can be either interior nodes or border nodes.

Let denote $\tau^u := \mu k$ ($\mu \in \mathbf{N}$), where k is a fixed positive number. We will further on call it a time level μ, or a μ-th time level.

Our goal is to determine the value of u_i ($i = 1, 2$) in the nodal points, i.e. the values $u_i\left(\tau^\mu, x^M\right)$ ($i = 1, 2, \mu \in \mathbf{N}, M = \mathsf{M}$). For the sake of the simplicity of next notations we introduce the denotation:

$$u_i^{\mu,M} := u_i\left(\tau^\mu, x^M\right) \quad (i = 1, 2, \mu \in \mathbf{N}, M \in \mathsf{M}). \tag{21}$$

At the beginning let define difference operators, which will be used for the approximation of the partial derivatives of the first and second order, occurring

in equation (13) and in boundary conditions (15).
For $\mu \in \mathbf{N}$, $M \in \mathsf{M}^0$, $(i, j = 1, 2)$ we determine:

$$\frac{\partial u_i}{\partial \tau}(\tau^\mu, x^M) \approx k^{-1}\left(u_i^{\mu+1,M} - u_i^{\mu,M}\right) =: u_i^{+\mu,M}, \tag{22}$$

$$\frac{\partial u_i}{\partial x_j}(\tau^\mu, x^M) \approx (h_j + h_{-j})^{-1}\left(u_i^{\mu,j(M)} - u_i^{\mu,-j(M)}\right) =: u_i^{\mu,Mj}, \tag{23}$$

$$\begin{aligned}\Delta u_i(\tau^\mu, x^M) &\approx \sum_{j=1}^{2} \frac{h_j^{-1}\left(u_i^{\mu,j(M)} - u_i^{\mu,M}\right) + h_{-j}^{-1}\left(u_i^{\mu,M} - u_i^{\mu,-j(M)}\right)}{0.5(h_j + h_{-j})} = \\ &= : u_i^{\mu,M\oplus}.\end{aligned} \tag{24}$$

Let notice, that on a basis of initial conditions (14) we have:

$$u_i^{0,M} = u_{i0}(x^M) \quad (i = 1, 2, M \in \mathsf{M}). \tag{25}$$

Further on the problem is, that knowing the values of the functions u_1 $(i = 1, 2)$ in the points (τ^μ, x^M) $(M \in \mathsf{M})$, i.e. on the μ-th time level, find the values of this functions in the points $(\tau^{\mu+1}, x^M)$ $(M \in \mathsf{M})$, i.e. on the $(\mu + 1)$-th time level.

Let assume than, that the values of $u_i^{\mu,M}$ $(i = 1, 2,\ M \in \mathsf{M})$ are known. In nodes $x^m \in \mathsf{S}^0$ we approximate the differential equations (6) using difference equations:

$$\begin{cases} u_1^{+\mu,M} = f_1\left(u_2^{\mu,M}, u_2^{+\mu,M}, u_1^{\mu,M\oplus}\right), \\ u_2^{+\mu,M} = f_2\left(u_2^{\mu,M}, u_2^{\mu,M\oplus}\right) \end{cases} \tag{26}$$

These equations can be written in an equivalent form:

$$\begin{cases} u_1^{\mu+1,M} = u_1^{\mu,M} + kf_1\left(u_2^{\mu,M}, u_2^{+\mu,M}, u_1^{\mu,M\oplus}\right), \\ u_2^{\mu+1,M} = u_2^{\mu,M} + kf_2\left(u_2^{\mu,M}, u_2^{\mu,M\oplus}\right) \end{cases} \tag{27}$$

Let notice, that $\forall M \in \mathsf{M}^0$ known are all the values on the right hand side of the second of specified equations. It permits us to calculate all the values of $u_2^{\mu+1,M}$ $(M \in \mathsf{M}^0)$. Then, however $\forall M \in \mathsf{M}^0$, known are all the values on the right hand side of the first of the specified equations, what allows us to calculate the values of $u_2^{\mu+1,M}$ $(M \in \mathsf{M}^0)$.

On the 'left hand' boundary of the set H (Fig. 5), i.e. in the nodes $x^M \in \{0\} \times (0, a_2)$ we approximate the boundary conditions (14) employing equations :

$$-h_1^{-1}\left(u_i^{\mu+1,1(M)} - u_i^{\mu+1,M}\right) = 0 \quad (i = 1, 2), \tag{28}$$

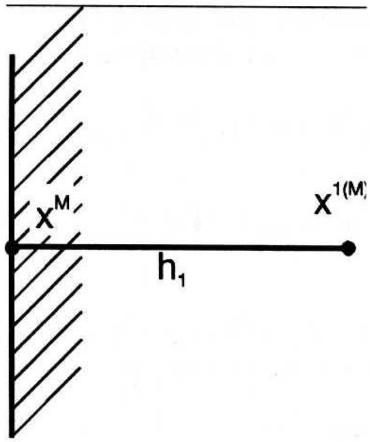

Fig. 5. The surrounding of the node on the left hand limit of the set H

which induce the equalities:

$$u_i^{\mu+1,M} = u_i^{\mu+1,1(M)} \quad (i = 1, 2,\ x^M \in \{0\} \times (0, a_2)). \tag{29}$$

In the same way we obtain the following equalities:

$$u_i^{\mu+1,M} = u_1^{\mu+1,-1(M)} \quad (i = 1, 2,\ x^M \in \{a_1\} \times (0, a_2)), \tag{30}$$

$$u_i^{\mu+1,M} = u_i^{\mu+1,2(M)} \quad (i = 1, 2,\ x^M \in [0, a_1] \times \{0\}). \tag{31}$$

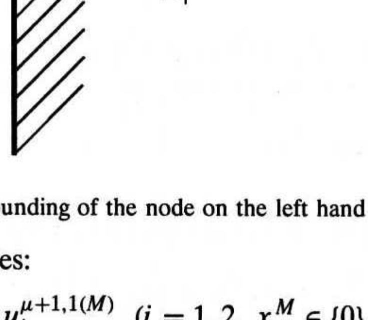

Fig. 6. The surrounding of the node on the upper limit of the set H

On the 'upper' side of the rectangle H (Fig. 6), i.e. in the nodes $x^M \in [0, a_1] \times \{a_2\}$ we approximate the boundary conditions (15) using equations:

$$-h_{-2}^{-1}\left(u_i^{\mu+1,M} - u_i^{\mu+1,-2(M)}\right) = g_i\left(u_1^{\mu+1,M}, u_2^{\mu+1,M}\right) \quad (i = 1, 2). \tag{32}$$

The unknown values, which shall be determined are $u_i^{\mu+1,M}$ $(i = 1, 2)$. Therefore in each of the nodes $x^M \in [0, a_1] \times \{a_2\}$ it is necessary to solve a system of two non-linear equations with two unknowns.

The values $u_i^{\mu+1,M}$ in the nodal points $x^M \in O((s_1, s_2), r)$ still need to be determined.

At the beginning we shall notice, that if $x^M \in (x_1^M, x_2^M) \in O((s_1, s_2), r)$, then $\mathbf{n}_M$ normal versor outward to the boundary exposed at the point x^M is equal to

$$\mathbf{n}_M = (n_{1,M}, n_{2,M}) := = \left(\frac{s_1 - x_1^M}{\sqrt{(s_1 - x_1^M)^2 + (s_2 - x_2^M)^2}}, \frac{s_2 - x_2^M}{\sqrt{(s_1 - x_1^M)^2 + (s_2 - x_2^M)^2}} \right). \tag{33}$$

Let define a node x^P placed on the circle $O((s_1, s_2), r)$. With this node is matched one node x^{M_0}, which we call a neighbour of the node x^P and three nodes adjacent to the node x^{M_0} different from the node x^P (Fig. 7), which we denote $x^{M_1}, x^{M_2}, x^{M_3}$. Among the nodes $x^{M_1}, x^{M_2}, x^{M_3}$ we choose that one (denoted x^{M_*}), which meets the condition:

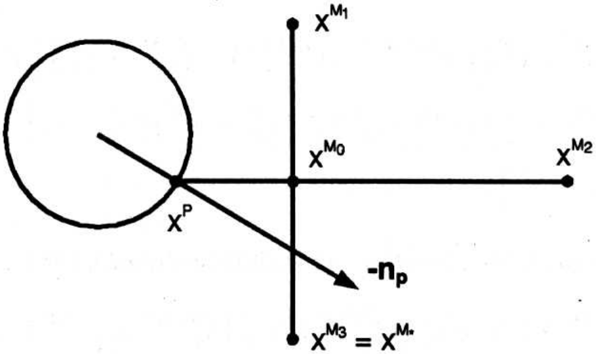

Fig. 7. surrounding of the node placed on the edge of the heating pipe

$$\left| \cos \left(\overrightarrow{x^P x^{M_*}}, -\mathbf{n} \right) \right| = \max_{i=1,2,3} \left| \cos \left(\overrightarrow{x^P x^{M_*}}, -\mathbf{n} \right) \right|. \tag{34}$$

Planes described by the equation

$$z_i(x_1, x_2) = u_i^{\mu+1,P} + \begin{vmatrix} x_1^{M_0} - x_1^P & x_1^{M_*} - x_1^P \\ x_2^{M_0} - x_2^P & x_2^{M_*} - x_2^P \end{vmatrix}^{-1} \times \\ \times \left(\begin{vmatrix} x_1 - x_1^P & x_1^{M_*} - x_1^P \\ x_2 - x_2^P & x_2^{M_*} - x_2^P \end{vmatrix} \left(u_i^{\mu+1,M_0} - \mu_1^{\mu+1,P} \right) + \\ + \begin{vmatrix} x_1^{M_0} - x_1^P & x_1 - x_1^P \\ x_2^{M_0} - x_2^P & x_2 - x_2^P \end{vmatrix} \left(u_i^{\mu+1,M_0} - \mu_1^{\mu+1,P} \right) \right) \quad (i = 1, 2) \tag{35}$$

crosses the points $\left(x^P, u_i^{\mu+1,P}\right)$, $\left(x^{M_0}, u_i^{\mu+1,M_0}\right)$, $\left(x^{M_*}, u_i^{\mu+1,M_*}\right)$.

Using the derivative $\frac{\partial u_1}{\partial \mathbf{n}}\left(\tau^{\mu+1}, x^P\right)$ we approximate the derivatives $\frac{\partial z_i}{\partial \mathbf{n}}\left(x^P\right)$ $(i = 1, 2)$ i.e.:

$$\frac{\partial u_i}{\partial \mathbf{n}}\left(\tau^{\mu+1}, x^P\right) \approx \frac{\partial z_i}{\partial \mathbf{n}}\left(x^P\right) = n_{1,P}\frac{\partial z_i}{\partial x_2}\left(x^P\right) + n_{2,P}\frac{\partial z_i}{\partial x_2}\left(x^P\right) \quad (i = 1, 2) \tag{36}$$

Values of the derivatives $\frac{\partial z_i}{\partial x_1}(x^P)$, $\frac{\partial z_i}{\partial x_2}(x^P)$ $(i = 1, 2)$ are given by the relationships:

$$\begin{aligned}\frac{\partial z_i}{\partial x_1}(x^P) &= \left(\left(x_2^{M_*} - x_2^P\right)\left(u_i^{\mu+1,M_0} - u_i^{\mu+1,P}\right) - \left(x_2^{M_0} - x_2^P\right)\left(u_i^{\mu+1,M_*} - u_i^{\mu+1,P}\right)\right) : \\ &: \left(\left(x_1^{M_0} - x_1^P\right)\left(x_2^{M_*} - x_2^P\right) - \left(x_2^{M_0} - x_2^P\right)\left(x_1^{M_*} - x_1^P\right)\right) = \\ &=: \varphi_1\left(u_i^{\mu+1,P}\right),\end{aligned} \tag{37}$$

$$\begin{aligned}\frac{\partial z_i}{\partial x_2}(x^P) &= \left(\left(x_1^{M_0} - x_1^P\right)\left(u_i^{\mu+1,M_*} - u_i^{\mu+1,P)}\right) - \left(x_1^{M_*} - x_1^P\right)\left(u_i^{\mu+1,M_0} - u_i^{\mu+1,P}\right)\right) : \\ &: \left(\left(x_1^{M_0} - x_1^P\right)\left(x_2^{M_*} - x_2^P\right) - \left(x_2^{M_0} - x_2^P\right)\left(x_1^{M_*} - x_1^P\right)\right) = \\ &=: \varphi_2\left(u_i^{\mu+1,P}\right).\end{aligned} \tag{38}$$

In the node x^P boundary conditions (14) induce therefore the equations

$$n_{1,P}\ \varphi_1\left(u_i^{\mu+1,P}\right) + n_{2,P}\ \varphi_2\left(u_i^{\mu+1,P}\right) = g_i\left(u_1^{\mu+1,P}, u_2^{\mu+1,P}\right) \quad (i = 1, 2) \tag{39}$$

This is a system of two strongly non-linear algebraic equations with the unknown

$$u_i^{\mu+1,P} \quad (i = 1, 2).$$

By that the process of construction the $u_i^{\mu+1,M}$ $(i = 1, 2, M \in \mathrm{M}$ values has been completed.

The presented difference scheme is an explicite scheme in all interior nodes and implicite in all boundary nodes. The difference scheme must be compatible, convergent and stable, to be useful for calculations - meaning that the values generated by it approach at the limit the solution of differential equations approximated by that scheme (Keller 1971). We employ here only some mental short cuts. The compatibility of the scheme means, that it approximates the given differential problem. In case of the scheme presented here this property is quite easy to prove, assuming the regularity of the functions f_i, g_i $(i = 1, 2)$ and compatibility of the boundary conditions and initial conditions at the time $\tau = 0$. The stability of the difference scheme, because of the non-linearity of the differential problem, is here considered to be a continuous relationship between the solution of differential problem and the right hand sides of the equations as well as the initial and boundary conditions. This is also relatively easy to prove. Unfortunately – for nonlinear problems the compatibility and stability of the difference scheme do not guarantee its convergence. The formal proof of the convergence of the presented scheme has not been made yet, although all the premises suggest that the scheme is convergent, provided that the ratio of the time step k to the shortest spatial step is sufficiently small ($<< 1$).

While conducting the calculations, one shall take care not to allow too large disproportion of the spatial steps. The inconvenient situation can crop up practically only in the neighbourhood of nodes located on the circle $O((s_1, s_2)r)$. In such situation it shall be enough to change slightly the position of the circle or to change the length of the spatial step.

3. Numerical Example

Simulation studies on the distribution of temperature and water content in soil beds were performed for:

- technical data of the installation: $d = 0.05$ [m], $h = 0.3$ [m]
- physical parameters of heated and ambient air:
  $T = 32$ [°C], $T_1 = 13$ [°C],
  $c = 0.0135$ [kgm^{-3}], $c_1 = 0.0079$ [kgm^{-3}],

Thermo, hydrophysical characteristics and soil water potential of the presented substrate were determined in a laboratory (Kurpaska et al., 1996; Kurpaska, 1993). The obtained experimental data pertaining to the thermal and hydro-physical features were fitted with approximation curves of obtained values. The course of such changes is defined by following relations:

$a = 0.95 + 9.8u_g 18.4u_g^2 + 5.7u_g^3$

$\lambda = 0.033 + 0.28u_g + 5.5u_g^2 9.2u_g^3$

$\kappa_w = 620.1 + 4001.7u_g$

$K = \exp(-27.17 + 90.7u_g)$

$D = \exp(-22.38 + 27.6u_g)$

Isotherms of temperature and water content are shown on Fig. 8; they were obtained from the numerical solution upon expiration of periods of 32.4; 108.6 and 156 h (i.e. upon reaching the quasi-stationary state).

Tests verifying changes of temperature and water content in analyzed soil beds were executed in a soil canal of 5 m length, 1.2 m width, and 0.6 m height. The results of tests verifying the bed humidity and bed temperature measurements, made in spots located in the soil canal, are presented on Fig. 9. These results were compared with quantities computed as a solution for spots exhibiting the

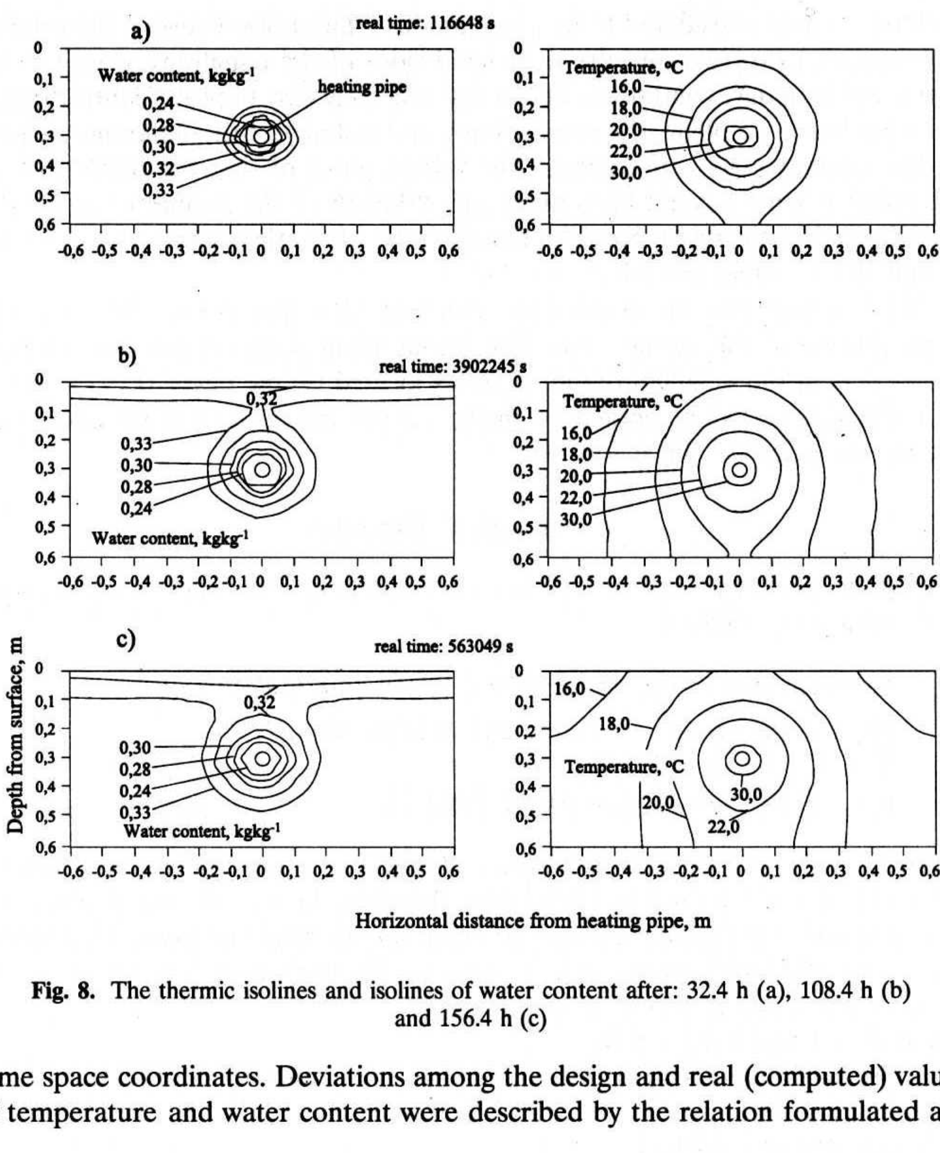

Fig. 8. The thermic isolines and isolines of water content after: 32.4 h (a), 108.4 h (b) and 156.4 h (c)

same space coordinates. Deviations among the design and real (computed) values of temperature and water content were described by the relation formulated as:

$$\sigma = \left[\sum_{i=1}^{n} \frac{(E_{calc.}, i(\tau) - E_{meas.}, i(\tau))^2}{n} \right]^{0.5}, \tag{40}$$

where: $E_{calc.}, i(\tau)$ – and $E_{meas.}, i(\tau)$ – values of temperature and water content obtained from the numerical model ($E_{calc.}$) and from measurements ($E_{meas.}$) after the same time (τ), and n is the number of comparison.

It is apparent that the mean square error (σ) for temperature and water content is 0.85°C and 0.02 kg/kg, respectively. A relatively bigger difference between the measured and computed quantities of water content in soil beds should be

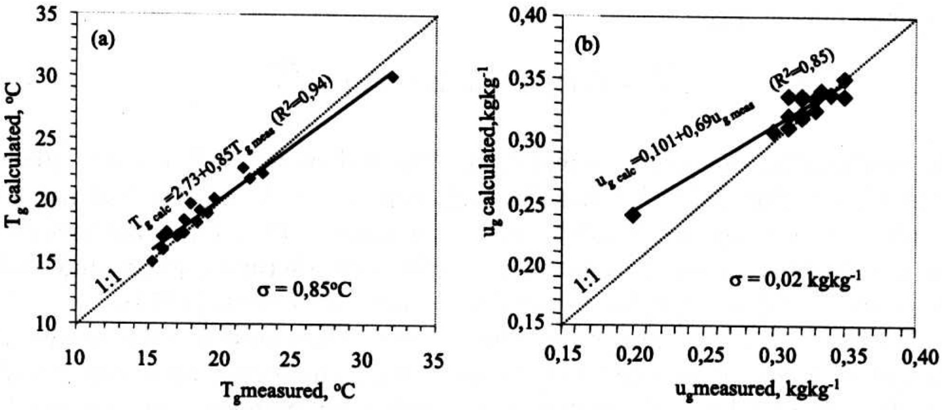

Fig. 9. Comparison of calculated values from the model with measuerd values for (a) temperature and (b) water content of the soil

referred to the fact that a mass stream has not been included in the thermodiffusive model. In a general case, heat transfer through soil pores and capillaries in a bed being dried occurs by convection, and the mass stream - by convection in a liquid and vapour phases. A theory interpreting this process, the evaporation-condensation hypothesis (Przesmycki, Strumiłło, 1983), assumes that in a period with a constant drying speed, the simultaneous heat and mass transfer in the boundary layer (mainly in a liquid phase) plays the dominant role, whereas the transfer (convection) conditions inside the soil bed (in a gaseous phase) are decisive while the drying speed is decreasing.

4. The Physical Parameters of Air Heated Soil, Obtained from the Proposed and the Thermal-diffusion Model of Heat and Mass Exchange

In order to check to what extent the proposed model (drying model) reflects correctly the processes occurring in the course of heating soil with warm air, comparative analysis was effected of temperature changes and soil water content levels obtained from the solution of Philip and de Vries model. The selection of this model was motivated by the fact of significant divergence of results obtained from the solution of such model - and the empirical measurements (see e.g. Ahmed et al. 1983; Puri 1987). The heat movement in Philip and de Vries model is defined by the following formula:

$$\frac{\partial T}{\partial t} = a\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) - \frac{L}{C}\nabla(D_{\theta_V}\nabla\theta) \tag{41}$$

and mass flow in form:

$$\frac{\partial \theta}{\partial t} = \nabla(D_T \nabla T) + \nabla(D_\theta \nabla \theta) + \frac{\partial K}{\partial y}. \tag{42}$$

The comparative analysis of temperature and soil moisture levels was effected for sand. The thermal and hydro-physical properties (D_θ – isothermal moisture diffusivity as vapour and liquid or only as vapour – $D_{\theta V}$; D_T – thermal moisture diffusivity as vapour and liquid; C – volumetric thermal capacity and earlier mentioned a and K) were taken from Jurry and Miller study (1973).

The above discussed model (equations 4 and 6) was used in order to show the changes of temperature and soil moisture. These equations (equations 6 and 4, as well as 41 and 42) were supplemented with initial-boundary conditions (third type boundary conditions), identical to equations applied in the simplified model (equations 7-12).

The difference between both applied models lies exclusively in the values of heat and mass penetration coefficients, since – in the simplified model - effective coefficients were used reflecting the interfering elementary flows of heat and mass. These coefficients were obtained from correlation equations applied in dehydration theory involving the simultaneous flow of heat and mass. The comparison of both models (equations 6 and 4 , 41 and 42) leads to the conclusion that the difference between them lies also in their internal structure, since the equation defining the mass flow (Eq. 4) neglects the thermal diffusion of soil water (the thermal diffusion effect) and the gravitation element; since the soil moisture (θ) was used instead of weight moisture (u_g), the thermal capacity of sand (C) was used instead of the specific heat (κ_w).

The differential method was used also to solve the Philip and de Vries model with the attached border terms; the algorithm of applied method is discussed in detail in Chapter 2. Figures 10a and 10b show the results of simulation analysis of temperature changes (Fig. 10a) and water content changes (Fig. 10b) in sand. The iso-lines show the changes of physical parameters of sand for pseudo-stable condition of heat exchange. The calculations were made for the following model parameters: simulation time (τ); $\tau = 120$ hours; the intensity of the flow of the warm air (Q); $Q = 20 \times 10^{-5}$ m^{-3}s^{-1}, temperature of the heated air (T); $T = 35$°C; ambient temperature (T_1); $T_1 = 13$°C; initial temperature of the sand (T_{g0}); $T_{g0} = 14.3$°C; initial soil moisture (θ_0); $\theta_0 = 0.34$ cm^{-3}cm^{-3}; humidity of ambient air – 70% and heated air 40%. The other values of technical parameters were adopted at the level of garden substrate. The analysis of temperature and water content iso-lines shows that while the temperature values in studied soil space have similar shape, the water content change analysis in the case of Philip and de Vries model indicates a more homogeneous field of such values. Undoubtedly, it is the consequence of the inclusion of thermal diffusion effect in the description of soil water flows. In order to assess the diversity of obtained

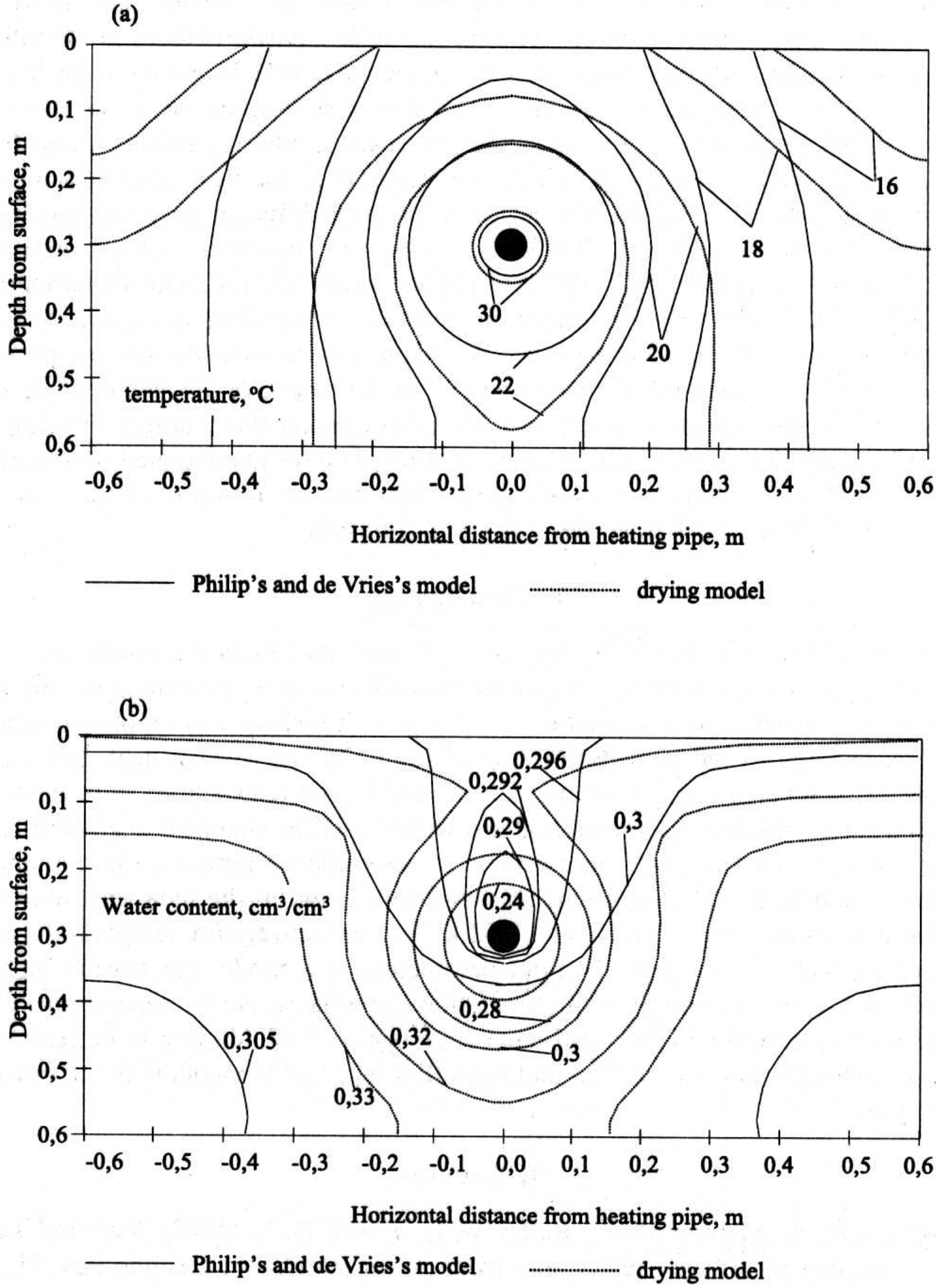

Fig. 10. The thermic isolines (a) and isolines of soil moisture (b) in the sand after 120 h calculated from Philip-de Vries and drying models

temperature and moisture content results (in pseudo-stable condition of heat exchange) schematic points were identified , the co-ordinates of which are shown in Fig. 1. The mean square error term (σ) between the analysed values in identified points (obtained from both models) is not higher than 0.53 K and 0.006 cm^3cm^{-3} – in the case of temperature changes. This fact corroborates the correctness of the adopted simplified conditions in the presented model (equations 4 and 6 $\div$ 12). The simplifications introduced in the model (e.g. the neglected flow of soil water in gravitation element, the neglected thermal diffusion element (equation 4) proved to be insignificant. Doubtless, it was the consequence of e.g. applying the most correct (physically) third type border terms, the insignificant temperature differentials between the heated air and the surrounding soil space, as well as the adoption of the situation when the heated air flowing though the porous space absorbs any amount of steam released in the course of air flow through the capillary-porous substance. The possibility of neglecting the thermal diffusion in describing the flow of soil moisture is corroborated by results obtained by Zaradny and Sutor (1974); these authors found that the thermal diffusion on the average has twice smaller values than the isothermal diffusion.

5. Conclusions

A numerical algorithm solving the issue of heat and mass transport in a soil heated by warm air with no additional humidification is presented in this paper it. To describe mass transport, a soil water diffusivity was employed which has an analogy to the heat diffusion coefficient. The convective heat and warm transport coefficients (for both the natural and forced convection) were determined from correlation equations that are widely used in chemical engineering. It was certified that there was an essential correspondence between the numerical solution and empirical measurements; deviations between the measured and calculated quantities were 0.85°C and 0.02 kg/kg, respectively for temperature and water content. The changes of input parameters of a model are feasible in the developed algorithm and thus, the simulation experiments can be carried out. This algorithm appeared to be a constructive investigative tool allowing to better study the analyzed phenomena of heat and mass flow in a capillary-colloidal and porous medium.

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