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Large-Scale Coastal Morphodynamics: a Simplified Mathematical Model Demonstrating the Possibility of Chaotic Behaviour

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Abstract

A simplified discrete-time mathematical model, demonstrating the possibility of intrinsically chaotic morphodynamic behaviour of coastal zones with strongly pronounced seasonality is presented. The dynamic state of the considered zone is characterized by mean concentration of the sediment transported.

Assuming a temporarily constant wave climate and using some physical arguments based upon known approximate interdependencies between the variations of sediment concentration, grain size, mean bed slope, and the wave-caused morphodynamic effects, a non-linear (logistic-type) evolution relation with only one parameter is obtained.

The influence of this parameter on the behaviour of the dynamic system considered is examined by means of numerical calculations. In particular, a parameter interval corresponding to some kind of chaotic behaviour has been found.

1. Introduction

Mathematical models describing various aspects of temporal and spatial behaviour of coastal morphodynamic systems are, as a rule, non-linear. On the other hand, such dynamic systems are nowadays known to manifest, in many cases, some kind of apparently irregular behaviour, called "deterministic chaos" (Schuster 1988). By reason of such circumstances, it cannot be *a priori* excluded that in some cases (especially those connected with large time scales) such kind of behaviour may occur in coastal morphodynamic systems (see e.g. De Vriend et al. 1993), the moreso as numerous field observations seem to confirm such a conclusion.

Application of the contemporary chaos theory in atmospheric sciences has been known for many years (see e.g. Zeng et al. 1993), in fact exactly this field of knowledge happened to be the 'birthplace' of the considered approach. The appearance of analogous application in coastal morphodynamics would thus seem to be only a question of time.

The purpose of the present contribution is to construct an greatly simplified mathematical model which would be:

- connected as closely as possible with large-scale coastal morphodynamics, and
- able to describe the apparently chaotic behaviour of the dynamic system considered.

2. Formulation of the Mathematical Model

The proposed mathematical model has been based upon the following reasoning:

(i) First and foremost it is assumed that the dynamic state of the coastal region considered may be characterized by only one dependent variable: the effective volume of the sediment transported (in the total load sense) per unit length of shoreline. This parameter is interpreted as the linear concentration of the moving sediment and denoted by C. Generally speaking, the dependence of the parameter considered on the wave climate, granulometric properties of the movable sediment and bottom configuration of the coastal zone is assumed.

(ii) For each wave climate the existence of a special characteristic value C_* of the considered concentration is postulated, which is assumed to:

- correspond to the generally accepted concept of an average state of equilibrium of the long-term coastal bottom configuration, and
- determine the direction of the forthcoming concentration changes: $C < C_*$ is assumed to result in an increase, $C > C_*$ – in a decrease of the concentration.

(iii) The coastal zone considered is assumed to be characterized by strong seasonality with well-pronounced winter (or storm) and summer (or swell) conditions and the respective bed profiles. Time t is therefore considered in the often applied discrete manner, viz.: $t = nT$, where n – number of time periods, T – length of the period (usually not less than 1 year). Concentration value at the end of the n-th period is denoted by C_n.

(iv) The concentration change within the $n + 1$ period, viz. $C_{n+1} - C_n$, is simply assumed to be proportional to the difference $C_* - C_n$, i.e.:

$$C_{n+1} - C_n = K_n(C_* - C_n), \tag{1}$$

or

$$c_{n+1} - c_n = K_n(1 - c_n), \tag{2}$$

where $c_i = C_i/C_*$ $(i = n,\ n+1)$ are dimensionless concentrations. K_n is a positive coefficient (some kind of stirring parameter). Values $K_n > 1$ (resulting in "overshoots") are not excluded; on the contrary, such values seem to play an important role in the generation of intrinsic chaos.

(v) The value of K_n depends of course on the wave acitivity in the surf zone, therefore its proportionality to the dimensionless concentration c_n is assumed, i.e.

$$K_n = kc_n, \tag{3}$$

where k – a positive, more universal parameter, depending only on the grain size and characterizing the sensitivity of the sediment to the impact of waves and wave-generated currents (i.e. its mobility).

The final evolution relation of the proposed mathematical model may therefore be written in the following form:

$$c_{n+1} = c_n + kc_n(1 - c_n). \tag{4}$$

(vi) From the physical point of view the model is assumed to work in the following manner: low average sediment concentration during the n-th period corresponds to locally coarser grains and greater bed slopes, leading to more intensive local accumulation of fine-grained sediment, smaller slopes and higher sediment concentration during the $n+1$ period. Higher concentration of fine sediment leads, in turn, to more intensive local erosion and to low average concentration during the next period. In the case of small k-values the considered process attains eventually equilibrium. In the contrary situation, high k-values lead to much more complex behaviour. The variations in seasonally averaged sediment concentration are assumed to be accompanied by some changes in the shoreline location.

3. Fundamental Properties of the Proposed Model

From the mathematical point of view the proposed model surprisingly happens to be equivalent to the discrete version of the well-known long-term population model of Verhulst (1838). The results of numerical computations, carried out for sufficiently high period numbers and various values of the parameter k, are shown in Fig. 1.

Basing on these computations the behaviour of the analysed system may be characterized as follows:

a) For $0 < k \leq 2$ the dynamical system considered is asymptotically stable with c_n tending to 1, i.e. independent of its initial value C_0 the concentration C tends with increasing n to its equilibrium value C_*.

b) At $2 < k < 2.4$ the dimensionless concentration oscillates periodically between two values (one higher than 1, the second – lower).

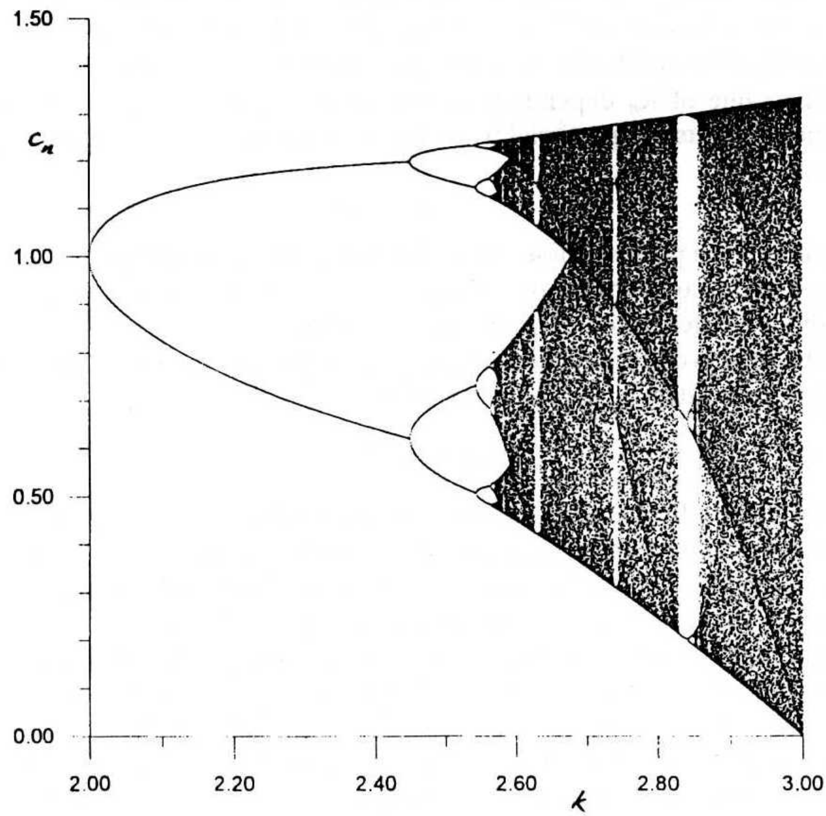

Fig. 1. Numerical evaluation of the fundamental properties of the proposed model ($1000 < n \leq 1200$)

c) At $2.4 < k < 2.57$ the number of "visited" values equal to 4, 8, 16, 32 and so on (the higher the k-value, the larger the number of values).

d) At $2.57 < k < 3$ the phenomenon of so-called deterministic chaos can be observed.

e) In case of k-values greater than 3 the model loses its physical meaning due to the appearance of negative concentrations.

In Fig. 2 the dependence of the averaged values of c_n, defined as:

$$\langle c_n \rangle = \frac{1}{N} \sum_{n=n_0}^{n_0+N} c_n, \qquad (n_0 = 10000,\ N = 10000) \tag{5}$$

on parameter k is depicted. In the interval of asymptotic stability ($0 < k \leq 2$) the mean value of the dimensionless concentration is, of course, equal to 1. It should be noticed, however, that for $k > 2$ the mean values considered are definitely smaller than 1, with a minimum value of about 0.7.

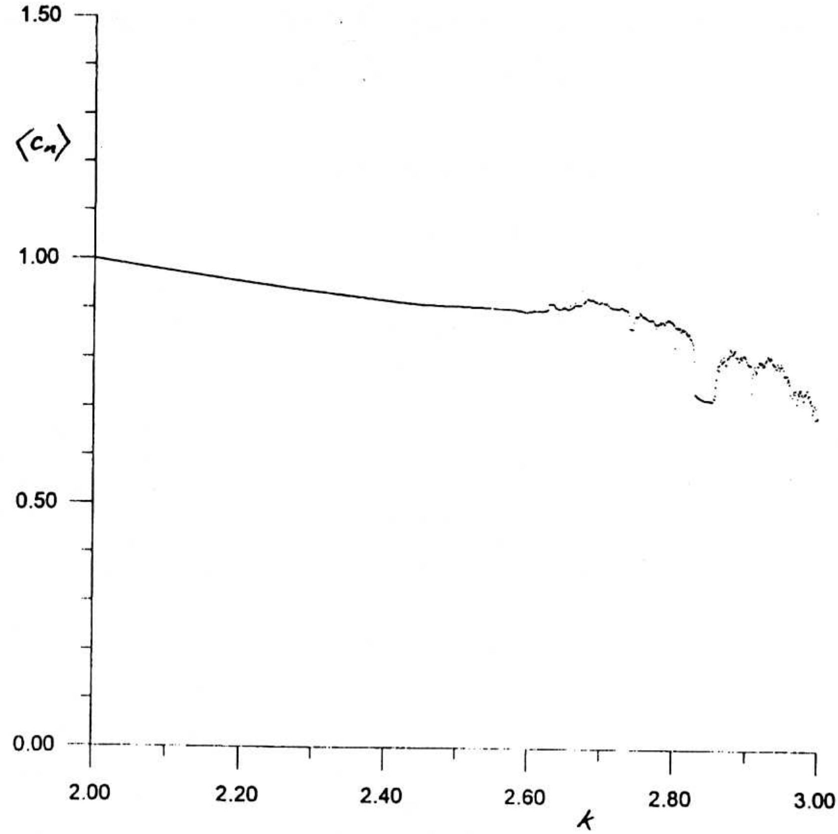

Fig. 2. The dependence of (numerically calculated) average c_n-values on parameter k

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