Archives of Hydro-Engineering and Environmental Mechanics
Vol. 47 (2000), No. 1–4, pp. 51–73

Determination of Elastic Moduli of Sands from Triaxial Compression Test

Waldemar Świdziński

Institute of Hydro-Engineering of the Polish Academy of Sciences, ul. Kościerska 7, 80–953 Gdańsk

(Received December 10, 1999; revised October 23, 2000)

Abstract

In the paper an empirical method of determination of elastic constants of non-cohesive soils on the basis of the experimental data from conventional triaxial compression tests, is proposed. The method is based on a new interpretation of triaxial tests during which samples of dry sands are subjected to several cycles of loading and unloading. The test results of all strain and stress components measured in the experiment are presented in terms of deviatoric stress versus deviatoric strain and mean effective pressure versus volumetric strain diagrams. It is assumed that after any stress reversals the material exhibits purely elastic response that obeys Hooke's linear law. Elastic moduli are determined from the first stage of unloading, which is different from the other methods commonly accepted in soil mechanics. The main advantage of the method proposed was isolating linear behaviour of the material that corresponds to elastic response and including in the analysis the lateral deformation of a sample. In the paper several examples of test results for various confining pressures and initial void ratios are presented and analysed. Comparison with other methods is made and discussed.

1. Introduction

Despite many proposals for determination of elastic properties of non-cohesive soils, this problem has not been sufficiently well solved. Experimental isolation of elastic response of non-cohesive soils is in general a very difficult task due to development of both reversible (elastic) and irreversible (plastic) deformation during the loading. There exists extensive literature on this subject, but all the methods proposed seem not to be sufficiently convincing to produce the most representative values describing real elastic response. The most common proposals for the determination of elastic moduli of particulate materials were extensively analysed and discussed by Świdziński, 2000b. A study embraced various methods using of different laboratory equipment and different approaches to experimentally isolate elastic response of particulate materials.

In general, the elastic deformation of non-cohesive soils is much smaller than irreversible deformation of the material. However, the information regarding the soil elastic response is of great importance for the appropriate description of soil behaviour and thus cannot be disregarded. The elasticity theory is still widely used in geotechnical engineering and the application of more-complex models based on an elasto-plastic concept must include elastic strain components in order to invert the stress-strain relations.

Experimental results indicate that the elastic behaviour of non-cohesive soils can be considered to be isotropic. Thus determination of elastic properties of a given non-cohesive soil reduces to the experimental determination of two elastic constants which can be Young's modulus and Poisson's ratio.

There exist a number of various direct and indirect methods to determine elastic constants of non-cohesive soils. Direct methods usually make use of such conventional geotechnical laboratory devices as oedometer with additional measurement of lateral stresses (Sawicki, 1994; Sawicki and Świdziński, 1997), triaxial compression apparatus (cf. e.g. Duncan and Chang, 1970; Lade and Nelson, 1987), three-dimensional cubical triaxial tests (Lade and Nelson, 1987); hollow cylinder (Dakoulas and Yuanhui Sun, 1992). Another group of direct methods adopts testing of non-cohesive soils at very small ranges of strains. Such tests are carried out in conventional, specially adapted laboratory apparatus for the measurement of small deformations (Hardin and Black, 1969; Hicher, 1996; Tatsuoka and Shibuya, 1992).

Indirect methods are usually based on dynamic testing embracing measurement of wave propagation velocities. Such tests can be performed either in the laboratory (resonant columns) or filed conditions (cross-hole or down-hole tests).

The conclusion resulting from the study of various developments in theoretical and experimental determining of elastic properties of non-cohesive soils is that at present stage of knowledge none of the methods discussed could be considered appropriate. Due to various shortcomings and inconsistencies it is difficult to indicate the most convincing method of determining elastic constants for particulate materials.

One of the main weaknesses of methods based on the interpretation of experimental data from triaxial compression or extension tests is excluding from the determination of Young's modulus the lateral deformation of the sample. It has been shown (Świdziński, 2000b) that such deformations are of the same order as axial strains and therefore cannot be ignored.

Another matter of issue regards isolation of a zone from the loading curve that could have been identified with almost elastic response of the material tested. Due to the non-linear character of the behaviour of non-cohesive soils subjected to various states of stress this problem causes a lot of trouble, this leading sometimes to inconsistencies with classical elasticity theory. A typical example of such incorrect interpretation is initial tangent modulus defined as the slope of loading curve at the origin of the stress-strain diagram. It has been shown experimentally (Świdziński, 2000b) that even for the early stage of loading both irreversible and reversible strains develop within the sample. Basing on triaxial compression and extension tests at very small ranges of strains Hicher, 1996 found that sands exhibit reversible linear behaviour as long as strains amplitudes are below 1 to 3×10^{-5}. Such small strains cannot be measured in conventional triaxial apparatuses. Additionally, in the definition of initial tangent modulus, lateral strains are neglected, thus only part of the information about the material behaviour is taken into account. Therefore, the initial tangent modulus cannot be identified as elastic property of the material.

Another common approach assumes that elastic modulus can be determined from the reloading-unloading cycle from the triaxial compression test (cf. Duncan and Chang, 1970; Lade and Nelson, 1987). In this case it is described as a secant modulus determined by extreme points on the "thin" hysteresis loop. However, Świdziński, 2000b has shown that for some sands the hysteresis loop produced by subsequent loading and unloading of the sample may be more pronounced with clearly curvilinear sectors. Additionally, again in the analysis of elastic modulus only the axial strain is involved.

The aim of the present paper is to propose an empirical method of determination of elastic constants of non-cohesive soils, based on a new interpretation of experimental data from a conventional triaxial compression test. The method incorporates the assumption that only the first stage of unloading can be considered as elastic response of the material. Such an approach is the outcome of previous studies of the behaviour of particulate materials in oedometric conditions with the measurement of lateral stresses (cf. Sawicki, 1994; Sawicki and Świdziński, 1997a, 1997b, 1998).

The present analysis includes all stress and strain components measured during the tests, which significantly differs from the methods commonly accepted in soil mechanics. The method is verified in terms of triaxial compression tests carried out in IBW PAN geotechnical laboratory as well as for some experimental data available from the literature. The results of elastic moduli obtained are compared with those determined by other methods.

2. Experimental Investigations

The triaxial compression tests were carried out in the triaxial apparatus, which is an integral part of a computer-controlled hydraulic triaxial testing system from GDS Instruments Ltd. The detailed description of the system, which was schematically shown in Fig. 1, can be found in Menzies, 1988 and Świdziński, 2000a. It consists of a Bishop and Wesley's triaxial cell for controlled stress path testing linked to a desktop computer via three microprocessor controlled hydraulic actuators called "digital pressure controllers". The controllers regulate pressure and volume change of deaerated water supplied to the cell to control axial load or axial deformation, cell pressure, and back pressure precisely. For triaxial testing of soils the volumetric capacity of pressure controllers is 200 cm^3 and the pressure range is from 0 to 2000 kPa. Pressure measurement is resolved to 0.2 kPa, and pressure is controlled to 0.5 kPa. The triaxial cell allows for testing specimens 38 and 50 mm in diameter. The axial force is exerted on the test specimen by the piston actuated hydraulically from an integral lower chamber in the base of the cell, which contains deaerated water. Such a system of loading means that the specimen is not subjected to any vibrations which sometimes arise with conventional loading frames.

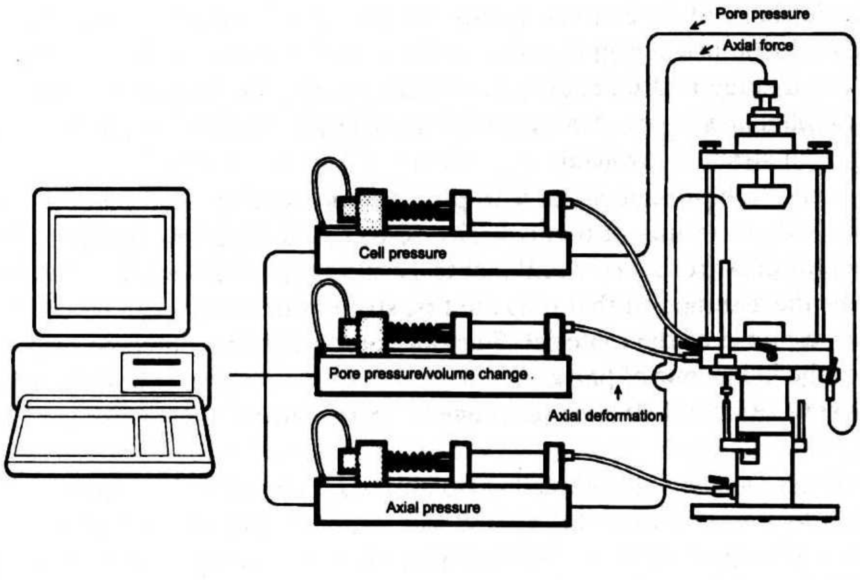

Fig. 1. Schematic layout of the computer-controlled hydraulic triaxial testing system

The system has been additionally equipped with a special gauge for the measurement of lateral deformation of a sample. It consists of a special semiconductor, which makes use of the Hall Effect. The semiconductor, together with two diametrically opposed pads creates a kind of calliper, mounted in the middle part of the sample by adhesive, bonding the device to the rubber membrane. The gauge has been designed so that self-weight is partly counteracted by buoyant uplift. A typical way of installing the gauge onto the sample is shown in Fig. 2. The gauge enables highly accurate measurement of lateral deformation of a sample. Practical resolution of the gauge for a sample with an average diameter of 38 mm corresponds to a change of radial strain $d\varepsilon_3 = 10^{-4}$.

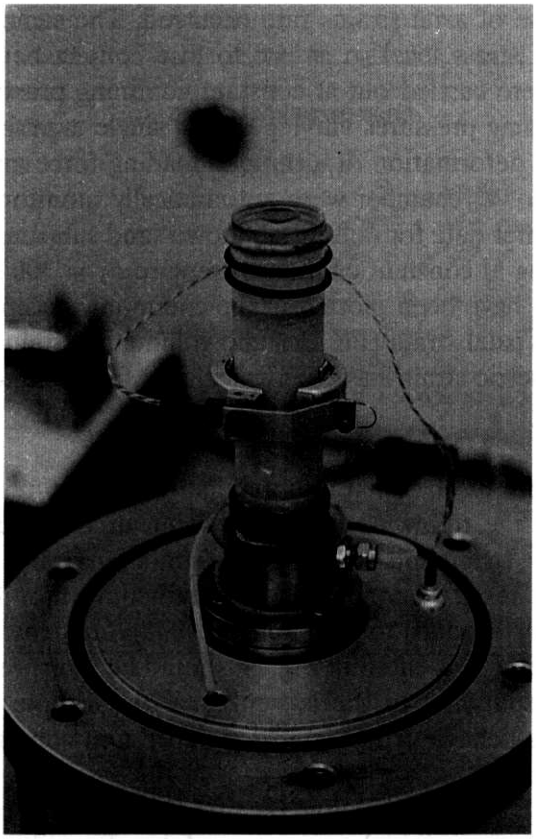

Fig. 2. Photo of the sample with installed gauge for the measurement of lateral deformation

All tests were performed on Lubiatowo sand, which is composed of medium subrounded grains. The characteristics of this sand are summarised as follows: mean diameter $D_{50} = 0.25$, minimum void ratio $e_{min} = 0.49$, maximum void ratio $e_{max} = 0.74$, coefficient of uniformity $c_u = 1.5$, specific gravity $G = 2.65$.

The tests presented here were performed on dense and loose air-dry cylindrical samples 38 mm in diameter and 78 mm high, on average. Specimens were prepared by dry pluviation from heights which varied, depending on the type of density of the sand tested. Dense samples were additionally tamped in several layers. The loosest state of the sample corresponded to a relative density of $D_r = 25\%$. Attempts to prepare looser specimens failed due to installation on the samples gauge for the measurement of radial strains, which usually caused its deformation. Such samples could not have been accepted for further investigations.

All successfully prepared samples were next subjected to several cycles of loading and unloading controlled by axial deformation with a constant velocity of 2 mm/hour. In order to obtain an appreciation of the magnitude and composition (elastic versus plastic) of the measured strains each loading cycle was maintained until obvious decrease of axial strains rate occurred. The samples were then unloaded to the lowest stress level so as not to lose contact between loading cell and top cap. Tests were carried out at constant confining pressure σ_3 from 20 to 400 kPa or the confining pressures varying during single experiments. During the tests axial and radial deformation of a sample, loading force and pressure within the triaxial cell and lower chamber were automatically monitored.

Typical experimental data for dense Lubiatowo sand subjected to four cycles of loading and unloading at constant confining pressure $\sigma_3 = 300$ kPa are presented in Fig. 3. Test results have been plotted in the commonly accepted configuration: stress deviator versus axial strain (Fig. 3a) and volumetric strain versus axial strain (Fig. 3b). The volumetric strain has been calculated from the following formula:

$$\varepsilon_v = \varepsilon_1 + 2\varepsilon_3, \tag{1}$$

where $\varepsilon_1, \varepsilon_3, \varepsilon_v$ denote axial, radial and volumetric strains, respectively. For the sake of convenience the following units have been used throughout this paper: stress unit – 10^5 N/m^2, strain unit – 10^{-3}, Young's modulus unit – 10^8 N/m^2. For the interpretation of test results a commonly accepted convention has been assumed where the '+' sign corresponds to compression. Therefore, the negative values of volumetric change curve presented in Fig. 3b should be interpreted as an increase in the volume corresponding to the loosening of the sample after compaction. This is a well known behaviour observed for dense non-cohesive soils subjected to shearing (Craig, 1992).

For purposes of comparison in Fig. 3a some commonly accepted definitions of elastic modulus such as initial tangent modulus E_{in} or reloading-unloading modulus E_{ur} which is represented by AB line as a secant of the first hysteresis loop, have been shown.

Let us interpret the test results shown in Fig. 3 in terms of two diagrams: deviatoric stress versus deviatoric strain and mean effective pressure versus volumetric strain, where mean effective stress is defined as $p = (\sigma_1 + 2\sigma_3)/3$. Such interpretation has been presented in Fig. 4. Comparing the diagrams shown in Figs. 3a and 4a it can be noticed that the inclination of the reloading-unloading curves differs and the curves have a non-linear character. The second conclusion that can be drawn from the analysis of Fig. 4a is that the subsequent unloading-reloading curves are very similar and are linear within the first phase after any stress reversals. These linear sectors are parallel to each other. The similar character of unloading-reloading suggests that the sample behaves elastically, at least during the first stage and this part of the curve may be treated as being the linear elastic response of the material. Note that the inclination of this linear approximation differs from other proposals related to the distinguishing of the phase of loading responsible for elastic behaviour.

The same trend may be observed when analysing the changes of volumetric strains ε_v versus mean effective stress p presented on the diagram in Fig. 4b.

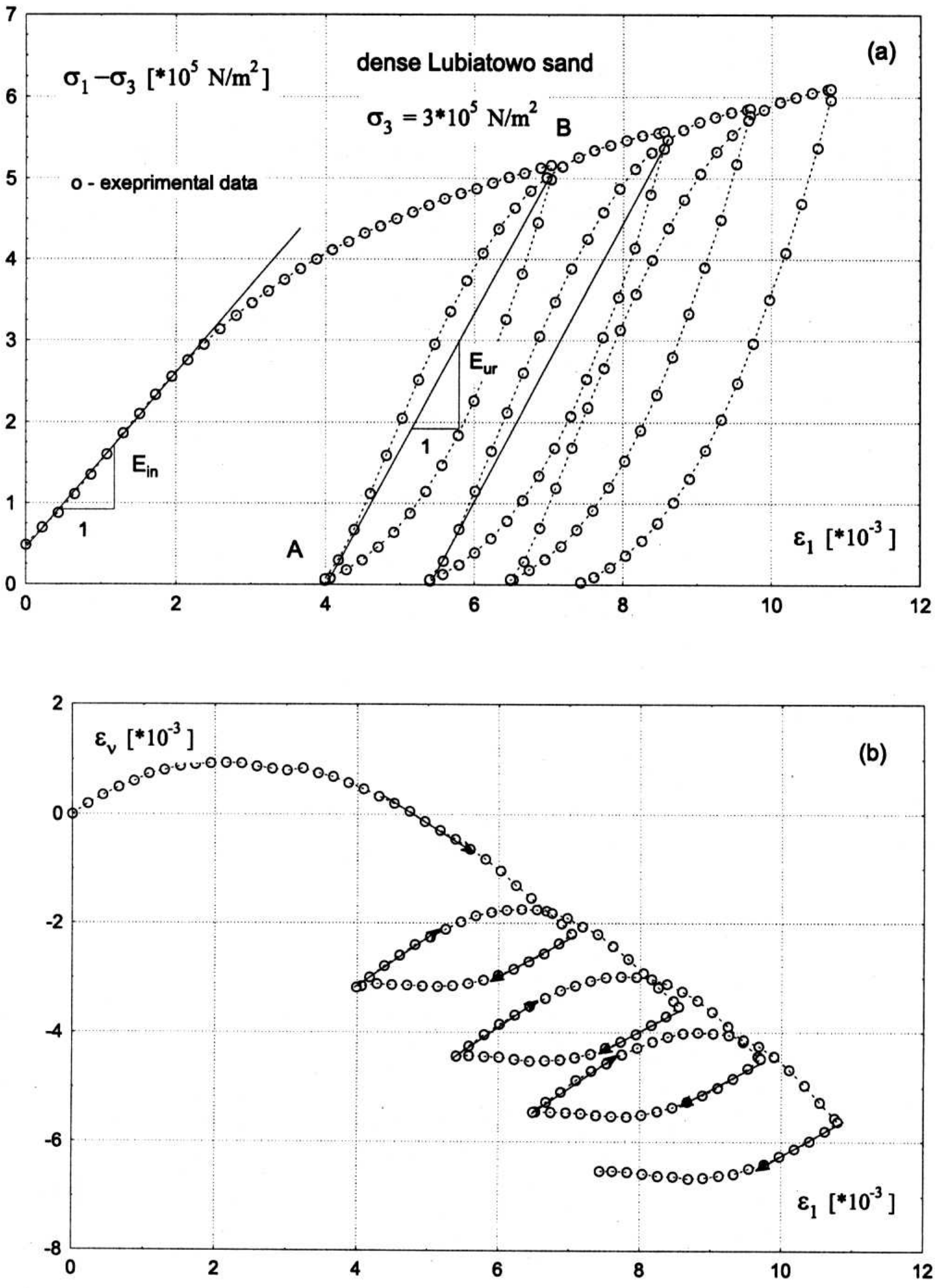

Fig. 3. Stress-strain and volume change behaviour of dense Lubiatowo sand in triaxial compression tests with several unloading-reloading cycles

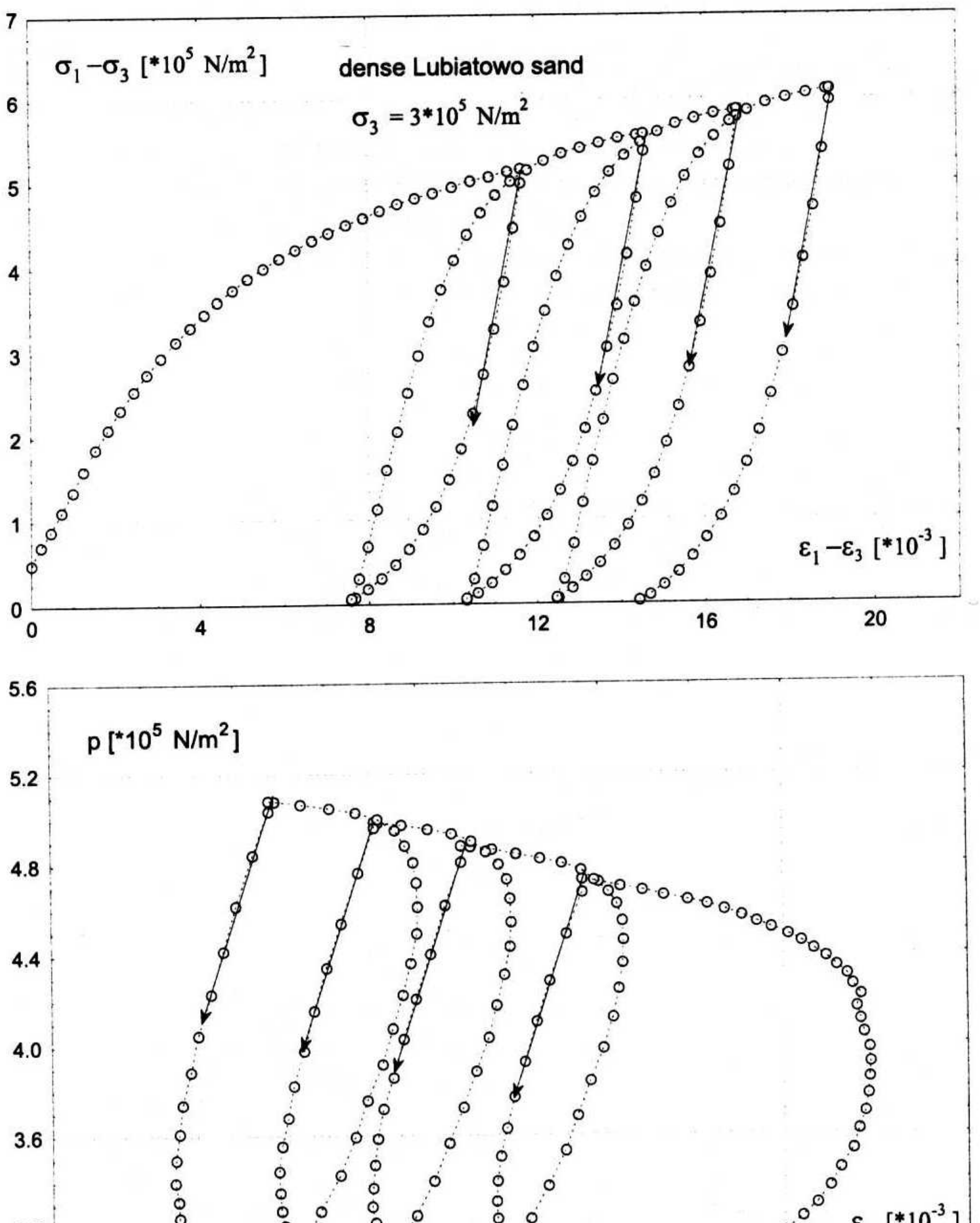

Fig. 4. Triaxial compression test results of dense Lubiatowo sand in another interpretation: a) deviatoric stress – deviatoric strain, b) mean effective pressure – volumetric strain

However, in this case although the first phases of unloading and reloading can be approximated by sectors which are parallel for every hysteresis loop, the sectors corresponding to unloading and reloading are not parallel to each other. The first phase of reloading cannot be approximated by the same sector as for unloading stage.

Such an assumption of the first stage of unloading as the elastic response of the material is consistent with a similar approach regarding the interpretation of oedometric tests with additional measurement of lateral stresses (Sawicki, 1994, Sawicki and Świdziński, 1998). In the interpretation it has been assumed that the unloading stress path, as well as stress-strain unloading curve can be approximated by two linear sectors. The first sector of the unloading curve corresponds to purely elastic response of the material in oedometric conditions. The bilinear approximation of unloading was the key to the determination of elastic moduli of non-cohesive granular materials on the basis of oedometric test results.

3. Theoretical Description of Elastic Response

During the loading of a sample in the triaxial conditions both the reversible and irreversible strains develop in the soil. It can be expressed by the following general formula:

$$\varepsilon_{ij} = \varepsilon_{ij}^{e} + \varepsilon_{ij}^{p}, \tag{2}$$

where ε_{ij} denotes tensor of total strains developing in the sample. The superscripts e and p stand for the elastic (reversible) and plastic (irreversible) parts of total strain tensor.

Let us assume that elastic response of the material obeys Hooke's law which for linearly elastic isotropic material can be expressed by the following stress-strain relationship (cf. Fung, 1965):

$$\sigma_{ij} = \left[\lambda tr\left(\varepsilon_{ij}^{e}\right)\right]\delta_{ij} + 2G\varepsilon_{ij}^{e}, \tag{3}$$

where σ_{ij} denotes stress tensor and δ_{ij} is the Konecker delta (Einstein's summation convention is used). The quantities λ and G are well-know Lame constants which describe the elastic properties of the material. These two elastic constants can be expressed by another pair of elastic moduli i.e. Young's modulus E and Poisson's ratio ν in terms of the following relationships:

$$G = \frac{E}{2(1+\nu)}, \qquad \lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}. \tag{4}$$

For the triaxial conditions Eq. 3 reduces to the two stress-strain relationships:

$$\begin{aligned} \sigma_1 &= (\lambda + 2G)\varepsilon_1 + 2\lambda\varepsilon_3, \\ \sigma_3 &= \lambda\varepsilon_1 + 2(\lambda + G)\varepsilon_3, \end{aligned} \tag{5}$$

where subscripts "1" and "3" denote axial and radial principle directions, respectively. Simple algebraic transformations of Eqs. 5 lead to the following formulae:

$$\begin{aligned} \sigma_1 - \sigma_3 &= 2G(\varepsilon_1 - \varepsilon_3), \\ \sigma_1 + 2\sigma_3 &= (3\lambda + 2G)(\varepsilon_1 + 2\varepsilon_3). \end{aligned} \tag{6}$$

The left hand sides in Eqs. 6 are stress deviator and triple mean effective stress, respectively whereas corresponding strain terms on the right hand side refer to strain deviator and volumetric strain (cf. Eq. 1).

Consequently, the respective ratios of deviatoric stress and strains and triple mean effective pressure and volumetric strain take the following form:

$$\begin{aligned} \frac{\sigma_1 - \sigma_3}{\varepsilon_1 - \varepsilon_3} &= 2G = a, \\ \frac{\sigma_1 + 2\sigma_3}{\varepsilon_1 + 2\varepsilon_3} &= (3\lambda + 2G) = b. \end{aligned} \tag{7}$$

Let us assume that the first stage of any unloading can be identified with linear elastic response of the material (long arrows approximating parts of unloading curves in Figs. 4a and 4b). We have:

$$\begin{aligned} \Delta q &= \Delta(\sigma_1 - \sigma_3) = q^A - q^B, \\ \Delta\hat{\varepsilon} &= \Delta(\varepsilon_1 - \varepsilon_3) = \hat{\varepsilon}^A - \hat{\varepsilon}^B, \end{aligned} \tag{8}$$

and

$$\begin{aligned} \Delta p &= p^A - p^B, \\ \Delta\varepsilon_v &= \varepsilon_v^A - \varepsilon_v^B, \end{aligned} \tag{9}$$

where q and denote deviatoric stress and strain, respectively.

It follows from Eqs. 7–9 together with the definition of mean effective pressure that

$$\begin{aligned} \frac{\Delta q}{\Delta\hat{\varepsilon}} &= a, \\ \frac{\Delta_p}{\Delta\varepsilon_v} &= 3b, \end{aligned} \tag{10}$$

where a and b are slopes of first stage of unloading paths of deviatoric stress versus deviatoric strain and mean effective pressure versus volumetric strains, respectively (cf. Figs. 4a and 4b). Note, that the coefficients a and b have the same units as Young's modulus, 10^{-8} N/m^2.

Simple algebraic transformations of Eqs. 4, 7 and 10 lead to the closed formulae for Young's modulus and Poisson's ratio:

$$E = \frac{9ab}{a + 6b}, \tag{11}$$

and

$$\nu = \frac{3b - a}{a + 6b}. \tag{12}$$

4. Determination of Elastic Constants

According to the relationships described by Eqs. 10, 11 and 12 the following values of coefficients and elastic moduli for experimental data corresponding to dense Lubiatowo sand shown in Fig. 4 have been obtained: $a = 2.41 \times 10^8$ N/m^2, $b = 1.37 \times 10^8$ N/m^2, $E = 2.8 \times 10^8$ N/m^2, $n = 0.14$.

Let us compare the values of elastic moduli determined in terms of the proposed method with those based on another definitions. The initial tangent modulus calculated from Fig. 3a is $E_{in} = 1.08 \times 10^8$ N/m^2 and the elastic modulus corresponding to the reloading-unloading curve (sector AB in Fig. 3a) is $E_{ur} = 1.72 \times 10^8$ N/m^2. Thus both values are lower than Young's modulus determined from the first stage of unloading. Poisson's ratio is often determined from similar triaxial compression tests with several cycles of loading and unloading and constant confining pressure according to the formula of Lade and Nelson, 1987:

$$\nu = \nu_\nu = -\frac{\dot{\varepsilon}_3}{\dot{\varepsilon}_1} = \frac{1}{2}\left(1 - \frac{\dot{\varepsilon}_\nu}{\dot{\varepsilon}_1}\right), \tag{13}$$

where the term $\dot{\varepsilon}_\nu/\dot{\varepsilon}_1$ denotes a slope of the volumetric change curve immediately after stress reversal. Indeed, when analysing the experimental data shown in Fig. 3b it can be found that for first phases of unloading, subsequent slopes of volumetric curve plotted against vertical strain are parallel to each other. Similar parallel sectors can be drawn for corresponding reloading phases, however an inclination of reloading sectors differs from unloading sectors.

The calculation of Poisson's ratio in terms of the first phase of unloading according to Eq. 13 gives the value $\nu_\nu = 0.12$.

Elastic moduli of dense Lubiatowo sand determined in terms of the method based on the interpretation of oedometric test (Sawicki, 1994) are the following: Poisson's ratio $\nu = 0.17$, Young's modulus $E = 3.35 \times 10^8$ N/m^2. Thus, the value of Young's modulus is approximately 30% higher than corresponding elastic constant based on the interpretation of the triaxial compression test whereas the values of Poisson's differ 20%. The difference of Young's moduli determined by oedometric and triaxial methods can be easily explained. In oedometric conditions the lateral strains are prevented, which results in somewhat 'higher stiffness' of the material tested, as compared with the triaxial compression test in which the lateral deformation is permitted.

In Fig. 5 are presented the results of triaxial compression test of loose Lubiatowo sand (D_r = 26%) tested at the same confining pressure as for dense sand. The experimental data presented in terms of stress and strain deviators are qualitatively similar to those corresponding to dense sand. However, the diagrams of mean effective stress and volumetric strain are different. First of all in the case of loose sand during whole test the current volume was always smaller than volume of sample corresponding to the initial state (compaction). Secondly, there are almost no hysteresis loops or they are of "residual" character. Nevertheless, for loose sand one can also easily distinguish linear phases of unloading curves, which are parallel to each other for subsequent cycles of reloading and unloading, for both representations shown in Figs. 5a and 5b. Calculations of elastic constants according to the Eqs. 10–11 give the following values: $a = 1.96 \times 10^8$ N/m^2, $b = 1.08 \times 10^8$ N/m^2, $E = 2.25 \times 10^8$ N/m^2, $\nu = 0.15$.

The values of Young's modulus in the case of loose sand tested in the same pressure conditions are somewhat lower than for dense sand (approximately 20%). However, the analysis of Figs. 4 and 5 may suggest that the elastic moduli are independent of density changing during subsequent cycles of loading (the same slopes of first stage of unloading). The calculation of sand density change corresponding to volume change the sample experienced after four cycles of loading and unloading according to Fig. 4b gives the value of approximately 1% and is even smaller for loose sand. Therefore, such small density change can not influence the values of elastic moduli, which may however vary for densities corresponding to loose and dense states of non-cohesive soils.

At this stage of analysis it is difficult to assess to what stress level the behaviour of the material during the first phase of unloading corresponds to the elastic response. For oedometric conditions the first phase of unloading was restricted by so-called failure in extension line (FEL), Sawicki, 1994. This line had its oedometric representation in stress space. Results presented suggest that a similar line could be drawn in the case of a triaxial compression test, however this problem requires further comprehensive experimental studies and theoretical considerations.

5. Dependence of Elastic Moduli on Stress Level

In most of the models serving to determine elastic moduli of non-cohesive soils it is assumed that Young's modulus is a power function of mean effective stress or confining pressure (cf. e.g. Duncan and Chang, 1970; Hardin and Drnevich, 1972; Seed et al., 1985; Lade and Nelson, 1987; Hicher, 1996). This dependence is also supported by some theoretical considerations (Gassmann, 1951; Duffy and Mindlin, 1957). It r is not however, perfectly clear. Let us assume that Young's modulus is defined as reloading-unloading modulus E_{ur} (sector AB in Fig. 3a). The value of an elastic modulus so defined is constant along the stress path for which the mean effective pressure changes from 4.7×10^5 N/m^2 (point A) to zero stress level (point B). Studies made by Sawicki and Świdziński, 1998 on various particulate materials subjected to one-dimensional compression with the measurement of lateral stresses did not confirm such strong dependence of elastic moduli on stress level. In the method proposed it is also assumed that Young's modulus is independent of mean effective pressure within the range of stress levels corresponding to the first stage of unloading.

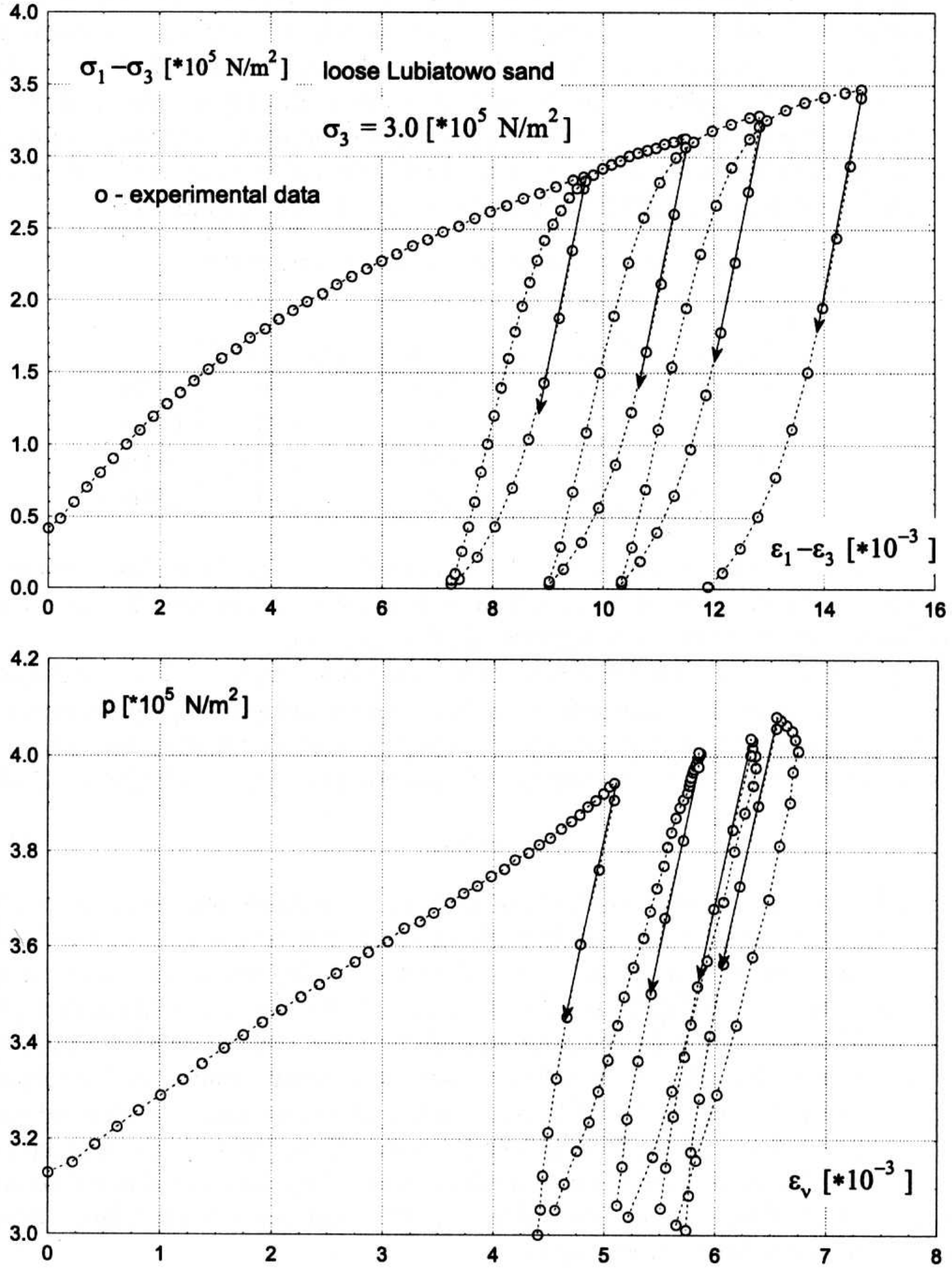

Fig. 5. Triaxial compression test results of loose Lubiatowo sand in another interpretation: a) deviatoric stress – deviatoric strain, b) mean effective pressure – volumetric strain

In order to analyse this problem for triaxial conditions let us consider experimental results shown in Fig. 6. These empirical data correspond to four cycles of loading and unloading in which the same sample of very dense Lubiatowo sand ($D_r = 92\%$) was tested at four different confining pressures: 1, 2, 3, 4 $\times 10^5$ N/m^2. Computer graphics software support allowed for very careful analysis of first phases of unloading which for various confining pressures appeared not to be parallel to each other. According to Eqs. 10–12, the calculation results of elastic moduli for the experimental data from Fig. 6 are as follows (Table 1):

Table 1. Elastic parameters for test results shown in Fig. 6

$p = \sigma_3$ [10^5 N/m^2]	a [10^5 N/m^2]	b [10^5 N/m^2]	E [10^5 N/m^2]	ν
1	1.64	2.23	1.80	0.097
2	2.33	3.04	2.52	0.084
3	2.50	3.40	2.74	0.097
4	2.86	3.87	3.13	0.095

The analysis of the results from Table 1 shows that Young's modulus varies with confining pressure whereas Poisson's ratio is practically constant. These results confirm the experimental observations of other authors.

Apart from to the test described above some other tests on dense and loose Lubiatowo sand were carried out at constant and changing confining pressures.

Let us assume that a dependency of Young's modulus upon the mean effective stress is given by the following formula (Hardin and Drnevich, 1972; Hicher, 1996):

$$E = E_0 p^n \tag{14}$$

where E_0 and n are some coefficients which have to be determined experimentally. Least square analysis shows that best fitting of the test results collected in Table 1 and results from separate tests at different confining pressures, in terms of function described by Eq. 14 is achieved for the following values of coefficients: $E_0 = 1.788$ and $n = 0.422$. The approximation of the Young's moduli values by power function described in Eq. 14 for loose and dense samples of Lubiatowo sand is shown in Fig. 7. It can be seen that there is an apparent dependence of Young's modulus on mean confining pressure. Thus, the opposite conclusion derived from the analysis of elastic moduli determined by the method based on the interpretation of oedometric tests (Sawicki, 1994; Sawicki and Świdziński, 1998) has not been confirmed in this case.

Some words of explanation are required by the contradiction between the assumption that Young's modulus is constant during the first stage of unloading corresponding to some changes of stress level and, experimentally proved, its dependence on mean confining pressure. When analysing Figs. 4 and 5 it can be

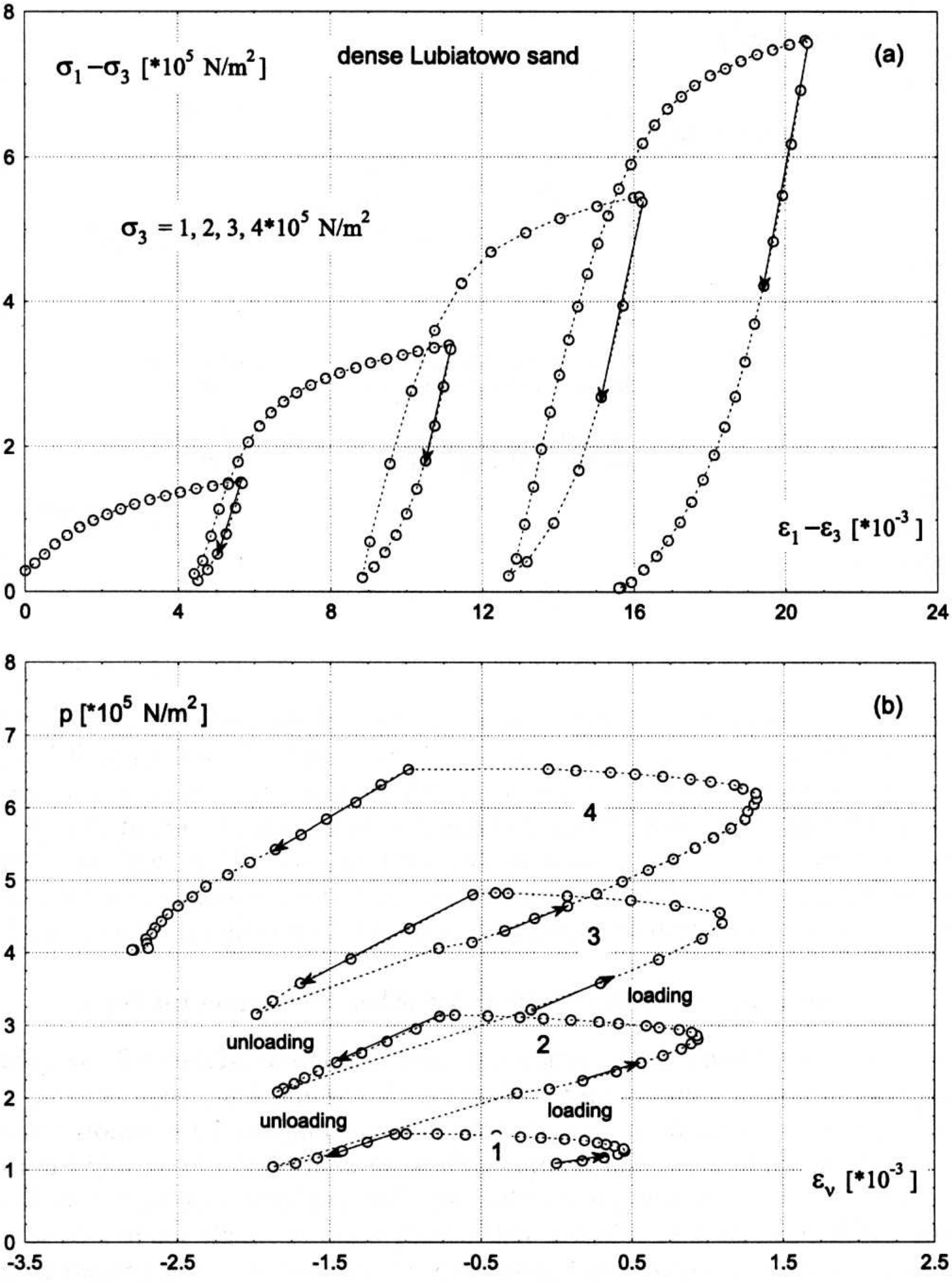

Fig. 6. Triaxial compression test results of dense Lubiatowo sand tested at different confining pressures

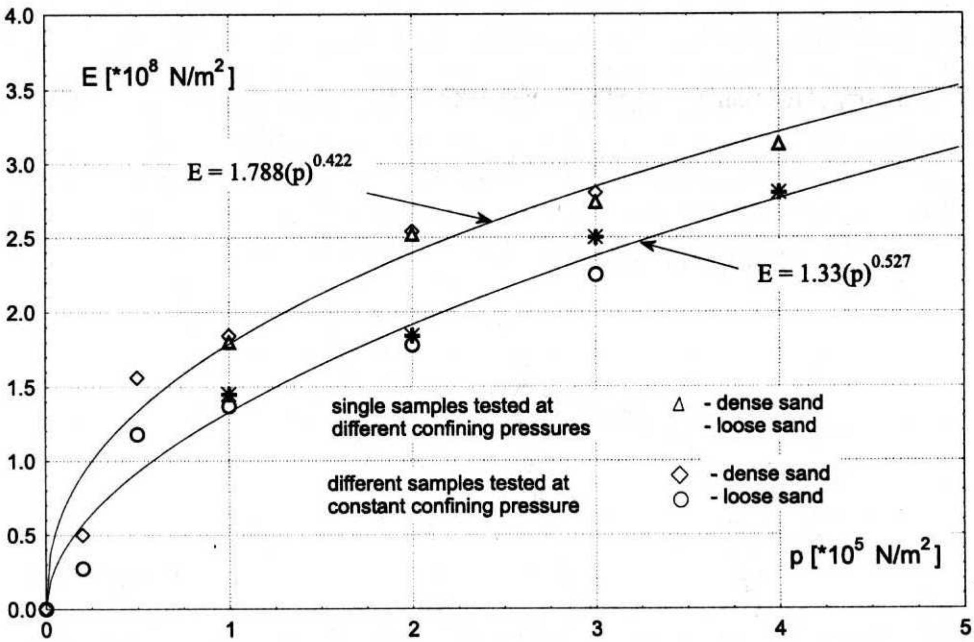

Fig. 7. Approximation of Young's moduli for different confining pressures by power function

seen that the ranges of confining pressure changes within which Young's modulus is assumed to be constant are 3.7 to 5.1 × 10^5 N/m^2 and 3.45 and 4.1 × 10^5 N/m^2 for dense and loose sands, respectively. As follows from Fig. 7 the dependence of elastic constant on mean effective pressure for such ranges is not so strong and as a simplification can be reduced to the constant value. Thus, the assumption concerning the independence of Young's modulus and confining pressure within the stress level range corresponding to first stage of unloading can be justified.

6. Verification of the Method for other Experimental Data

There are not too many experimental data available in the literature from triaxial compression tests on non-cohesive materials that would have been suitable for verification of the method proposed. This requires complete information regarding all stress and strain tensor components being controlled or measured during subsequent cycles of loading and unloading. The available empirical data from static triaxial tests on non-cohesive soils usually concern simple loading or sometimes single cycle of loading and unloading. The second reason is related to the lack of lateral deformation measurements, especially of dry sandy samples. Usually, this measurement is made indirectly by the measurement of volume changes of water coming out of or flowing into a fully saturated specimen in drained conditions.

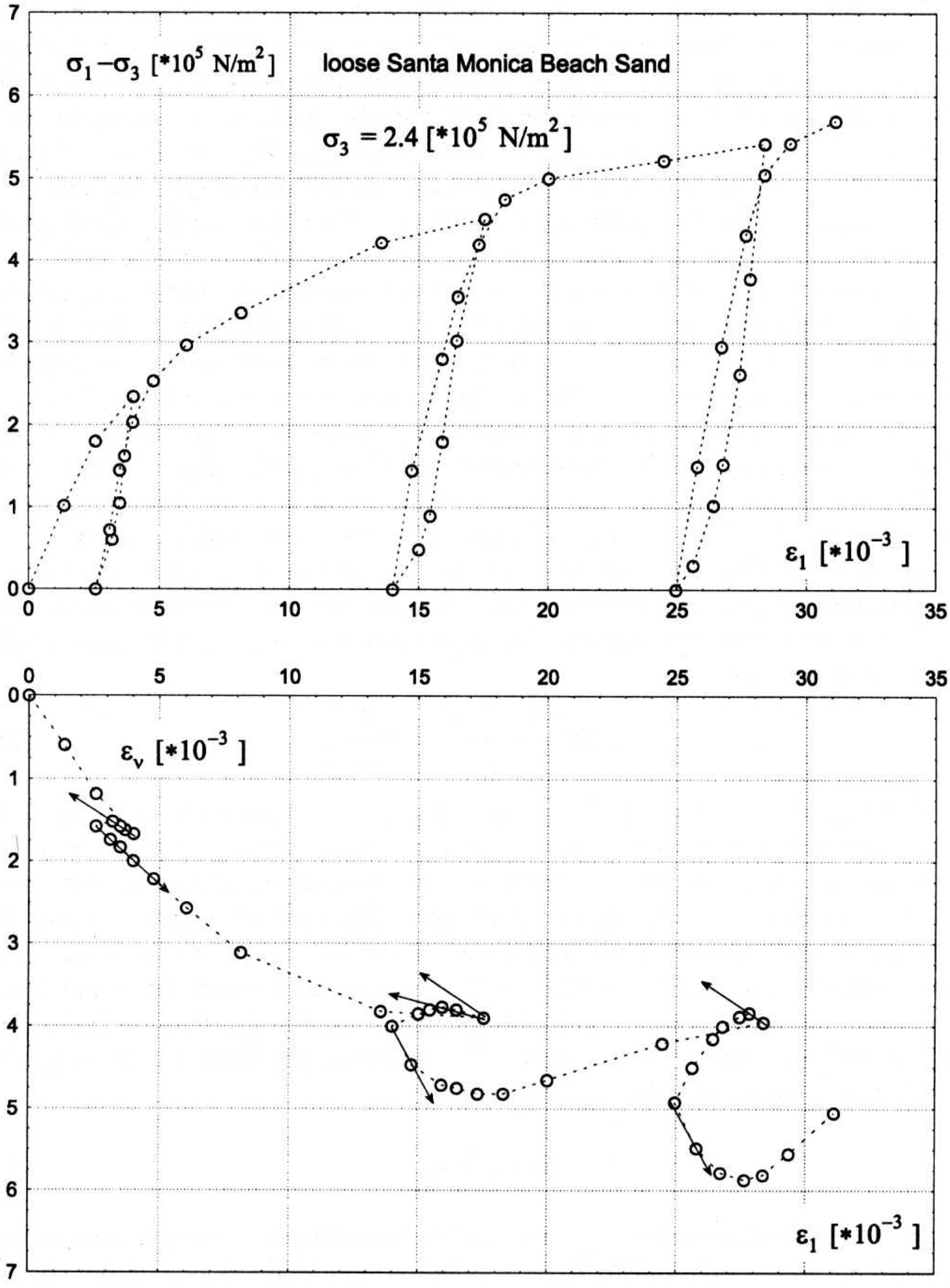

Fig. 8. Stress-strain and volume change behaviour of loose Santa Monica Beach Sand in triaxial compression tests with several unloading-reloading cycles

Suitable experimental data from triaxial compression tests on loose Santa Monica Sand with several unloading-reloading cycles at constant confining pressure $\sigma_3 = 2.4 \times 10^5$ N/m^2 have been provided by Lade and Nelson, 1987. These data have been reproduced in Fig. 8. The experimental results were shown in the original interpretation given by the authors, namely: deviatoric stress versus axial strain (Fig. 8a) and volumetric strain versus axial strain (Fig. 8b). In Fig. 9 these results have been shown in the representation required for the determination of elastic moduli by the proposed method as in Fig. 4. It can be seen that the diagram of axial strains against deviatoric stress (Fig. 9a) is qualitatively similar to that for loose Lubiatowo sand (Fig. 5a) however, in Lade and Nelson triaxial compression test much larger deformations developed in the sample. The diagram of volumetric strains versus axial strain is somewhat different from the corresponding plot for loose Lubiatowo sand presented in Fig. 5b, however, the trends are the same. Despite these differences for both representations shown in Figs. 9a and 9b, first phases of unloading may be nicely approximated by parallel linear sectors. Calculations of elastic constants according to Eqs. 10–11 give the following values: $a = 4.27 \times 10^8$ N/m^2, $b = 1.80 \times 10^8$ N/m^2, $E = 2.52 \times 10^8$ N/m^2, $\nu = 0.40$.

Direct comparison of these values with those proposed by Lade and Nelson is rather difficult due to the general form of expression for calculation of Young's modulus proposed by the authors. The expression has been given in terms of the following power law:

$$E = Mp_a \left[\left(\frac{I_1}{p_a} \right)^2 + R \frac{J_2'}{p_a^2} \right]^\lambda , \tag{15}$$

in which p_a is the atmospheric pressure expressed in the same units as E, I_1 is the first invariant of the stress tensor and reflects the dependence of E on mean normal stress, whereas J_2' is the second invariant of the deviatoric stress tensor corresponding to the deviatoric changes. The modulus number M and the exponent λ are constant, dimensionless numbers which have to be determined experimentally from any type of tests with a measurement of all stress and strain components (triaxial compression tests including unloading-reloading cycles, cubical triaxial tests, etc.). The parameter R is a function of Poisson's ratio and can be calculated from the following formula:

$$R = 6\frac{1+\nu}{1-2\nu}. \tag{16}$$

In the method proposed by Lade and Nelson Poisson's ratio is assumed to be constant and is determined from a slope of volumetric change curve corresponding to first phase of unloading or reloading (cf. arrows in Fig. 8b and Eq. 13). According to the authors opinion so defined Poisson's ratio was $\nu_v = 0.26$. However, very careful analysis with the aid of computer graphics showed that neither the initial phases of subsequent reloading-unloading curves are parallel

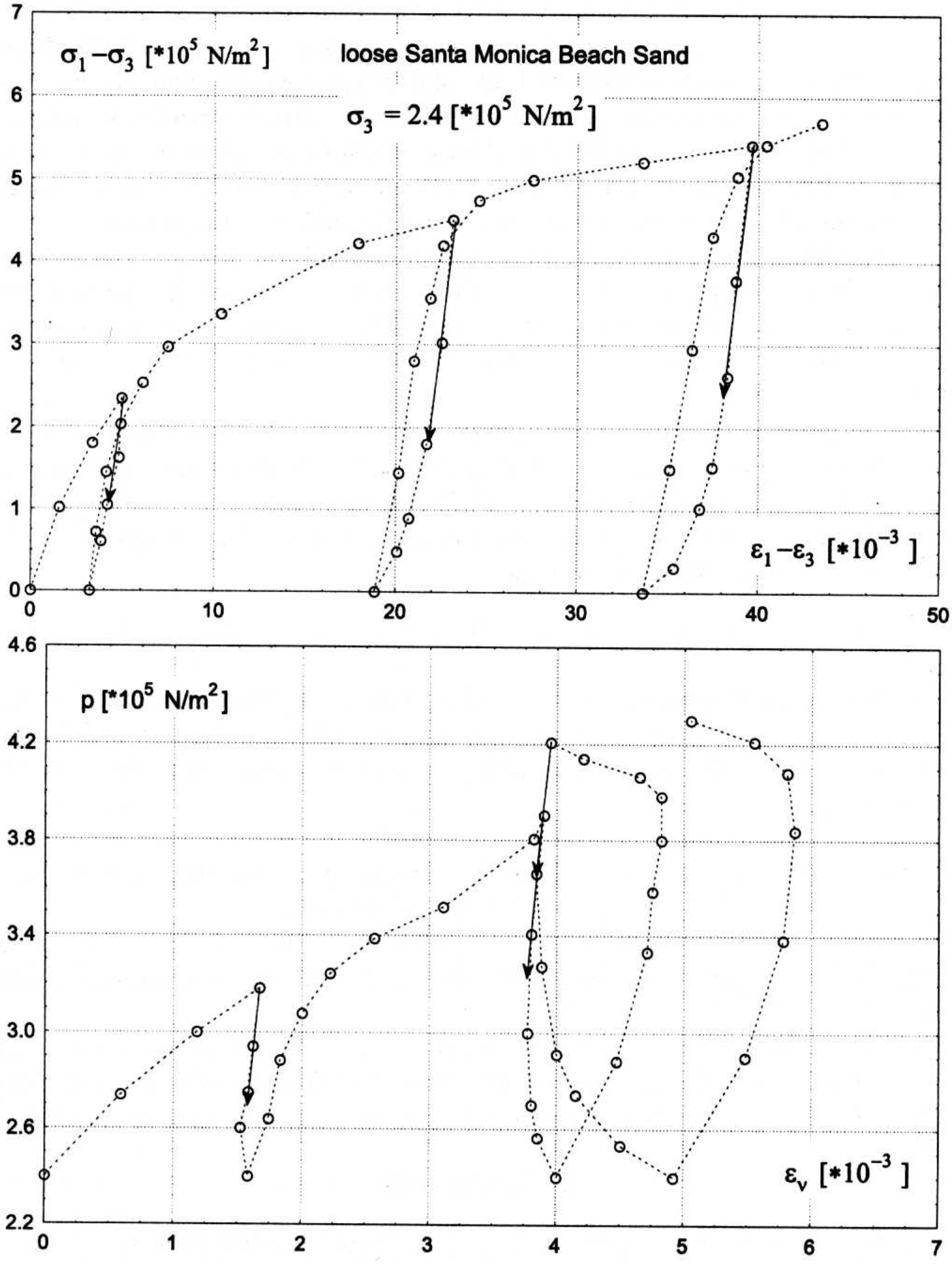

Fig. 9. Triaxial compression test results of loose Santa Monica Beach Sand in another interpretation: a) deviatoric stress – deviatoric strain, b) mean effective pressure – volumetric strain

nor do the values of Poisson's ratio calculated from the slope take similar value as that given by Lade and Nelson. Poisson's ratio calculated for the first and third unloading cycle is $\nu_{v13} = 0.4$ and for second unloading cycle $\nu_{v2} = 0.46$. These values differ from the estimation of Lade and Nelson but are close to the Poisson's ratio determined by the method proposed here. This observation shows that determination of Poisson's ratio from slopes of volumetric change curve corresponding to first phases of unloading and reloading may produce wrong results due sometimes to, too small resolution of experimental data measured.

In order to compare the values of elastic moduli of loose Santa Monica Beach Sand obtained from Eq. 14 with those determined in terms of the method proposed the average value of Young's modulus will be calculated. The average value of deviator stress for ranges varying from 2×10^5 N/m^2 to 5×10^5 N/m^2 (cf. Fig. 9a) is 3.5×10^5 N/m^2.

The respective values of model parameters for loose Santa Monica Beach Sand obtained from a number of tests performed in both triaxial apparatus as well as cubical triaxial device are: $R = 15.75$ (for $\nu_v = 0.26$), $M = 600$, $\lambda = 0.27$, Lade and Nelson, 1987. The respective invariants for triaxial conditions can be calculated from the following formulae:

$$I_1 = \sigma_1 - 2\sigma_3 \qquad J_2' = \frac{1}{3}(\sigma_1 - \sigma_3)^2. \tag{17}$$

For average deviatoric stress and model parameters given above the Young's modulus value calculated by Eqs. 15–17 is $E_{avr} = 2.43 \times 10^8$ N/m^2, which is very close to the value of the corresponding Young's modulus determined by the method proposed.

The direct determination of Young's moduli defined as initial tangent modulus E_{in} and reloading-unloading modulus E_{ur} yielded the following values: $E_{in} = 0.73 \times 10^8$ N/m^2 and $E_{ur} = 1.25 \times 10^8$ N/m^2, respectively.

Examples of some values of elastic moduli determined in terms of various definitions for two sands of different densities and confining pressures are shown in Table 2.

Analysing the results from Table 2 it can be seen that there are clear trends of dependencies of Young's modulus on both mean effective stress and soil states where Young's modulus increases with the increase of density and stress level.

7. Conclusions

The paper presents the original empirical method of determination of elastic moduli of non-cohesive soils. The method is based on a new interpretation of experimental data from triaxial compression tests. In this method test results are represented in terms of two diagrams: deviatoric stress versus deviatoric strain and mean effective pressure versus volumetric strains. Such a representation requires information of lateral strains developing in a sample. The key point of the

Table 2. Values of elastic moduli determined on the basis of various definitions for chosen triaxial compression tests)

Type of sand	D_r %	σ_3 10^5 N/m^2	E_{in} 10^8 N/m^2	E_{ur} 10^8 N/m^2	ν_v	E 10^8 N/m^2	ν
loose	36	1.0	0.15	0,77	0.14	1.37	0.21
Lubiatowo	40	2.0	0.49	1.39	0.17	1.78	0.18
sand	32	3.0	0.48	1.61	0.12	2.25	0.15
dense	76	1.0	0.83	1.11	0.14	1.84	0.15
Lubiatowo	76	2.0	1.22	1.52	0.14	2.54	0.14
sand	80	3.0	1.08	1.72	0.12	2.80	0.14
loose Santa Monica Beach Sand	–	2.4	0.73	1.25	0.41	2.52	0.40

interpretation of experimental data is the assumption that during first phase of unloading the material exhibits linear elastic response and can be described by Hooke's linear elastic law. This assumption is of a purely empirical character. The linear approximation of the first stage of unloading enables the determination of Young's modulus and Poisson's ratio in terms of respective slopes of linear sectors. The concept of such interpretation of triaxial compression test results differs from other concepts encountered in the literature. The advantage of this approach is including in the calculations of elastic moduli of all strain components which is consistent with the theory of elasticity. In almost all commonly accepted definitions for calculations of Young's modulus of non-cohesive soils, these deformations are neglected.

The values of Young's modulus based on the new interpretation of triaxial compression tests carried out for Lubiatowo sand are several times higher than those of initial tangent modulus and also somewhat higher than modulus corresponding to the reloading-unloading curve which is a secant of a hysteresis loop. The respective values of Poisson's ratio are almost the same as those calculated from the volumetric change curve corresponding to the first phase of unloading. However, detailed analysis of such a method of determination shows that it can lead to some errors due to small resolution of test results, especially for loose samples which reveal relatively small changes in volume.

Analysis of values of elastic moduli corresponding to various initial states of sand and various confining pressures at which the samples were tested, confirms the commonly accepted observation that the value of Young's modulus is a function of soil density and state of stress acting on the soil, whereas the value of Poisson's ratio depends only on the initial void ratio. It has been shown that for dense Lubiatowo sand this dependence can be expressed in terms of power law.

The application of the method proposed for experimental data corresponding to independent triaxial compression tests described in the literature shows that the method can be a reliable and powerful tool when determining elastic moduli of non-cohesive soils.

The designation of limits for the first phase of unloading corresponding to the elastic response of the material, requires further studies.

References

Craig R. F. (1992) *Soil Mechanics, Fifth Edition*, Chapman & Hall, University and Professional Division, London, Glasgow, New York, Tokyo, Melbourne, Madras, 427 pp.

Dakoulas P., Sun Y. (1992) Fine Ottawa Sand: Experimental Behavior and Theoretical Predictions, *J. Geotech. Engrg.*, Vol. 1188, No. 12, pp. 1906–1923.

Duffy J., Mindlin R. D. (1957) Stress-strain relations and vibrations of granular media, *J. Appl. Mech., Trans. ASME*, 24(4), pp. 585–593.

Duncan J. M., Chang C-Y. (1970) Nonlinear analysis of stress and strain in soils, *J. Soil Mech. Found. Div., ASCE*, Vol. 96, No. SM5, pp. 1630–1653.

Fung Y. C. (1965) *Foundations of Solid Mechanics*, Prentice-Hall, Englewood Cliffs, NJ.

Gassmann F. (1951) Elastic waves through a packing of spheres, *Geophysic*, Vol. 16, pp. 673–685.

Hardin B. O., Black W. L. (1969) Closure to "Vibration modulus of normally consolidated clay", *J. Soil Mech. Found. Div., ASCE*, 95(SM6), pp. 1531–1537.

Hardin B. O., Drnevich V. P. (1972) Shear modulus and damping in soils: design equations and curves, *J. Soil. Mech. and Found. Div., ASCE*, 89(1), pp. 33–45.

Hicher P. Y. (1996) Elastic Properties of Soils, *J. Geotech. Engrg., ASCE*, Vol. 1228, No. 8, pp. 641–648.

Lade P. V., Nelson R. B. (1987) Modelling the Elastic Behaviour of Granular Materials, *Int. J. Num. Anal. Methods Geomech.*, Vol. 11, pp. 521–542.

Menzies B. K. (1988) A Computer Controlled Hydraulic Triaxial System, [in:] *Advanced Triaxial Testing of Soil and Rock*, ASTM STP 977, R. T. Donaghe, R. C. Chaney and M. L. Silver, eds., ASTM, Philadelphia, pp. 82–94.

Sawicki A. (1994) Elasto-plastic interpretation of oedometric test, *Arch. Hydro-Eng. Environ. Mech.*, 41 (1–2), pp. 111–131.

Sawicki A., Świdziński W. (1997a) Moduły "geotechniczne", moduły sprężystości i charakterystyki zagęszczania gruntów niespoistych, *Inżynieria i Budownictwo*, nr 12, pp. 633–636.

Sawicki A., Świdziński W. (1997b) Moduły sprężystości ośrodka analogowego – porównanie oszacowań teoretycznych z eksperymentem, *XI Kr. Konf. Mech. Gruntów i Fund.*, Gdańsk, pp. 301–307.

Sawicki A., Świdziński W. (1998) Elastic moduli of non-cohesive particulate materials, *Powder Techn.*, 96, pp. 24–32.

Seed H. B., Wong R. T., Idriss I. M., Tokimatsu K. (1985) Moduli and Damping Factors for Dynamic Analysis of Cohesionless Soils, *J. Geotech. Engrg.*, Vol. 112, No. 11, pp. 1016–1033.

Świdziński W. (2000a) Sterowany komputerem system do badań w aparacie trójosiowego ściskania, (in Polish), *Inż. Morska i Geotechnika*, No. 1/2000, pp. 18–24.

Świdziński W. (2000b) Methods of determination of elastic moduli of particulate materials – brief review, *Arch. Hydro-Eng. Environ. Mech.*, Vol. 47, No. 1–4, pp. 27–50.

Tatsuoka F., Shibuya S. (1992) Deformation characteristics of soils and rocks from field and laboratory tests, Keynote Lecture for Session No. 1, *Proc. 9th Asian Regional Conference on SMFE*, Bangkok, 1991, Vol. 2, pp. 101–170.