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Dynamics of Variable Declining Rate Filters during Backwash

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Abstract

Variable declining rate (VDR) filters are usually controlled by the interaction of laminar head losses in the filter beds and turbulent head losses in the drainage systems and orifices installed at the outflow of each filter. The non-linear turbulent head losses are much higher for clean filters, but as the filter media becomes clogged the flow rate is reduced and the turbulent head losses decline. The disconnection of one filter during backwash causes a sudden increase in the water table level above the remaining filters, and so produces a rapid increase in the flow rates through them. This change in flow rate can have a significant impact on effluent quality. A simple and efficient method of calculating acceleration at the beginning of a backwash has been developed and verified by numerical solutions of the equations governing the hydraulics of VDR filters. The results show that the flow rate increases are approximately proportional to the flow rates during normal plant operation. A practical method of calculating the highest possible water table fluctuation during backwash has been developed.

Notations

a – surface area of a single filter,

A – surface of z filters during normal operation, or $z-1$ filters when one is out of service,

b – coefficient determined empirically,

$f(t)$ – a function of time describing additional head loss Δh created by a flow rate controller installed as shown in Fig. 2,

h – total head loss $h(t)$ through a filter including the drainage and the orifice,

h_0 – highest water table fluctuation between backwashes,

H – total head loss through a filter on commencement of disconnecting one of the units from the bank for backwashing,

K_e – coefficient of turbulent friction created by drainage and an orifice,

K_e – coefficient of laminar linear head losses in filter media $K_i(t)$, dependent on its initial constant value K_0 and on volume of filtered water $V_i(t)$,

Q – flow rate of inflow to the filter plant,

q_i – flow rate through i-th filter,

q_i^* – flow rate through i-th filter at the end of backwash, when new equilibrium is established,

t – time,

V_i – volume of water passed through filter i,

i – subscript denoting filter i $(i = 1, \dots, z)$,

z – number of filters in a plant,

α, β – empirical coefficients dependent on both quality of raw water and filter media,

Δh_i – increase of water table level above filters during backwash.

1. Introduction

Variable Declining Rate (VDR) Filters were developed almost thirty years ago (Cleasby 1969) and since then have become increasingly popular, especially in South and North America. Thirteen years ago (Arboleda et al. 1985) more than 500 filter plants were reported to be operated under Variable Declining Rate in North and South America. Application of this system of operation enabled the capacities of many filter plant to be significantly increased, sometimes with simultaneous improvements in filtrate quality (Cleasby 1969; Cleasby, Di Bernardo 1980; Cornwell, Bishop and Dunn 1984). These filters were not equipped with flow rate controllers, but instead relied on the turbulent friction of the drainage and orifices installed at outflows from the filters, as well as parameters of filter operation (total head loss before backwash, highest water level fluctuation above filters) to determine the design and operation. In Poland many water filter plants are operated without flow rate controllers and application of the strict rules developed for Variable Declining Rate Filters operation may improve the filters' performance and afford economic benefits comparable with those achieved at the water treatment plant in Żywiec (Dąbrowski and Niemiec 1995) at the beginning of the nineteenth. VDR Filters are essentially identical with Constant Rate (CR) Filters, except for the lack of flow rate controllers. These are replaced by orifices (Arboleda 1974, Cleasby and Di Bernardo 1980), or simple devices preventing one hundred per cent opening of butterfly valves installed at outflows from the filters (Cornwell et al. 1984). Moreover, this kind of filter operation requires low hydraulic friction losses in the pipes distributing raw water to the filters, and the location of inflows below the lowest water level in filter boxes to ensure that the water table is at the same height for all filters at any moment of operation (Cleasby 1969). Turbulent non-linear head losses created by orifices are approximately proporional to the square of the flow rate (Arboleda et al. 1985). For proper coagulant doses and flocculation, the accumulated deposit consists of about 2% solids, but the flow friction is still not visibly affected by possible agglomeration and cementation processes (Dąbrowski 1993), therefore head losses in the filter media are laminar and linearly dependent upon the flow rate. Recently backwashed filters are operated under higher hydraulic loads restricted by head losses created by orifices, while clogged filters produce less water due to high resistance of filter media. Laboratory (Cleasby and Di Bernardo 1980), pilot and full scale plant experiments (Arboleda et al. 1985), as well as theoretical considerations

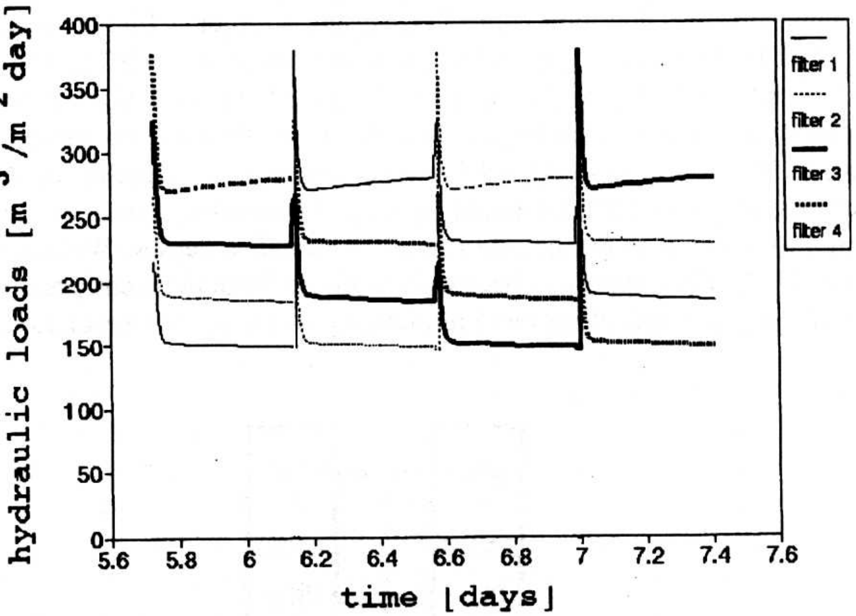

Fig. 1. A VDR Filter plant operation according to experimental data (dotted lines) published by Arboleda et al. (1985) after a report by Gregory and Yadaw. Unbroken lines denote results of computations

(Chaudhry 1987), demonstrated that flow drops in VDR Filters occur only during and just after a backwash in a plant. This behaviour of VDR Filters is presented in Fig. 1, developed here from computations following the mathematical model of Arboleda et al. (1985) modified by Dąbrowski (1994). When the most clogged filter is backwashed the water surface level rises sharply above all other filters, causing a sudden increase in filtration velocities for plants consisting of a small number of filters. This results in a temporary degradation of filtrate quality, the extent of which depends on the rate of change in filtration velocity (Cleasby, Williamson and Baumann 1963), deposit properties and concentration in the filter media. When organic polymers instead of trivalent metal salts are used as coagulants, the increase in filtrate turbidity is usually significantly reduced. However, for small filter plants the flow rate changes are more pronounced and degradation of filtrate quality is an important factor. This is especially so if the surface potential conditions are unfavourable for attachment and may result in inadequate control of pathogens in drinking water, rendering disinfection less efficient. This inefficiency occurs as bacteria use pores in the solid particles as shelters actively when exposed to hydrochloric acid or other disinfectants. US statistics confirm that many waterborne diseases outbreaks are the result of inadequate filters performance. However, it is possible to make the changes of filtration velocities smoother by installing a single flow rate controller causing additional friction at the very beginning of backwash in a plant (Dąbrowski and Dziopak 1992).The installation of this flow rate controller as shown in Fig. 2 is possible only in those plants from which filtrate is collected by a single main, this being the most common solution in Europe. It is well known from laboratory experiments (Cleasby et al. 1963) that the degradation of filtrate quality does not depend so much on the value of filtration velocity u as on the rate of increase du/dt (Cleasby, Williamson and Baumann 1963). The impact of a backwash on du/dt for both recently backwashed and significantly clogged filters has been investigated here in computations.

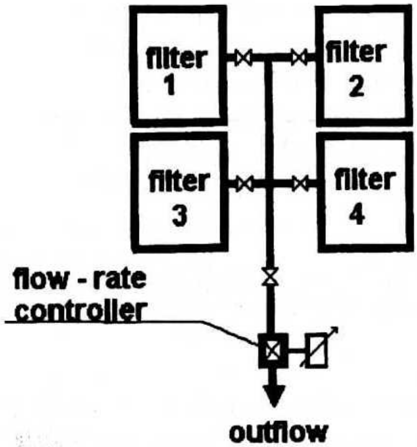

Fig. 2. An example of a water treatment plant arrangement

2. Formulation of the Model

Despite of extensive investigations into deep filtration theory after the work of Yao (1971) successfully describing transport mechanisms within the filter media, no direct practical application of the theory for filter design is known. This is because of complexity of the filtration process, and the non-homogeneity of the suspension and media. There are some simplified phenological methods of modelling deep filtration in non-homogeneous porous media, and a few stochastic and Unit Bed Element models of non-homogeneous suspension flow, but their application in water technology is limited because of unpredictable floc non-homogeneity in size, shape, density and surface charge. In this situation mathematical models of VDR Filters (Chaudhry 1987) are based on the somewhat urealistic assumption of uniformity of both filter media and flowing suspension. Concluding, at present there is no realistic mathematical model describing the kinetics of clogging of VDR Filters. Fortunately for the purpose of the present study it is important only to predict flow rates through VDR filters and the resistance to flow created by the filter media at the very beginning of a backwash in the plant. The work by Di Bernardo (1986, 1987) and later Dąbrowski (1994)showed that this data can be predicted independent of the kinetics of clogging, and result from general hydraulic rules to be fulfilled by the filters. To be more precise, the flow rate through the most recently backwashed filter may easily be calculated (Di Bernardo 1987) as the coefficients describing the hydraulic friction in filter media, orifice, and drainage are known. Then, taking notice of the fact that the flow rates through VDR filters (see Fig. 1) remain almost constant between backwashes and decrease after a backwash, it is possible to determine flow rates through all filters and account for head losses created by the filter media and by the orifices (Dąbrowski 1994) just before and just after each backwash in the plant. Supported by these findings a mathematical model of Arboleda et al. (1985) has been chosen here to investigate the sudden rise in filtration velocity when one of the filters is disconnected from the bank. This model of VDR filters, in contrast to all others, takes into account an accumulation of water above the filter media, negligible in comparison with water production during a filter run but being an important element of our particular considerations. Assuming that a flow wate controller installed between the filter plant and a treated water tank is operated in such a way that during one of the backwashes it creates an additional head loss Δh, this being a function of time $f(t)$, a system of equations describing a plant consisting of z filters, according to the mathematical model of Arboleda et al. (1985), may be written in the form (Dąbrowski 1994):

$$h = \frac{K_i}{a} q_i + K_e q_i^n + f(t) \qquad (z \text{ equations}) \qquad (1)$$

$$K_i = K_0 (1 + \alpha V_i)(1 - \beta V_i)^{b V_i} \qquad (z \text{ equations}) \qquad (2)$$

$$Q = \sum_{i=1}^{i=z} q_i + A\frac{dh}{dt} \qquad \text{(1 equation)} \tag{3}$$

$$q_i = \frac{dV_i}{dt} \qquad \text{(z equations)} \tag{4}$$

where:

- a – surface of a single filter,
- A – surface of z filters during normal operation, or $z-1$ filters when one of them is out of service,
- b – empirical coefficient,
- h – total head loss $h(t)$ of flow through a filter including the drainage and the orifice,
- i – subscript denoting a filter i $(i = 1, \ldots, z)$,
- K_e – coefficient of turbulent friction created by drainage and an orifice,
- K_i – coefficient of laminar linear head losses in filter media $K_i(t)$, dependent on its initial constant value K_0 and on the volume of filtered water $V_i(t)$,
- n – empirical coefficient $(1 < n \leq 2)$,
- Q – flow rate of inflow to the filter plant,
- q_i – flow rate through i-th filter,
- t – time,
- V_i – volume of water passed through filter i,
- z – number of filters in a plant,
- α, β – empirical coefficients dependent on both quality of raw water and filter media.

Equations (1) describe the fluctuation in time of head loss $h(t)$. This consists of the head loss created by the filter media $(K_i/a)q_i$, that at the drainage and the orifice $K_e q_i^n$, and finally by a single flow rate controller $f(t)$ located at the main between the plant and the treated water tank. The head loss $h(t)$ is supposed to be the same at any moment for all filters because one of the important principles of Variable Declining Rate Filters plant design states that friction created by piping should be negligible in comparison with the loss of head in filters, so the water table is expected to fluctuate in the same manner above all filters. The head loss $h(t)$ increases continuously between the subsequent backwashes in a plant due to clogging of the filter media, then rapidly rises when one of the filters is taken out of service and sharply falls down after bringing it back into service again. The head loss created by the filter media $(K_i/a)q_i$ is affected by the volume of raw water V_i passed through a filter i and its quality described by coefficients b, α, β

included in equations (2). Equation (3) describes the mass balance, taking into account an accumulation of water above filters. Equations (4) relate flow rates through the filters with the derivative of volume of water passing through the filters with respect to time and complete the set of $3z + 1$ equations of $3z + 1$ unknown variables K_i, V_i, q_i, h.

3. Boundary Conditions

To find specific solutions to the system of equations (1)–(4) it is necessary to assume boundary conditions. Unfortunately the real boundary conditions are a part of the unknown solution and cannot be predicted in advance. Two different approaches to these difficulties are visible. In the first approach the boundary conditions are calculated from another mathematical model of Variable Declining Rate (VDR) Filters, such as the one proposed by Di Bernardo (1986, 1987) or Di Bernardo and Cleasby (1980), but this procedure may cause an instability at the very beginning of computations. A longer but safer approach has been originally proposed by Arboleda et al. (1985) assuming that a plant operation stars from an unusual situation when all z filters are clean. In practice such a situation may occur only once, just after putting a new plant to work. For this particular case the boundary conditions to the system of equations (1)–(4) are the following:

$$q_i(t = 0) = \frac{Q}{z} \tag{5}$$

$$V_i(t = 0) = 0 \tag{6}$$

These primary boundary conditions provide the initial condition necessary to solve the set of equations (1)–(4), which is shown in Fig. 3. In many computations the same behaviour of the solution has been observed. First the flow rates through all filters remain the same in spite of an increase in the head loss of flow through the plant causing an increase of the water table level. Then when the head loss reaches an assumed level one of the filters is taken out of work for backwash and at this moment a sharp rise in flow rates through the remaining units is observed. After the first backwash the flow rate of the recently backwashed filter is higher than previously while all remaining filter behaved identically and they decreased in the flow rates. After several filter cycles the solution is characterised by repeating values of flow rates and the same periods between backwashes. In this way the solution and new boundary conditions are found.

4. Solution of Governing Equations

Arboleda et al. (1985) solved the system of equations describing their model numerically. Dąbrowski (1994) used an analytical solution to the system of equations (1)–(4) at the beginning to reduce the number of equations and variables. He then

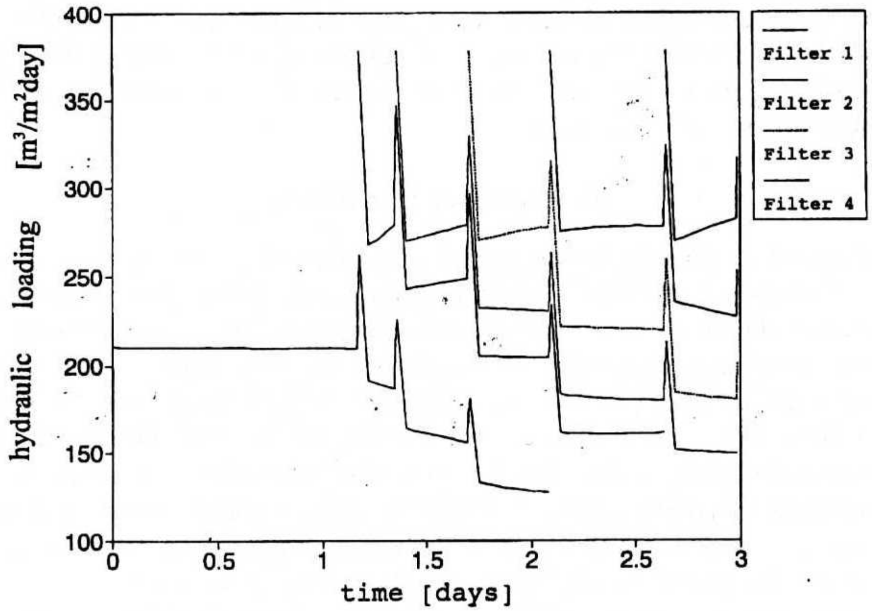

Fig. 3. An illustration to an application of primarily assumed boundary conditions (5), (6). In the computations the data describing the results of full scale VDR Filter plant in Medmenham (Great Britain) have been used after the paper by Arboleda et al. (1985)

followed a numerical approach. According to the monograph (Dąbrowski 1994) the set of equations (1)–(4) is equivalent to (7), (8):

$$q_i = \frac{dV_i}{dt} \tag{7}$$

$$\frac{dq_i}{dt} = \frac{1}{B(V_i, q_i)} \left\{ \frac{Q - \sum_{i=1}^{i=z} q_i}{A} - C(V_i, q_i) - D(V_i, q_i) - f'(t) \right\} \tag{8}$$

where:

$$B(V_i, q_i) = \frac{K_0}{a}(1 + \alpha V_i)(1 - \beta V_i)^{bV_i} + nK_e q_i^{n-1} \tag{9}$$

$$C(V_i, q_i) = \frac{q_i^2}{a} K_0 \alpha (1 - \beta V_i)^{bV_i} \tag{10}$$

$$D(V, q_i) = \frac{K_0}{a}(1 + \alpha V_i)(1 - \beta V_i)^{bV_i} q_i^2 b \left\{ \ln(1 - \beta V_i) - \frac{V_i \beta}{1 - \beta V_i} \right\} \tag{11}$$

The same analytically derived formulas have been used here for further numerical computations but this time the numerical program was written in C^{++} (Marzec and Dąbrowski 1996).

5. Disturbance of Flow-rates

Most small treatment plants do not have reliable flow meters suitable for reading non-stationary flow rates through filters during a backwash. On the other hand, the impact of disconnection of a filter from a bank on flow rates through the others is important only for small treatment plants consisting of a few filters. Only in these plants is a degradation of filtrate quality likely to be seen as the result of a sudden rise in flow rate. This is why a more detailed theoretical study on the impact of backwash on other filters is required.

The most rapid changes of flow rates occur at the very beginning of backwash so it is important to predict derivatives dq_i/dt for all filters at the moment when that most highly clogged is being disconnected from the bank. To make the computations of dq_i/dt first the procedure developed originally by Di Bernardo (1986, 1987) and then simplified by Dąbrowski (1994) has been used here to find flow rates q_i through filters in a plant. The following assumptions have been made:

1. flow rates through all filters are subject to change exclusively during and just after a backwash, and remain almost constant between subsequent backwashes in a plant,
2. the period of backwash is short enough to ensure that it is reasonable to assume that hydraulic resistance of the other filters does not change significantly during this time.

Under these assumption the following systems of equations (12), (13), (14) describes the flow rates through the VDR Filters (Dąbrowski 1994):

$$\frac{H - h_0 - c_2 q_{i+1}^n}{q_{i+1}} = \frac{H - c_2 q_i^n}{q_i} \qquad (z-1 \text{ equations}) \qquad (12)$$

$$H - h_0 = c_1 q_1 + c_2 q_1^n \qquad (1 \text{ equation}) \qquad (13)$$

$$Q = \sum_{i=1}^{i=z} q_i \qquad (1 \text{ equation}) \qquad (14)$$

where i denotes the number of filter, H head loss through a filter plant at the moment of disconnecting one of units for backwashing. The mass balance equations (3) and (14) differ by the term $A(dh/dt)$ which has been neglected by Di Bernardo (1987) as being small in comparison with Q. From the above system of $z + 1$ equations both flow rates through all z filters q_i and the highest water table fluctuation h_0 can be computed if the head loss of flow through the plant H and coefficients describing linear laminar resistivity c_1 of clean porous media and turbulent non-linear resistivity of drainage and orifices c_2 are known. Fast and accurate methods of solution are described elsewhere (Dąbrowski 1994). After predicting the flow rates through the filters and taking into account that just before a backwash head loss of flow through filters reaches its highest value H the resistivities of clogged filter media c_{ci} can be predicted from equation (15):

$$H = c_{ci}q_i + c_2 q_i^n \qquad (z \text{ equations}) \qquad (15)$$

After computing these resistivities and assuming them to be unaffected during a backwash, the following equation approximates equation (1) at the very beginning of backwash ($t = 0$) when $h = H$:

$$h(t = 0) = H = c_{ci}q_i + c_2 q_i^n + f(t) \qquad (z \text{ equations}) \qquad (16)$$

After differentation of this equation with respect to time the following formula results:

$$\frac{dh}{dt} = \frac{c_{ci}dq_i}{dt} + nc_2 q_i^{n-1}\frac{dq_i}{dt} + f'(t) \qquad (z \text{ equations}) \qquad (17)$$

In this formula both derivatives dh/dt and dq_i/dt are unknown at present but the first of them can easily be determined from the mass balance equation (3):

$$\frac{dh}{dt} = \frac{Q - \sum_{i=1}^{i=z-1} q_i}{A} \qquad (18)$$

where A denotes the total surface of $z - 1$ filters at the moment of backwash.

Now flow rates at the end of continuous operation q_i are known from the set of equations (12), (13), (14), filter media resistivities just before a backwash c_{ci} from the set of equations (15), the derivative $(dh/dt)_{t=0}$ from formula (18) and finally equations (17) may be used for predicting $(dq_i/dt)_{t=0}$ as follows:

$$\frac{dq_i}{dt} = \frac{dh/dt - f'(t)}{c_{ci} + nc_2 q_i^{n-1}} \qquad (19)$$

Until now consideration has focused on the highest possible rate of increase of flow rate $(dq_i/dt)_{t=0}$ occurring just after disconnection of one of the units from a bank for backwashing. Equally important is the highest possible water table increase at the end of backwash. This height is limited by a new state of equilibrium which may be reached at the end of a long backwash between an increase in water level of Δh and new flow rates q_i^* through the $z - 1$ filters staying in operation (Di Bernardo 1987). According to measurements from water treatment plants it is unlikely that this equilibrium is reached and Δh should be treated as the upper limit for the real rise in water level. To predict this limit let us assume again that the period of a filter disconnection is short enough to ensure that the filter media is unaffected by clogging during this time. This assumption allows us to compare the ratio of laminar head losses through filter media to flow rates at the beginning and then at the end of backwash. If the total head loss at the moment of a filter disconnection is denoted by H, the head loss through the media, of let us say filter i, may be calculated as equal to $H - c_2 q_i^n$, where $c_2 q_i^n$ is a turbulent head loss created by the drainage and the orifice. Similarly, at the end of a backwash the laminar head loss through the media is equal to $H + \Delta h - c_2 (q_i^*)^n$. From this $z - 1$ equations (20) result:

$$\frac{H - c_2 q_i^n}{q_i} = \frac{H + \Delta h - c_2 (q_i^*)^n}{q_i^*} \qquad (z - 1 \text{ equations}) \qquad (20)$$

The accumulation of water above the filter is negligible in comparison to the total flow, therefore:

$$Q = \sum_{i=1}^{i=z-1} q_i^* \qquad (1 \text{ equation}) \qquad (21)$$

From the system of z equations (20), (21) flow rates through filters at the end of backwash q_i^* and the highest possible water increase Δh can be calculated as the values of q_i are known from the set of equations (12), (13), (14).

6. Numerical Experiments

The method of calculating $dq_i/dt_{t=0}$, q_i^*, Δh developed here has been tested and illustrated using computations based on the mathematical model (1), (2), (3), (4) of Arboleda et al. (1985) supported with empirical data reported by them after Gregory and Yadav from a water treatment plant in Medmenham, UK. The following set of data enabled numerical simulation of the results of their experiments: $z = 4$, $H = 1.6$ m, $b = 0.00205$ [1/m^3], $K_0 = 0.00174$ [(m H_2O) day/m], $K_e = 0.0000066$ [(m H_2O) day^2/m^2], $\alpha = 0.00513$ [l/m^3], $\beta = 0.00513$ [l/m^3], $h_0 = 0.385$ [m], $f(t) = 0$. In the computations, speeded up here by partially analytical solutions (7), (8), the total period of filter disconnection was assumed not to exceed 20 minutes (0.014 days). Solving the system of equations (12), (13), (14) for the set of data: $z = 4$, $c_1 = 0.00253$ [(m H_2O) day/m] the following hydraulic loads (flow rates per square metre of filter surface) were obtained: $q_i = 274$ [m^3/m^{2}day], $q_4 = 153$ [m^3/m^{2}day], $q_3 = 188$ [m^3/m^{2}day], $q_2 = 229$ [m^3/m^{2}day]. These are almost identical with the values resulting from a numerical solution to equations (7), (8), developed from the original model by Arboleda et al. (1985) described by equations (1), (2), (3), (4). According to equations (15) resistivities of filter media before a backwash in a plant have been calculated as equal to: $c_{c1} = 0.003915$ [(m H_2O) day/m], $c_{c4} = 0.009863$ [(m H_2O) day/m], $c_{c3} = 0.007559$ [(m H_2O) day/m], $c_{c2} = 0.005523$ [(m H_2O) day/m]. Then for the beginning of backwash the derivative $dh/dt_{t=0}$ has been calculated from equation (18) and used in equation (19) to find the highest possible rate of flow change $dq_i/dt_{t=0}$. The following results were obtained: $dq_1/dt_{t=0} = 6724$ [m^3/m^{2}day^2], $dq_3/dt_{t=0} = 5044$ [m^3/m^{2}day^2], $dq_2/dt_{t=0} = 5934$ [m^3/m^{2}day^2]. These values of the derivatives $dq_i/dt_{t=0}$, are similar to those resulting from a solution to equations (7), (8), (9), (10), (11). The results are presented in Fig. 4.

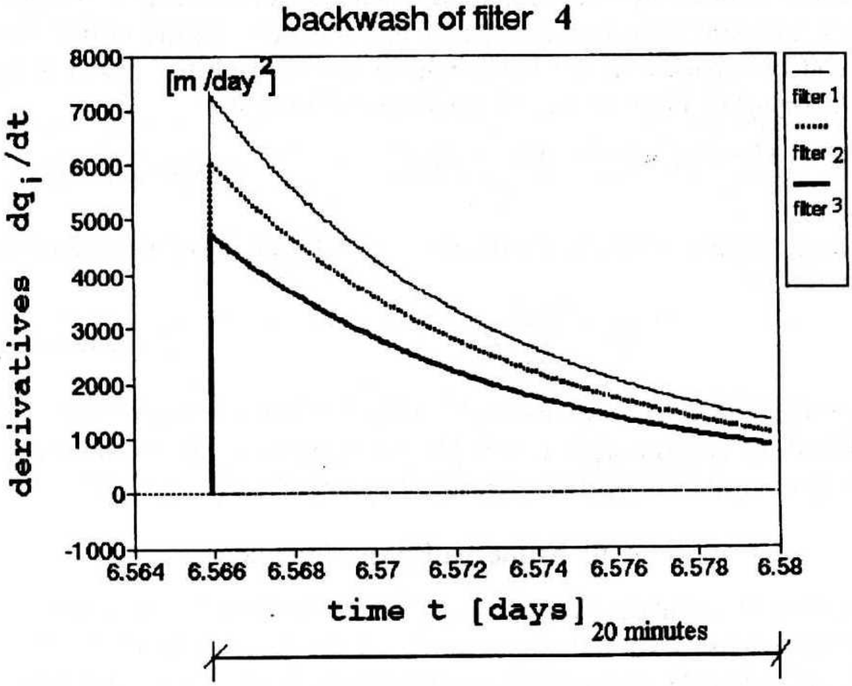

Fig. 4. Derivatives dq_i/dt at the beginning of backwash ($t = 0$) according to the numerical solution of equations (7), (8) developed from the model by Arboleda et al. (1985) – see equations (1), (2), (3), (4)

Good conformity between the values of the derivative dq_i/dt has been achieved, including the initial value of $dq_i/dt_{t=0}$. If head losses created by orifice and drainage are small in comparison with those created by filter media the resistivities c_{ci} calculated for the beginning of backwash from equation (15) are almost proportional to flow rates q_i. If then the term $nc_2q_i^{n-1}$ in equation (17) is negligible as compared with c_{ci} and $n = 2$, the derivatives dq_i/dt are almost proportional to flow rates ar the beginning of backwash. The rate of water table rise decrease inversely in time, as flow rates through operating filters approach a new equilibrium the derivatives dq_i/dt become smaller, but in numerical simulations it was found that they were still proportional to q_i at the moment of backwash. To show this effect for the same set of data as for Fig. 1 the ratio of $(dq_i/dt)/(q_i)$ has been calculated and plotted in Fig. 5, giving an almost the same values for all filters.

From the numerical solution to equations (1), (2), (3), (4) it can be seen that even 20 minutes after disconnection of a filter from the bank the new equilibrium between flow rates has not been fully established and a tendency to increase is still visible. Hydraulic loads at the end of backwash have reached the following val-

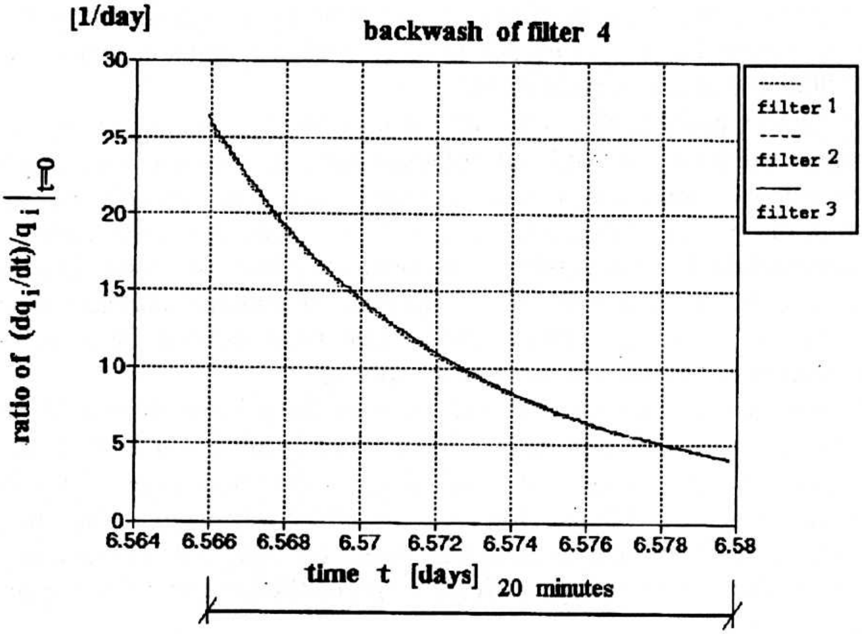

Fig. 5. Changes of the ratio $(dq_i/dt)/(q_i)$ at time of backwash

ues: $q_1 = 327$ [m^3/m^{2}day], $q_3 = 215$ [m^3/m^{2}day], $q_2 = 268$ [m^3/m^{2}day] while from the set of equations (20), (21) at the state of equilibrium should be equal to: $q_1^* = 330.6$ [m^3/m^{2}day], $q_3^* = 233.1$ [m^3/m^{2}day], $q_1^* = 280.3$ [m^3/m^{2}day], for $\Delta h = 0.455$ m.

7. Conclusions

Filtrate degradation caused by sudden rises in flow rates may happen during backwash in small filter plants consisting of several units. To calculate the highest possible rate of change in flow rates $dq_i/dt_{t=0}$ through each of the filters the following procedure may be followed:

1. calculate flow rates through the filters from the set of equations (12), (13), (14) (according to the Di Bernardo (1987) model modified by Dąbrowski (1994)),
2. predict resistivities of filter media just before a backwash from equations (15),
3. calculate $dh/dt_{t=0}$ from formula (18),
4. calculate $dq_i/dt_{t=0}$ from equations (19).

The derivatives $dq_i/dt_{t=0}$ are approximately proportional to the flow rates through the filters at the beginning of backwash. Numerical computations confirmed approximate proportionality of dq_i/dt from q_i until the moment when the backwashed filter is put into service again.

The highest possible flow rates q_i^* through filters occur when one of them is disconnected from the bank for sufficient time to establish new equilibrium between inflow to and outflow from the plant. These flow rates q_i^* may be calculated from the system of equations (20), (21). The same system of equations can be used to find the highest possible water table level increase above filter Δh, due to backwash. In conclusion the most important parameters characterising filter plant behaviour during a backwash can be calculated without the need to know the coefficients describing the kinetics of clogging.

The work presented here does not consider the possible impact of variation in flow through the inlet pipe on the level of water table above the filters. This effect will be greatest in cases in which water is delivered to the filters by pipe under free surface conditions. However, the effect will be to reduce the water table fluctuation. Thus the methodology presented here will produce an upper boundary on the rate of change of dq_i/dt. The method therefore continues to be of practical use.

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