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Channels with Controlled Average Flow Velocity

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Abstract

A method of channels with controlled flow velocity designing is presented and experimentally verified. Proportional weirs at the outlets of the channel are used as velocity controlling devices. Studies are carried out for one and two-part weirs with discharge characteristic described by a power function. The conditions for practical shapes of the channel and weir are determined. This paper also presents particular solutions for the different discharge characteristic of the weir and velocity characteristic in a channel.

Notations

- a – height of the lower weir section,
- B – width of the channel at height $y = h_{max}$ for one-part channels and height $y = a$ for two-part channels,
- c – proportionality factor,
- C_d – coefficient of discharge,
- d – height of the location of the lower crest of the weir above the channel bottom,
- $f(y)$ – function determining the shape of the one-part weir,
- $f_1(y)$ – function determining the shape of the lower part of the two-part weir,
- $f_2(y)$ – function determining the shape of the upper part of the two-part weir,
- $F(y)$ – function determining the shape of the one-part channel,
- $F_1(y)$ – function determining the shape of the lower part of the two-part channel,
- $F_2(y)$ – function determining the shape of the upper part of the two-part channel,

g – acceleration of gravity,
h – head measured from the crest of the weir,
k, k_1, k_2, k_{30}, k_{31} – proportionality factors,
m, m_1, m_2, m_3 – exponents,
p – exponent,
Q, Q_1, Q_2 – discharge of the weir,
v, v_0 – average flow velocity in the channel,
λ – coefficient determining datum level for head.

1. Introduction

The control of flow velocity in a channel depends on satisfying the desired velocity – head relation. A proportional weir situated at the outlets of the channel can be used as a velocity controlling device. This method can be employed in model hydraulic studies and in technical devices, e.g. in grit chambers for sewage treatment.

Channels with controlled flow velocity designing requires the solving of two problems; the first refers to determining weir shapes for a given discharge characteristic of the weir and the other refers to determining the cross section shapes of the channel for a given velocity characteristic in a channel and discharge characteristic of the weir. The method of determining weir shapes was presented by Ricco (1963), Haszpra (1965), Rao et al. (1966, 1970, 1971), Banks et al. (1968), Murthy et al. (1968, 1969, 1977), Sreenivasulu and Raghavendran (1970), Grabarczyk and Majcherek (1973, 1974), Chandrasekaran and Rao (1976). The problem of determining channel shapes has not been entirely solved yet. The former study mainly concerned the control of flow velocity in grit chambers and was presented by Piotrowski et al. (1955, 1959), Stevens (1956), Rao and Chandrasekaran (1971, 1977), Majcherek (1985, 1990, 1997). The distribution of flow velocity in a cross section of a grit chamber above the weir controlling the velocity was examined by Majcherek (1972).

Continuation of this study is the purpose of the paper. The general method of determining the cross section shapes of the channel in which the average flow velocity is a specified function of the head with the use of weirs as velocity controlling devices is presented. The analytical solutions are compared to experimental results.

2. Shapes of Weirs with a Given Discharge Characteristic

The discharge characteristic of the free falling weir is discharge – head relation. The function determining the shape of the weir with a given discharge characteristic can be obtained from the equation describing the compatibility of weir discharge and discharge characteristic. Thus:

$$Q(h) = C_d\sqrt{2g}\int_0^h \sqrt{h-y}\,f(y)dy \qquad (1)$$

where:

- $Q(h)$ – discharge characteristic of the weir,
- C_d – coefficient of discharge,
- h – head measured from the crest of the weir,
- $f(y)$ – function determining the shape of the weir.

Differentiating Eq. 1 with respect to (h) we obtain the integral equation Abela type in the form:

$$\int_0^h \frac{1}{\sqrt{h-y}}f(y)dy = \frac{2}{C_d\sqrt{2g}}\frac{d}{dh}Q(h)$$

Solving the above equation in the manner presented by Grabarczyk and Majcherek (1973) gives:

$$f(y) = \frac{2}{C_d\pi\sqrt{2g}}\left\{\frac{1}{\sqrt{y-a}}\left[\frac{d}{dh}Q(h)\right]_{h=0} + \int_0^y \frac{1}{\sqrt{y-h}}\frac{d^2}{dh^2}Q(h)dh\right\} \qquad (2)$$

The shape of the weir is shown in Fig. 1.

Assuming the discharge characteristic of the weir in the form of a power function:

$$Q = kh^m \qquad (3)$$

one obtains from Eq. 2 the shape of the weir given by:

$$f(y) = \frac{2k}{C_d\sqrt{\pi}\sqrt{2g}}\frac{\Gamma(m+1)}{\Gamma(m-1/2)}y^{m-3/2} \qquad (4)$$

where:

- k – proportionality factor,
- m – exponent,
- $\Gamma(m+1)$, $\Gamma(m-1/2)$ – gamma functions.

From Eq. 4 we see that:

$f(y) < 0$	for	$0 < m < 1/2$
$f(y) = 0$	for	$m = 1/2$
$f(y) \to \infty$	for	$1/2 < m < 3/2$
$f(y) > 0$	for	$m \geq 3/2$

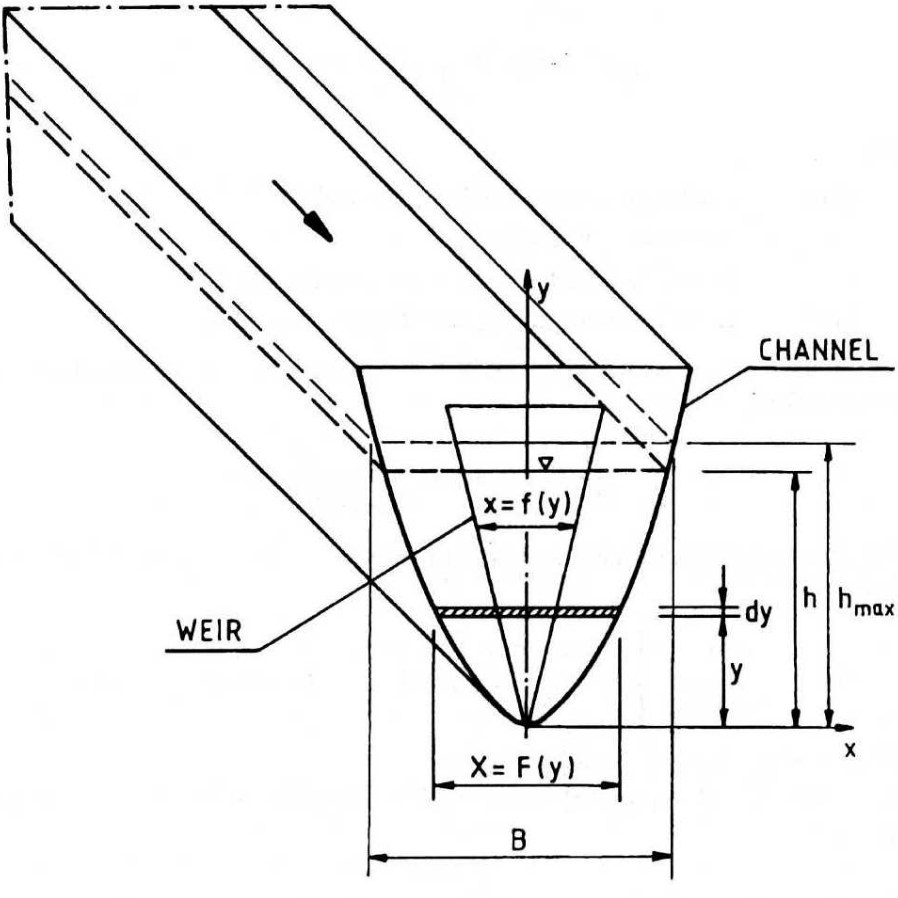

Fig. 1. One-part shapes of weir and channel

In the case of $m > 5/2$ the edge of the shape of the weir is at the beginning of the coordinates. Thus, the practical one-part shape of the weir described by one function can be obtained only for:

$$3/2 \leq m \leq 5/2 \tag{5}$$

For values of the exponent beyond the above range it is necessary to use a two-part weir: the lower one with the width described by the function $f_1(y)$ for $y \leq a$, and the upper one with the width described by the function $f_2(y)$ for $y \geq a$ (Fig. 2). This weir satisfies the discharge characteristic $Q_1(h)$ for $h \leq a$ and $Q_2(h)$ for $h \geq a$. In this case, the equations describing the compatibility of weir discharge and discharge characteristic can be written as:

$$Q_1(h) = C_d\sqrt{2g}\int_0^h \sqrt{h-y}\, f_1(y)dy \qquad \text{(for } h \le a\text{)} \qquad (6)$$

$$Q_2(h) = C_d\sqrt{2g}\left[\int_0^a \sqrt{h-y}\, f_1(y)dy + \int_a^h \sqrt{h-y}\, f_2(y)dy\right] \qquad \text{(for } h \ge a\text{)} \qquad (7)$$

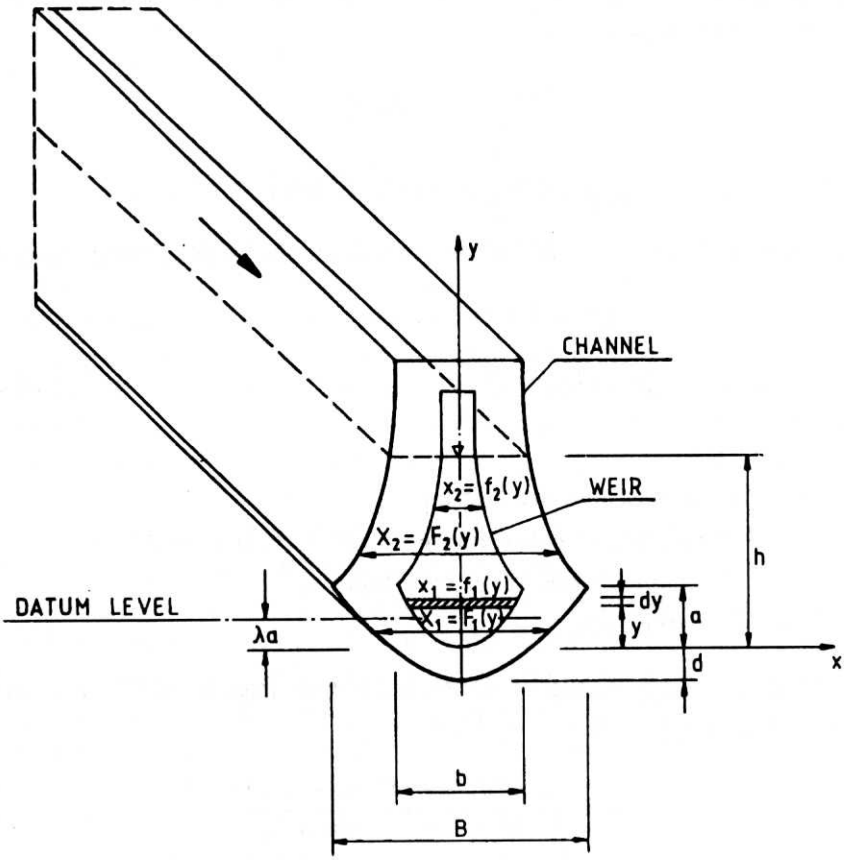

Fig. 2. Two-part shapes of weir and channel

Solving Eq. 6 one can obtain the function $f_1(y)$ which has the same form as Eq. 2:

$$f_1(y) = \frac{2}{C_d\pi\sqrt{2g}}\left\{\frac{1}{\sqrt{y-a}}\left[\frac{d}{dh}Q_1(h)\right]_{h=a} + \int_a^y \frac{1}{\sqrt{y-h}}\frac{d^2}{dh^2}Q_1(h)dh\right\} \qquad (8)$$

Subtracting both sides of Eq. 7 from Eq. 6, next differentiating both sides of the equation with respect to (h), and solving for $f_2(y)$, one obtains:

$$f_2(y) = f_1(y) - \frac{2}{C_d \pi \sqrt{2g}} \left\{ \frac{1}{\sqrt{y-a}} \left[\frac{d}{dh} (Q_1(h) - Q_2(h)) \right]_{h=a} + \right.$$
$$\left. + \int_a^y \frac{1}{\sqrt{y-h}} \frac{d^2}{dh^2} [Q_1(h) - Q_2(h)] \, dh \right\} \quad (9)$$

The equivalence of both functions $f_1(y)$ and $f_2(y)$ for $y = a$ is obtained when the following two conditions are satisfied:

$$[Q_1(h)]_{h=a} = [Q_2(h)]_{h=a} \quad (10)$$

$$\left[\frac{d}{dh} Q_1(h) \right]_{h=a} = \left[\frac{d}{dh} Q_2(h) \right]_{h=a} \quad (11)$$

Thus, discharge characteristic of the weir is in the form of power functions:

$$Q_1 = k_1 h^{m_1} \qquad \text{(for } h \le a\text{)} \quad (12)$$

$$Q_2 = k_2 (h - \lambda a)^{m_2} \qquad \text{(for } h \ge a\text{)} \quad (13)$$

where:

- k_1, k_2 – proportionality factors,
- λ – coefficient determining datum level for head,
- a – height of the lower weir section,
- m_1, m_2 – exponents.

Coefficients k_1, k_2 and λ should be determined from Eqs. 10 and 11 and from the condition that $f_1(y) = b$ for $y = a$. Thus:

$$k_1 = \frac{C_d \sqrt{\pi} \sqrt{2g} b}{2a^{m_1 - 3/2}} \frac{\Gamma(m_1 - 1/2)}{\Gamma(m_1 + 1)} \quad (14)$$

$$k_2 = \frac{C_d \sqrt{\pi} \sqrt{2g} b}{2a^{m_2 - 3/2}} \left(\frac{m_1}{m_2} \right)^{m_2} \frac{\Gamma(m_1 - 1/2)}{\Gamma(m_1 + 1)} \quad (15)$$

$$\lambda = \frac{m_1 - m_2}{m_1} \quad (16)$$

In that case Eqs. 8 and 9 can be expressed as:

$$f_1(y) = b \left(\frac{y}{a} \right)^{m_1 - 3/2} \quad (17)$$

$$f_2(y) = b\left\{\left(\frac{y}{a}\right)^{m_1-3/2} + \frac{m_1\Gamma(m_1-1/2)}{\sqrt{\pi}\Gamma(m_1+1)}\left[\frac{(m_2-1)}{a^{m_2-3/2}}\left(\frac{m_1}{m_2}\right)^{m_2-1}\times\right.\right.$$
$$\times\int_a^y \frac{1}{\sqrt{y-h}}\left(h-\frac{m_1-m_2}{m_1}a\right)^{m_2-2}dh - \frac{m_1-1}{a^{m_1-3/2}}\times \tag{18}$$
$$\left.\left.\times\int_a^y \frac{1}{\sqrt{y-h}}h^{m_1-2}dh\right]\right\}$$

Analysing Eq. 17 in an analogous manner to Eq. 4, it follows that the practical shape of the lower weir section can be obtained for values of the exponent:

$$3/2 \le m_1 \le 5/2 \tag{19}$$

3. Shapes of Channels with a Given Velocity Characteristic

The velocity characteristic in a channel describes the relationship between the average flow velocity and the head in the channel. In order to obtain the assumed flow velocity characteristic in the form of the function $v = v(h)$ when the flow changes one should control the head in the channel. A free falling weir installed at the channel's outlet which will adequately swell the head in a channel may be a controlling device. The purpose of this study is to determine the channel shape upstream of the weir, when the discharge characteristic of the weir and the velocity characteristic in the channel are given. It was assumed, that channels cooperating with one-part weirs should also have a one-part cross section shape described by a single function within the entire height range. Channels cooperating with two-part weirs should have a two-part cross section shape described by two different functions in the lower and the upper parts.

One-part weirs should be located at the channel bottom, whereas the two-part can be located both at the channel bottom and above. When the lower crest of the weir is situated at the channel bottom the desired velocity characteristic is satisfied within the entire range of the head. When the lower crest of the weir is situated above the channel bottom it is possible to achieve the desired velocity characteristic only when the head is bigger than the height of the lower part of the channel.

3.1. Channels with Weirs Located at Channel Bottom

Functions describing the cross section shape of the channel can be obtained from equations of the continuity of discharge passing through the weir and the channel. The velocity characteristic is given by the function:

$$v = ch^p \tag{20}$$

where:

c – proportionality factor,

p – exponent.

A. One-part channels and weirs (Fig. 1)

If a one-part weir is used, the shape of the channel is described by an unknown function $X = F(y)$. In this case, the equation of the continuity of discharge can be written as:

$$Q(h) = v(h) \int_0^h F(y)dy \tag{21}$$

where:

$v(h)$ – velocity characteristic in the channel,

$Q(h)$ – discharge characteristic of the weir.

Differentiating Eq. 21 with respect to (h) and solving for $F(y)$ gives:

$$F(y) = \frac{1}{[v(y)]^2}\left[v(y)\frac{d}{dy}Q(y) - Q(y)\frac{d}{dy}v(y)\right] \tag{22}$$

Substituting Eqs. 3 and 20, Eq. 22 can be expressed in the form:

$$F(y) = \frac{k(m-p)}{c}y^{m-p-1} \tag{23}$$

From Eq. 23 we see that:

$$\begin{array}{ll} F(y) = 0 & \text{for} \quad p = m \\ F(0) \to \infty & \text{for} \quad p > m-1 \text{ and } p \neq m \\ F(y) > 0 \text{ and } F(0) = 0 & \text{for} \quad p < m-1 \\ F(y) = \frac{k}{c} & \text{for} \quad p = m-1 \end{array}$$

In the case of $p < m-2$ the edge of the shape of the channel is at the bottom

This implies that the practical shape of the channel is for:

$$m-2 \leq p \leq m-1 \tag{24}$$

The practical shape of a one-part weir and channel can be obtained when conditions (5) and (24) are satisfied.

In order to connect the dimensions of the head and the width of the channel the parameter α from which the proportionality factor k can be calculated, was introduced. Thus:

$$\alpha = \frac{h_{\max}}{B} \tag{25}$$

where:

h_{max} – maximum head $h = h_{max}$ for maximum discharge $Q = Q_{max}$,
B – width of the channel $F(y) = B$ for $y = h_{max}$.

Substituting Eqs. 3 and 23 into Eq. 25 and solving for k, yields:

$$k = \left[\frac{c}{\alpha(m-p)}\right]^{\frac{m}{p+2}} Q_{max}^{\frac{p-m+2}{p+2}} \tag{26}$$

B. Two-part channels and weirs (Fig. 2)

If a two-part weir is used, the shape of the channel is also composed of two-parts: the lower one with the width described by the function $X_1 = F_1(y)$ for $y \leq a$ and the upper one with the width described by the function $X_2 = F_2(y)$ for $y \geq a$. The discharge characteristic of a two-part weir is taken as $Q_1(h)$ for $h \leq a$ and $Q_2(h)$ for $h \geq a$. Equations of the continuity of discharges are as follows:

$$Q_1(h) = v(h) \int_0^h F_1(y)dy \qquad \text{(for } h \leq a\text{)} \tag{27}$$

$$Q_2(h) = v(h) \left[\int_0^a F_1(y)dy + \int_a^h F_2(y)dy \right] \qquad \text{(for } h \geq a\text{)} \tag{28}$$

Solving Eqs. 27 and 28 in analogy to Eq. 21 gives:

$$F_1(y) = \frac{1}{[v(y)]^2}\left[v(y)\frac{d}{dy}Q_1(y) - Q_1(y)\frac{d}{dy}v(y)\right] \tag{29}$$

$$F_2(y) = \frac{1}{[v(y)]^2}\left[v(y)\frac{d}{dy}Q_2(y) - Q_2(y)\frac{d}{dy}v(y)\right] \tag{30}$$

Substituting Eqs. 12, 13 and 20 into Eqs. 29 and 30 gives:

$$F_1(y) = \frac{k_1(m_1 - p)}{c} y^{m_1-p-1} \tag{31}$$

$$F_2(y) = \frac{k_2}{cy^{p+1}}\left[m_2 y\,(y - \lambda a)^{m_2-1} - p\,(y - \lambda a)^{m_2}\right] \tag{32}$$

Analysis of Eq. (31) analogous to Eq. (23) shows that the practical shape of the lower channel section can be obtained for $m_1 - 2 \leq p \leq m_1 - 1$. On the basis of Eq. 32 one can show that the width of the upper channel section has positive values for $p < m_2$. Thus:

$$m_1 - 2 \leq p \leq m_1 - 1 \quad \text{and} \quad p < m_2 \tag{33}$$

The practical shape of a two-part weir and channel can be obtained when conditions (19) and (33) are satisfied.

The width of the weir $f_1(y) = b$ for $y = a$ can be calculated from the following condition:

$$\alpha = \frac{h_{\max} - \lambda a}{B} \tag{34}$$

where:

- $h_{\max}$ – maximum head $h = h_{\max}$ for maximum discharge $Q = Q_{\max}$,
- B – width of the channel $F_1(y) = B$ for $y = a$.

Substituting the head $(h_{\max} - \lambda a)$ according to Eq. 13 and the width (B) according to Eq. 31, one obtains the preceding condition in the form:

$$\alpha = \frac{c}{(m_1 - p)a^{m_2-p-1}} \left(\frac{m_1}{m_2}\right)^{m_2} \left(\frac{Q_{\max}}{k_2^{1+m_2}}\right)^{1/m_2} \tag{35}$$

Next substituting Eq. 15 and solving for b, yields:

$$b = \frac{2\Gamma(m_1 + 1)}{C_d\sqrt{\pi}\sqrt{2g}\,\Gamma(m_1 - 1/2)} \left[\frac{Q_{\max}}{a^{3/2-m_2(p+1/2)}} \left(\frac{m_2 c}{m_1 \alpha (m_1 - p)}\right)^{m_2}\right]^{1/(1+m_2)} \tag{36}$$

3.2. Channels with Weirs Located Above the Channel Bottom

When the weir is situated above the channel bottom, the shape of the lower channel section is described by the given function $F_1(y)$ and the shape of the upper channel section is described by the unknown function $F_2(y)$ which is to be derived from the flow continuity condition. Function $F_1(y)$ is given in the form:

$$F_1(y) = (k_{30}y + k_{31})^{m_3} \quad (\text{for} \quad -d \le y \le a) \tag{37}$$

where:

- k_{30}, k_{31} – proportionality factors,
- m_3 – exponent,
- d – height of the location of the lower crest of the weir above the channel bottom.

Function $F_2(y)$ can be derived from the flow continuity condition for the weir and for the channel for $h \ge a$. The flow rate in the channel is given by:

$$Q_I(h) = v_0(h) \int_{-d}^{h} F_1(y)dy \qquad (\text{for } 0 < h \le a) \tag{38}$$

$$Q_{II}(h) = v(h)\left[\int_{-d}^{a} F_1(y)dy + \int_{a}^{h} F_2(y)dy\right] \quad (\text{for } h \geq a) \tag{39}$$

where:

$v_0(h)$ – average flow velocity in the channel for $0 < h \leq a$,
$v(h)$ – desired velocity characteristic in the channel for $h \geq a$.

Equation of the flow continuity is:

$$Q_2(h) = Q_{II}(h) \qquad \text{for } h \geq a \tag{40}$$

where: $Q_2(h)$ – discharge characteristic of the weir for $h \geq a$.
Substituting Eq. 39 into Eq. 40 one obtains:

$$Q_2(h) = v(h)\left[\int_{-d}^{a} F_1(y)dy + \int_{a}^{h} F_2(y)dy\right] \tag{41}$$

Differentiating Eq. 41 with respect to h, function $F_2(y)$ takes the form of Eq. 30.

The equivalence of both functions $F_1(y)$ and $F_2(y)$ at height $y = a$ is obtained when two conditions defined for the flows in the channel $Q_I(h)$ and $Q_{II}(h)$ for $h = a$ are satisfied. The first condition can be written in the form:

$$\left[\frac{Q_I(h)}{v_0(h)}\right]_{h=a} = \left[\frac{Q_{II}(h)}{v(h)}\right]_{h=a} \tag{42}$$

To find the second condition one can subtract both sides of Eq. 39 from Eq. 38 and differentiate both sides of the equation with respect to h. Thus:

$$F_1(y) - F_2(y) = \frac{d}{dh}\left[\frac{Q_I(h)}{v_0(h)} - \frac{Q_{II}(h)}{v(h)}\right] \tag{43}$$

Expanding the right hand side of Eq. 43 in a Taylor's series as:

$$F_1(y) - F_2(y) = \left[\frac{d}{dh}\left(\frac{Q_I(h)}{v_0(h)} - \frac{Q_{II}(h)}{v(h)}\right)\right]_{h=a} + \\ + \sum_{n=1}^{\infty}\frac{(h-a)^n}{n!}\left[\frac{d^{n+1}}{dh^{n+1}}\left(\frac{Q_I(h)}{v_0(h)} - \frac{Q_{II}(h)}{v(h)}\right)\right]_{h=a} \tag{44}$$

one can obtain the second condition in the form:

$$\left[\frac{d}{dh}\left(\frac{Q_I(h)}{v_0(h)}\right)\right]_{h=a} = \left[\frac{d}{dh}\left(\frac{Q_{II}(h)}{v(h)}\right)\right]_{h=a} \tag{45}$$

Substituting Eq. 37 into Eq. 38 yields:

$$Q_I(h) = \frac{v_0(h)}{k_{30}(m_3+1)}\left[(k_{30}h + k_{31})^{m_3+1} - (k_{31} - k_{30}d)^{m_3+1}\right] \tag{46}$$

and assuming, according to Eqs. 13 and 40, the relation:

$$Q_{II}(h) = k_2(h - \lambda a)^{m_2} \tag{47}$$

the conditions described by Eqs. 42 and 45 can be expressed as:

$$\frac{1}{k_{30}(m_3+1)}\left[(k_{30}a + k_{31})^{m_3+1} - (k_{31} - k_{30}d)^{m_3+1}\right] = \frac{k_2}{v(a)}(a - \lambda a)^{m_2} \tag{48}$$

$$(k_{30}a + k_{31})^{m_3} = \frac{k_2}{v(a)}(a - \lambda a)^{m_2}\left\{\frac{m_2}{a(1-\lambda)} - \frac{1}{v(a)}\left[\frac{d}{dh}v(h)\right]_{h=a}\right\} \tag{49}$$

from which the factors k_{30} and k_{31} can be calculated.

The desired velocity characteristic $v = v(h)$ is assumed in the form:

$$v = c(h+d)^p \tag{50}$$

and the value of the exponent $m_3 = 1$.
In this case the shape of the channel, according to Eqs. 37 and 30 is described by the functions:

$$F_1(y) = k_{30}y + k_{31} \tag{51}$$

$$F_2(y) = \frac{k_2}{c(y+d)^{p+1}}\left[m_2(y+d)(y-\lambda a)^{m_2-1} - p(y-\lambda a)^{m_2}\right] \tag{52}$$

Making use of Eqs. 48 and 49, the factors k_{30} and k_{31} can be determined. Substituting according to Eqs. 14, 15 and 16 the following relations:

$$k_2 = k_1 a^{m_1-m_2}\left(\frac{m_1}{m_2}\right)^{m_2} \tag{53}$$

$$m_2 = m_1(1-\lambda) \tag{54}$$

and solving Eqs. 48 and 49 for k_{30} and k_{31}, yields:

$$k_{30} = \frac{2k_1a^{m_1-1}}{c(a+d)^{p+2}}\left[m_1(a+d) - a(p+1)\right] \tag{55}$$

$$k_{31} = \frac{k_1a^{m_1-1}}{c(a+d)^{p+2}}\left[ap(a-d) + 2a^2 - m_1(a^2-d^2)\right] \tag{56}$$

The function $F_1(y)$ should be rising ($k_{30} > 0$) and at the height of $y = -d$ should have positive values or should equal zero. Substituting Eqs. 55 and 56 into Eq. 51 shows that:

$$k_{30} > 0 \qquad \text{for } \frac{d}{a} > \frac{p+1}{m_1} - 1 \tag{57}$$

$$F_1(-d) \geq 0 \qquad \text{for } \frac{d}{a} \leq \frac{p+2}{m_1} - 1 \tag{58}$$

and also the ratio $d/a > 0$.

The function $F_2(y)$ should be positive at the whole range of height. The analysis of the equation (52) shows that:

$$F_2(y) > 0 \quad \text{for } \ p < m_2 \tag{59}$$

Taking into account the above conditions it can be written:

$$0 < \frac{d}{a} \leq \frac{p+2}{m_1} - 1 \tag{60}$$

$$m_1 - 2 < p < m_2 \tag{61}$$

where the value of the exponent m_1 should be taken according to condition 19.

The height of the lower part of the weir (a) should be calculated from the condition $h_{\min} \geq a$ for $Q_1 = Q_{\min}$ which according to the equation (12) is written as:

$$Q_{\min} \geq k_1 a^{m_1} \tag{62}$$

where, after taking into account Eq. 14, one can obtain:

$$a \leq \left[\frac{2Q_{\min}}{C_d\sqrt{\pi}\sqrt{2g}b} \frac{\Gamma(m_1+1)}{\Gamma(m_1-1/2)} \right]^{2/3} \tag{63}$$

Conditions 60, 61 and 63 guarantee the obtaining of practical solutions.

4. Examples

Examples of solutions are presented to illustrate the applicability of the method of channels with controlled flow velocity designing.

4.1. One-part Channels and Weirs

The velocity characteristic in a channel is described by Eq. 20:

$$v = ch^p$$

and the discharge characteristic of the weir is described by Eq. 3:

$$Q = kh^m$$

where proportionality factor k is given by Eq. 26.

The practical shape of a one-part channel and weir can be obtained for values of the exponent satisfying conditions 5 and 24. Thus:

$$3/2 \le m \le 5/2 \quad \text{and} \quad \begin{array}{ll} -1/2 \le p \le 1/2 & \text{for} \ \ m = 3/2 \\ 0 \le p \le 1 & \text{for} \ \ m = 2 \\ 1/2 \le p \le 3/2 & \text{for} \ \ m = 5/2 \end{array}$$

The shape of the weir is described by Eq. 4 and the shape of the channel by Eq. 23.

The results of the calculation of the shapes of the weir and channel for m = 3/2; 2; 5/2 are presented in Table 1 and Fig. 3.

Table 1. Functions describing one-part shapes of weirs and channels

Discharge characteristic of the weir		Velocity characteristic in a channel		Proportionality factor	Shape of the weir	Shape of the channel
m	$Q(h)$	p	$v(h)$	k	$x = f(y)$	$X = F(y)$
$\frac{3}{2}$	$Q = kh^{3/2}$	$\frac{1}{2}$	$v = ch^{1/2}$	$k = \left(\frac{c}{\alpha}\right)^{3/5} Q_{\max}^{2/5}$	$x = \frac{3k}{2C_d\sqrt{2g}}$	$X = \frac{k}{c}$
		0	c	$k = \left(\frac{2c}{3\alpha}\right)^{3/4} Q_{\max}^{1/4}$		$X = \frac{3k}{2c} y^{1/2}$
		$-\frac{1}{2}$	$ch^{-1/2}$	$k = \frac{c}{2\alpha}$		$X = \frac{2k}{c} y$
2	$Q = kh^2$	1	ch	$k = \left(\frac{c}{\alpha}\right)^{2/3} Q_{\max}^{1/3}$	$x = \frac{8k}{C_d \pi \sqrt{2g}} y^{1/2}$	$X = \frac{k}{c}$
		$\frac{1}{2}$	$ch^{1/2}$	$k = \left(\frac{2c}{3\alpha}\right)^{4/5} Q_{\max}^{1/5}$		$X = \frac{3k}{2c} y^{1/2}$
		0	c	$k = \frac{c}{2\alpha}$		$X = \frac{2k}{c} y$
$\frac{5}{2}$	$Q = kh^{5/2}$	$\frac{3}{2}$	$ch^{3/2}$	$k = \left(\frac{c}{\alpha}\right)^{5/7} Q_{\max}^{2/7}$	$x = \frac{15k}{4C_d\sqrt{2g}} y$	$X = \frac{k}{c}$
		1	ch	$k = \left(\frac{2c}{3\alpha}\right)^{5/6} Q_{\max}^{1/6}$		$X = \frac{3k}{2c} y^{1/2}$
		$\frac{1}{2}$	$ch^{1/2}$	$k = \frac{c}{2\alpha}$		$X = \frac{2k}{c} y$

4.2. Two-part Channels and Weirs

The velocity characteristic is given by Eq. 20 for $d = 0$ or Eq. 50 for $d > 0$:

$$v = ch^p \quad \text{for} \ \ d = 0$$
$$v = c(h+d)^p \quad \text{for} \ \ d > 0$$

The discharge characteristic of the weir is given by Eqs. 12 and 13:

$$Q_1 = k_1 h^{m_1} \quad \text{for} \ \ h \le a$$
$$Q_2 = k_2 (h - \lambda a)^{m_2} \quad \text{for} \ \ h \ge a$$

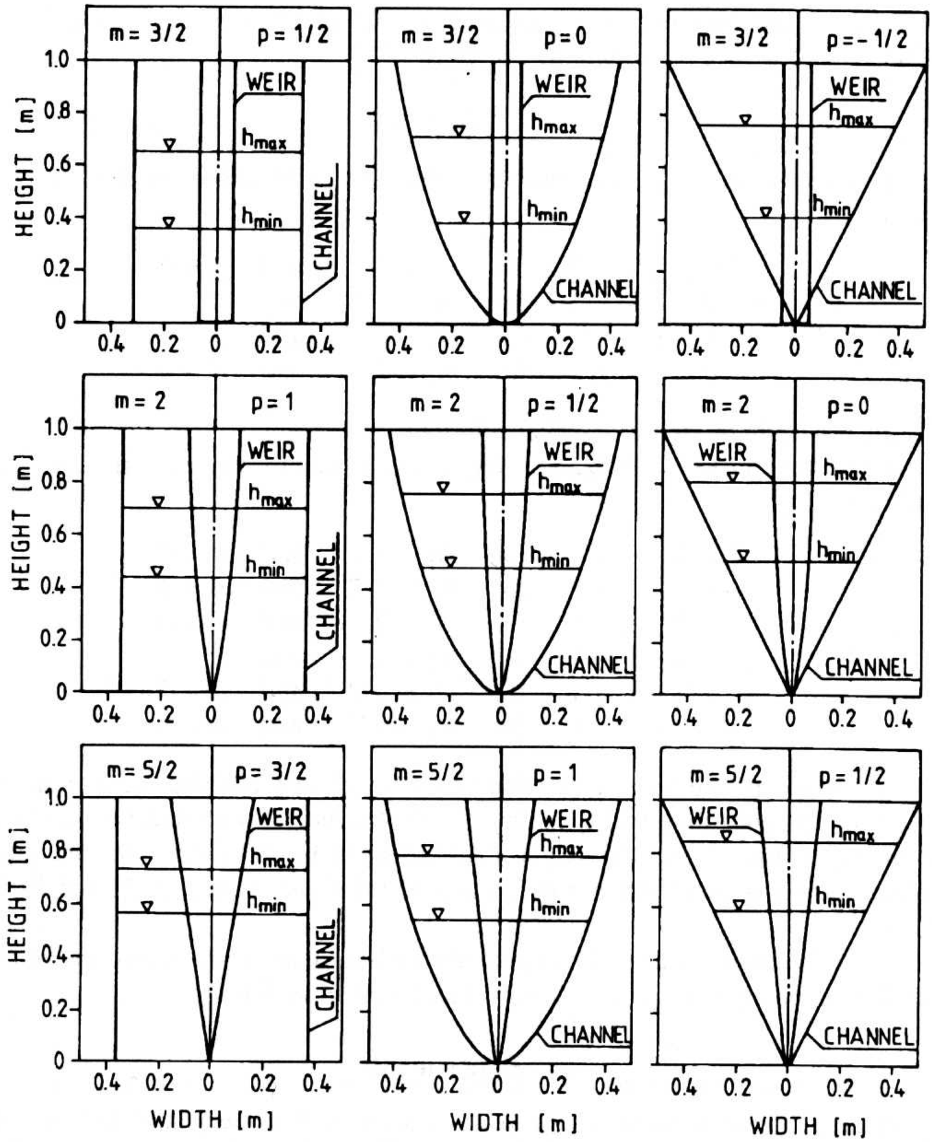

Fig. 3. One-part shapes of weirs and channels for $Q_{max} = 0.1$ m^3/s; $Q_{min} = 0.04$ m^3/s; $\alpha = 1$; $c = 0.30$; $C_d = 0.60$

where coefficients k_1, k_2 and λ are determined by Eqs. 14, 15 and 16.

Examples are presented for two cases: the first for $m_2 = 1/2$ and the second for $m_2 = 1$. The practical shape of a two-part weir and channel can be obtained for values of the exponents satisfying conditions 19 and 33 for $d = 0$ or conditions 19 and 61 for $d > 0$. According to Eq. 19 is:

$$3/2 \le m_1 \le 5/2$$

In the first case ($m_2 = 1/2$), according to Eqs. 33 and 61, there are the following limitations:

$$\begin{array}{rlll} -1/2 \le p < 1/2 & \text{for} \quad m_1 = 3/2 & \text{and} \quad d = 0 \\ -1/2 < p < 1/2 & \text{for} \quad m_1 = 3/2 & \text{and} \quad d > 0 \\ 0 \le p < 1/2 & \text{for} \quad m_1 = 2 & \text{and} \quad d = 0 \\ 0 < p < 1/2 & \text{for} \quad m_1 = 2 & \text{and} \quad d > 0 \end{array}$$

For $m_1 = 5/2$ there are no practical solutions.

In the second case $m_2 = 1$, according to Eqs. 33 and 61, there are the following limitations:

$$\begin{array}{rlll} -1/2 \le p \le 1/2 & \text{for} \quad m_1 = 3/2 & \text{and} \quad d = 0 \\ -1/2 < p < 1 & \text{for} \quad m_1 = 3/2 & \text{and} \quad d > 0 \\ 0 \le p < 1 & \text{for} \quad m_1 = 2 & \text{and} \quad d = 0 \\ 0 < p < 1 & \text{for} \quad m_1 = 2 & \text{and} \quad d > 0 \\ 1/2 \le p < 1 & \text{for} \quad m_1 = 5/2 & \text{and} \quad d = 0 \\ 1/2 < p < 1 & \text{for} \quad m_1 = 5/2 & \text{and} \quad d > 0 \end{array}$$

The shape of the weir is described by Eqs. 17 and 18. The width b should be calculated according to Eq. 36 for $d = 0$. The height a should be calculated according to Eq. 63 and the height d according to Eq. 60 for $d > 0$. The shape of the channel is described by Eqs. 31 and 32 for $d = 0$ or by Eqs. 51 and 52 for $d > 0$.

The calculation results of two-part channels and weirs are shown in Table 2 and Fig. 4 for $m_2 = 1/2$ and in Tables 3 and 4 and Fig. 5 for $m_2 = 1$.

5. Experiments

Experiments were conducted in a 0.60 m wide, 1.00 m deep and 23.8 m long rectangular channel in which a weir to control flow velocity was installed. The incoming water from the 18.0 m^3 volume upper tank entered the initial part of the channel. The discharge was controlled by a 90° triangular weir and was then experimentally tranquilized by perforated plates. Water levels in the channel were measured by point gages with the precision of ±0.1 mm. The purpose of these experiments was to verify the theoretical solutions for two cases.

Table 2. Functions describing two-part shapes of weirs and channels for $m_2 = 1/2$

Discharge characteristic of the weir		Shape of the weir	Velocity characteristic in the channel			Shape of the channel
m_1	$Q_1(h)$ $Q_2(h)$	$x_1 = f_1(y)$ $x_2 = f_2(y)$	p	d	$v(h)$	$X_1 = F_1(y)$ $X_2 = F_2(y)$
$\frac{3}{2}$	$Q_1 = k_1 h^{3/2}$ $Q_2 = k_2(h - \lambda a)^{1/2}$ where: $k_1 = \frac{2}{3} C_d \sqrt{2g} b$ $k_2 = \frac{2}{\sqrt{3}} C_d \sqrt{2g} ab$ $\lambda = 2/3$	$x_1 = b$ $x_2 = b\left[1 - \frac{2}{\pi} \arctan \times \right.$ $\left. \times \sqrt{\frac{y-a}{a}} - \frac{6}{\pi} \frac{\sqrt{a(y-a)}}{(3y-2a)}\right]$	0	0	c	$X_1 = \frac{3k_1}{2c}\sqrt{y}$ $X_2 = \frac{3k_1 a}{2c\sqrt{3y-2a}}$
				> 0	c	$X_1 = \frac{k_1\sqrt{a}}{2c(a+d)^2} \times$ $\times \left[a^2 + 3d^2 + \right.$ $\left. + 2y(a+3d)\right]$ $X_2 = \frac{3k_1 a}{2c\sqrt{3y-2a}}$
			$-\frac{1}{2}$	0	$ch^{-1/2}$	$X_1 = \frac{2k_1}{c} y$ $X_2 = \frac{k_1 a(3y-a)}{c\sqrt{y(3y-2a)}}$
2	$Q_1 = k_1 h^2$ $Q_2 = k_2(h - \lambda a)^{1/2}$ where: $k_1 = \frac{C_d \pi \sqrt{2g} b}{8\sqrt{a}}$ $k_2 = \frac{1}{4} C_d \pi \sqrt{2g} ab$ $\lambda = 3/4$	$x_1 = b\sqrt{y/a}$ $x_2 = b\sqrt{y/a} \times$ $\times \left[1 - \sqrt{\frac{y-a}{y}} \frac{(4y-a)}{(4y-3a)}\right]$	0	0	c	$X_1 = \frac{2k_1}{c} y$ $X_2 = \frac{2k_1 a^{5/2}}{cy\sqrt{4y-3a}}$

CASE 1

Assume:

– Velocity characteristic in the channel:

$$v = c \tag{64}$$

– Discharge characteristic of the weir to control the velocity in the channel:

$$Q_1 = k_1 h^{3/2} \qquad \text{for } h \leq a \tag{65}$$

$$Q_2 = k_2(h - \lambda a)^{1/2} \qquad \text{for } h \geq a \tag{66}$$

– Values of the parameters: $a = 0.15$ m; $b = 0.40$ m; $c = 0.30$ m/s; $C_d = 0.60$

In this case the shapes of the weir and the channel are described by the functions given in Table 2 for $m_1 = 3/2$ and $m_2 = 1/2$. The weir is described by

Table 3. Functions describing two-part shapes of weirs and channels for $m_2 = 1$ and $m_1 = 3/2$

Discharge characteristic of the weir		Shape of the weir	Velocity characteristic in the channel			Shape of the channel
m_1	$Q_1(h)$ $Q_2(h)$	$x_1 = f_1(y)$ $x_2 = f_2(y)$	p	d	$v(h)$	$X_1 = F_1(y)$ $X_2 = F_2(y)$
$\frac{3}{2}$	$Q_1 = k_1 h^{3/2}$ $Q_2 = k_2(h - \lambda a)$ where: $k_1 = \frac{2}{3} C_d \sqrt{2g} b$ $k_2 = C_d \sqrt{2g} \sqrt{a} b$ $\lambda = 1/3$	$x_1 = b$ $x_2 = b\left(1 - \frac{2}{\pi} \times \right.$ $\left. \times \arctan \sqrt{\frac{y-a}{a}}\right)$	$\frac{1}{2}$	0	$ch^{1/2}$	$X_1 = \frac{k_1}{c}$ $X_2 = \frac{k_1 \sqrt{a}(3y+a)}{4cy^{3/2}}$
				> 0	$c(h+d)^{1/2}$	$X_1 = \frac{k_1 \sqrt{a}}{2c(a+d)^{5/2}} \times$ $\times \left(2a^2 - ad + \right.$ $\left. + 3d^2 + 6dy\right)$ $X_2 = \frac{k_1 \sqrt{a}(3y+6d+a)}{4c(y+d)^{3/2}}$
			0	0	c	$X_1 = \frac{3}{2}\frac{k_1}{c}\sqrt{y}$ $X_2 = \frac{3}{2}\frac{k_1\sqrt{a}}{c}$
				> 0	c	$X_1 = \frac{k_1 \sqrt{a}}{2c(a+d)^2} \times$ $\times \left[a^2 + 3d^2 + \right.$ $\left. + 2y(a+3d)\right]$ $X_2 = \frac{3}{2}\frac{k_1\sqrt{a}}{c}$
			$-\frac{1}{2}$	0	$ch^{-1/2}$	$X_1 = \frac{2k_1}{c} y$ $X_2 = \frac{k_1 \sqrt{a}(9y-a)}{4c\sqrt{y}}$

the following functions:

$$f_1(y) = b \tag{67}$$

$$f_2(y) = b\left[1 - \frac{2}{\pi}\arctan\sqrt{\frac{y-a}{a}} - \frac{6}{\pi}\frac{\sqrt{a(y-a)}}{(3y-2a)}\right] \tag{68}$$

Substituting the factors k_1, k_2 and λ for $m_1 = 3/2$ and $m_2 = 1/2$ (Table 2) into Eqs. 65 and 66 gives:

$$Q_1 = \frac{2}{3} C_d \sqrt{2g} b h^{3/2} \tag{69}$$

$$Q_2 = \frac{2}{\sqrt{3}} C_d \sqrt{2g} a b \left(h - \frac{2}{3}a\right)^{1/2} \tag{70}$$

Table 4. Functions describing two-part shapes of weirs and channels for $m_2 = 1$ and $m_1 = 2$ and $m_1 = 5/2$

Discharge characteristic of the weir		Shape of the weir	Velocity characteristic in the channel			Shape of the channel
2	$Q_1 = k_1 h^2$ $Q_2 = k_2(h - \lambda a)$ where: $k_1 = \frac{C_d \pi \sqrt{2gb}}{8\sqrt{a}}$ $k_2 = \frac{1}{4} C_d \pi \sqrt{2g}\sqrt{ab}$ $\lambda = 1/2$	$x_1 = b\sqrt{\frac{y}{a}}$ $x_2 = b\sqrt{\frac{y}{a}}\left(1 - \sqrt{\frac{y-a}{a}}\right)$	$\frac{1}{2}$	0	$ch^{1/2}$	$X_1 = \frac{3k_1}{2c}\sqrt{y}$ $X_2 = \frac{k_1 a(2y+a)}{2cy^{3/2}}$
				> 0	$c(h+d)^{1/2}$	$X_1 = \frac{k_1 a}{2c(a+d)^{5/2}} \times$ $\times \left[a^2 - ad + 4d^2 +\right.$ $\left.+ 2y(a + 4d)\right]$ $X_2 = \frac{k_1 a(2y+a+4d)}{2c(y+d)^{3/2}}$
			0	0	c	$X_1 = \frac{2k_1}{c} y$ $X_2 = \frac{2k_1 a}{c}$
$\frac{5}{2}$	$Q_1 = k_1 h^{5/2}$ $Q_2 = k_2(h - \lambda a)$ where: $k_1 = \frac{4C_d\sqrt{2gb}}{15a}$ $k_2 = \frac{2}{3} C_d \sqrt{a}\sqrt{2gb}$ $\lambda = 3/5$	$X_1 = \frac{b}{a} y$ $X_2 = \frac{by}{a} \times$ $\times \left(1 - \frac{2a}{\pi y} \times\right.$ $\sqrt{\frac{y-a}{a}} - \frac{2}{\pi} \times$ $\left.\times \arctan\sqrt{\frac{y-a}{a}}\right)$	$\frac{1}{2}$	0	$ch^{1/2}$	$X_1 = \frac{2k_1}{c} y$ $X_2 = \frac{k_1(5y+3a)}{4c}\left(\frac{a}{y}\right)^{3/2}$

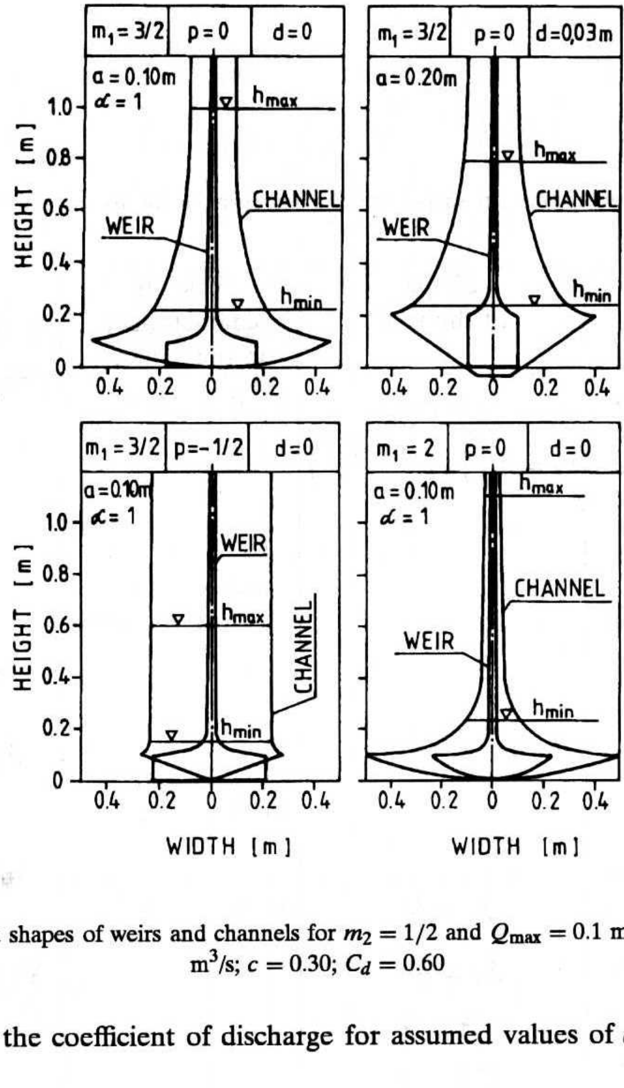

Fig. 4. Two-part shapes of weirs and channels for $m_2 = 1/2$ and $Q_{max} = 0.1$ m^3/s; $Q_{min} = 0.04$ m^3/s; $c = 0.30$; $C_d = 0.60$

Using Eq. 70, the coefficient of discharge for assumed values of a and b can be determined:

$$C_d = \frac{Q_2}{0.3069(h - 0.10)^{1/2}} \tag{71}$$

The average flow velocity in the channel for $h \geq a$ can be calculated from the following equations:

$$v = \frac{Q_2}{\int_0^a F_1(y)dy + \int_a^h F_2(y)dy} \quad \text{for } d = 0 \tag{72}$$

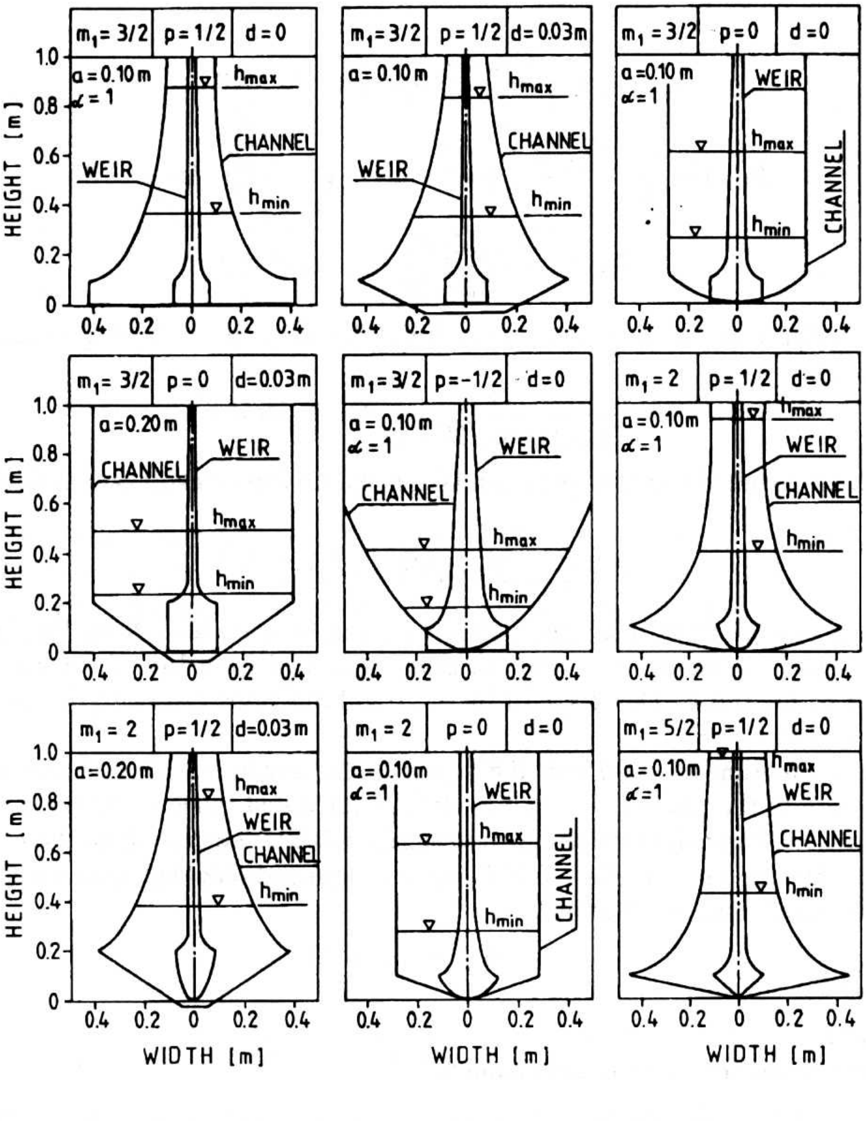

Fig. 5. Two-part shapes of weirs and channels for $m_2 = 1$ and $Q_{max} = 0.1\ m^3/s$; $Q_{min} = 0.04\ m^3/s$; $c = 0.30$; $C_d = 0.60$

$$v = \frac{Q_2}{\int\limits_{-d}^{a} F_1(y)dy + \int\limits_{a}^{h} F_2(y)dy} \qquad \text{for } d > 0 \tag{73}$$

where the functions $F_1(y)$ and $F_2(y)$ are given in Table 2:

$$F_1(y) = \frac{C_d\sqrt{2g}b}{c}\sqrt{y} \qquad \text{for } d = 0 \tag{74}$$

$$F_1(y) = \frac{C_d\sqrt{2g}\sqrt{ab}}{3c(a+d)^2}\left[2y(a+3d) + a^2 + 3d^2\right] \quad \text{for } d > 0 \tag{75}$$

$$F_2(y) = \frac{C_d\sqrt{2g}ab}{c\sqrt{3y-2a}} \qquad \text{for } d \geq 0 \tag{76}$$

Substituting Eqs. 74 or 75 and 76 into Eqs. 72 and 73 the following equation is obtained:

$$v = \frac{3c}{2C_d\sqrt{2g}ab\sqrt{3h-2a}}Q_2 \qquad \text{for } d \geq 0 \tag{77}$$

which for assumed values of parameters a, b and C_d can be expressed in the form:

$$v = \frac{2.8220}{\sqrt{3h-0.30}}Q_2 \qquad \text{for } d \geq 0 \tag{78}$$

The coefficient of discharge of the weir can be calculated from Eq. 71 and the average velocity in the channel can be calculated from Eq. 78 for measured values of Q_2 and h. The shape of the weir and functions $Q_2(h)$, $C_d(h)$ and $v(h)$ are given in Fig. 6.

The experiments show that the values of the coefficient of discharge of the weir were in the range $C_d = 0.5791 \div 0.5574$ for tested range of height h. These values were lower than the assumed value $C_d = 0.60$. The velocity in the channel was in the range $v = 0.2896 \div 0.2787$ m/s so it changed only slightly and was close the assumed value $v = 0.30$ m/s.

CASE 2

Assume:

– Velocity characteristic in the channel:

$$v = c \tag{79}$$

– Discharge characteristic of the weir to control the velocity in the channel:

$$Q_1 = k_1 h^{3/2} \qquad \text{for } h \leq a \tag{80}$$

$$Q_2 = k_2(h - \lambda a) \qquad \text{for } h \geq a \tag{81}$$

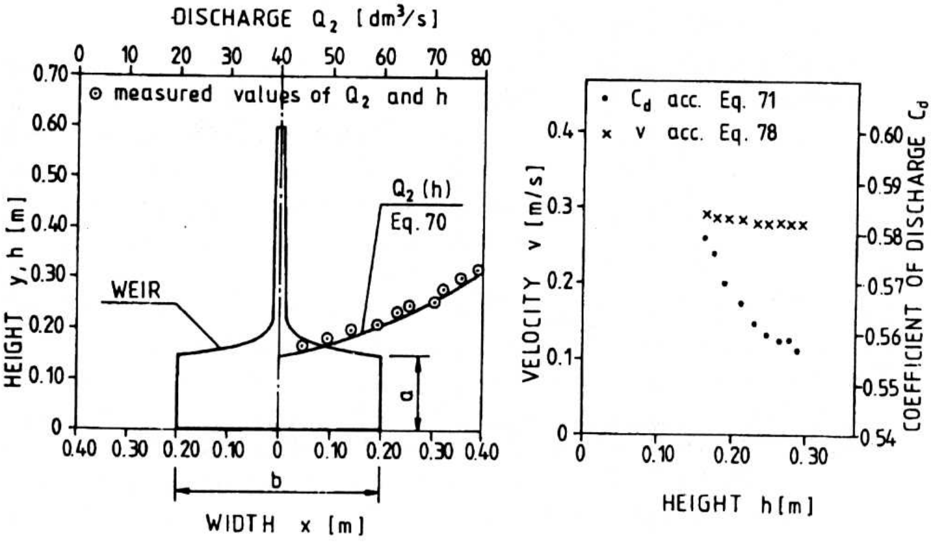

Fig. 6. The shape of weir and functions: $Q_2(h)$, $C_d(h)$, $v(h)$ for $m_2 = 1/2$ and $m_1 = 3/2$

– Values of the parameters: $a = 0.15$ m; $b = 0.40$ m; $c = 0.30$ m/s; $C_d = 0.60$

The shapes of the weir (type Sutro) and the channel are given by the functions in Table 3 for $m_1 = 3/2$ and $m_2 = 1$. The weir is described by the following functions:

$$f_1(y) = b \qquad \text{for } y \leq a \tag{82}$$

$$f_2(y) = b\left(1 - \frac{2}{\pi}\arctan\sqrt{\frac{y-a}{a}}\right) \qquad \text{for } y \geq a \tag{83}$$

The discharge characteristic after substituting the factors k_1, k_2 and λ given in Table 3 can be expressed in the form:

$$Q_1 = \frac{2}{3}C_d\sqrt{2g}bh^{3/2} \tag{84}$$

$$Q_2 = C_d\sqrt{2g}\sqrt{a}b\left(h - \frac{1}{3}a\right) \tag{85}$$

The coefficient of discharge according to Eq. 85 for assumed values of a and b is:

$$C_d = \frac{Q_2}{0.6862(h - 0.05)} \tag{86}$$

The average flow velocity in the channel for $h \geq a$ can be calculated from Eqs. 72 or 73 in which $F_1(y)$ is according to Eq. 74 for $d = 0$ or Eq. 75 for $d > 0$ and

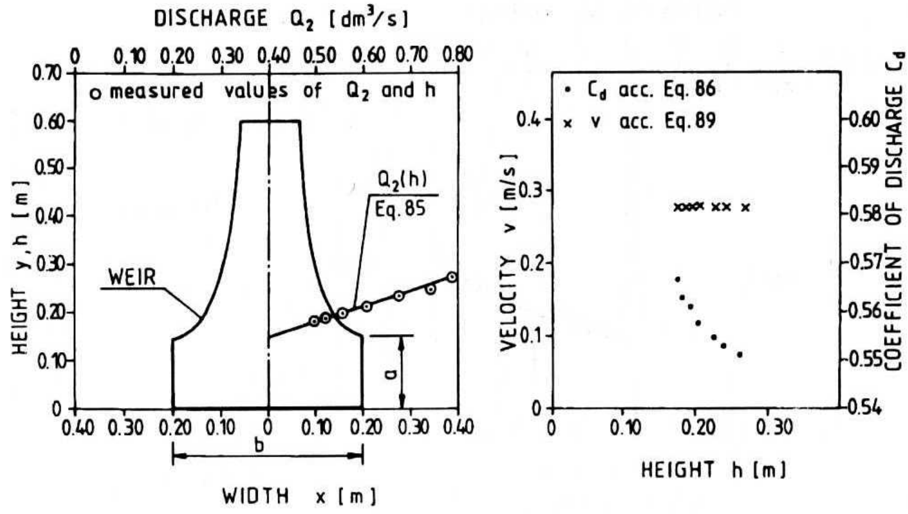

Fig. 7. The shape of weirs and functions: $Q_2(h)$, $C_d(h)$, $v(h)$ for $m_2 = 1$ and $m_1 = 3/2$

$F_2(y)$ is given by the function:

$$F_2(y) = \frac{C_d\sqrt{2g}\sqrt{a}b}{c} \tag{87}$$

In this case the following relation can be obtained:

$$v = \frac{3c}{C_d\sqrt{2g}\sqrt{a}b(3h-a)}Q_2 \qquad \text{for } d \geq 0 \tag{88}$$

or after substituting the assumed values of a, b and C_d in the form:

$$v = \frac{2.1859}{(3h-0.15)}Q_2 \qquad \text{for } d \geq 0 \tag{89}$$

The experimental values of the discharge coefficient of the weir and the velocity in the channel can be calculated in an analogous manner to case 1 for measured values of Q_2 and h. The results of the experiments are presented in Fig. 7. The value of the discharge coefficient of the weir was in the range $C_d = 0.5673 \div 0.5514$ and the velocity was in the range $v = 0.2836 \div 0.2757$ m/s for the tested range of height h.

6. Conclusions

The presented method of channels with controlled average flow velocity designing showed the possibility of using weirs as regulation devices. The application of weirs with various discharge characteristics enables the obtaining of variant solutions of the channel shape with the given velocity characteristic. The velocity characteristic can be satisfied in the whole range of the head only in the channels with weirs situated at the channel's bottom. In the case of two-part weirs situated above the channel's bottom the given velocity characteristic can be satisfied only in the range of the head bigger than the height of the lower part of the channel.

The practical channel and weir shapes, can be obtain only in a restricted range. The conditions which enable to obtain practical solutions were formulated in the paper.

From the point of view of practical applications making weirs of complex shapes does not present any difficulty. On the other hand making channels of a cross-section curvilinear shape may be difficult. Variant solutions of weirs' and channels' shapes for a given velocity characteristic in a channel which enable to choose the best shapes in given technical conditions were presented in the paper.

The experimental research confirms the correctness of theoretical solutions. The coefficient of discharge of the weir is a falling function of the head and its value depends on the weir shape.

In an engineering practice the assumption of a constant value of a discharge coefficient of a weir does not have a significant influence on the solutions correctness.

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