Archives of Hydro-Engineering and Environmental Mechanics
Vol. 44 (1997), No. 1–4, pp. 23–44

The Preliminary Analysis of Possibilities of the Application of Conventional Flow-meters in Measurements of Unsteady Flows

Janusz Pospolita

Technical University of Opole

(Received April 09, 1997; revised February 24, 1998)

Abstract

The paper presents the state-of-art concerning possibilities of measurements of unsteady flows with flowmeters applied in practice as well as the results of test made by the author in this field. One-dimensional mathematical models joining the fluid flux, changeable at time, with the flowmeters indications were derived. These models were applied for estimation of influence of unsteady flow states on the additional error of measurement of instatenous or mean value of the fluid flux. The analysis included periodically variable flows and flow changes in form of velocity jump. On the basis of the proposed mathematical models functions of flowmeter transition were derived, their time constants were determined and amplitude-frequency characteristics were presented.

The spatial and time distributions of fluid velocities and pressures near the flowmeters were used for analysis of simplifying assumptions, assumed for formulation of one-dimensional models and estimation of the variability change of coefficients of the models versus time. These quantities were obtained by solving Reynolds equations and k-e model of turbulence in the flow system having geometry corresponding to a given flowmeter. The finite difference method was applied.

Mathematical models for particular types of flowmeters, their time constants and values of dynamic errors were compared. The maximum ranges of model coefficients variability for flow changes (velocity jump) for different Strouhal numbers Sh, characterizing unsteady flows, were compared too. Conclusions of metrological character, determining possibilities of application of the flowmeters tested for measurements of unsteady flowmeters, were formulated.

1. Introduction

Unsteady flows constitute a frequent occurrence in industrial practice. Transitory processes are mainly accompanied by changes in flow and are also associated with the operation of control systems. Disturbances, occurring in installations, may also be of a periodic character as they are caused by the work of pumps and piston compressors. Also in the case of flows of heterogeneous fluids, e. g., gas – liquid mixtures, the changes of parameters in time are directly relevant to fluctuating pulsations specific for particular two-phase flow structures.

It is considered (Bhatt, Wedekind 1985; Kremlewskij 1989; Kuzniecov et al. 1981) that when it is necessary to measure rapidly-changing fluxes, electromagnetic or turbine flow-meters with low inertia of rotors should be used. Electromagnetic flow-meters are inertialess ones and in the case of the axial symmetry of flux, no additional error, relevant to the transient state of flow, occurs. However, it is not always possible to apply this measuring method, e. g., it may not be used in the case of transient gas flow or flows characterized by low value of the Reynolds number. Furthermore, industrial constructions of such a type of flow-meters, applied to measurements of transient flows, require some modification in their electronic part.

Turbine flow-meters with rotors of low inertia may be characterized by time-constants of the order of $0.01 \div 0.001$ s, which theoretically qualify them for measurements of rapidly-changing flows. However, the velocity profile, varying in time, may under certain conditions, be the source of an additional error, e. g., when pulsating flow of high pulse amplitude (Lee et al. 1975; Pospolita 1992) is measured by means of a turbine flow-meter. In cases of the applying of other types of flow-meters to measurements of rapidly-changing flows, an additional error of measurement occurs. The value of this error depends mainly on the dynamic properties of the applied measuring system and parameters characterizing the disturbance of flow.

Publications concerning the dynamic properties of flow-meters, comprise their particular types to a different degree. Relatively considerable much attention has been paid to constriction flow-meters (Dobrowolski et al. 1981; Dobrowolski et al. 1983; Dobrowolski et al. 1991; Kabza, Pospolita 1989; Kuzniecov et al. 1981) mainly considering their prevalence in technical practice. Both experimental (Baldin et al. 1983; Dijstelbergen 1964; Lee et al. 1975; Uri 1963) and theoretical investigations (Kuzniecov et al. 1981; Maki, Ikeda 1981; Pospolita 1992) embody turbine flow-meters and rotameters . On the other hand, there are relatively few publications concerning the influence of transient states of flux on the characteristics of rheometers (Torigoe 1987) or on ultrasonic flow-meters (Dorden 1979; Hamidulin 1989).

Still another problem is measurement of the mean value of pulsating mass flux. In the case of the application of industrial types of flow-meters, the value of an additional error of measurement, caused by pulsation, depends not only on the dynamic properties of particular types, but also on the linearity of their characteristics. Although this problem is broadly discussed in a number of publications, analysing the value of an additional error when pulsating mass flux is measured by different types of flow-meters (Dobrowolski et al. 1983; Dobrowolski et al. 1985; Lee et al. 1975; Maki, Ikeda 1981; Uri 1963), the above problem should, in fact still be treated as open.

In spite of the investigations, which have been carried out for many years, a complex appraisal of the influence of transient states of flux on the metrological properties of different types of flow-meters has been absent. This mostly results from the complexity of physics of the phenomenon of unsteady flow through meters. In the majority of cases the theoretical analysis is carried out on the basis of one-dimensional models of flow resulting from the theory of the perfect fluid (Bhatt, Wedekind 1985; Dijstelbergen 1964; Lee et al. 1975). It is connected with the adoption of a number of simplifying assumptions.

Also the coefficients, in proper equationsare, as a rule, accepted as those for steady flow. On the other hand, the models, when coming to a steady state, express the characteristics of the analysed flow-meters, carried out on the basis of one-dimensional mathematical models, must be supplemented by the analysis of the range of variations, in time, of model coefficients. Different values of the time function of these coefficients result from velocity fields and pressures within flow-meter variable in time . Such an analysis is possible only on the basis of well-known unsteady velocity and pressure distributions obtained from the solutions of basic equations of fluid mechanics (Locjanski 1973).

The unsteady problem considered is characterized by the fact that for a given fluid flux the profile of velocity results from both the form of variability of flux and the initial state (condition). A great number of transient states possible in flow and at the same time, the possible velocity distributions make it difficult to give the values of parameters characterizing unsteady flow which would determine the criteria for the range of applicability of simplified dimensional models. Therefore the dynamic properties of flow-meters and also an additional error of measurement, brought about by the presence of transient states, should be considered for such transient states which are as close as possible to those existing in technical practice. Hence the dynamic properties of flow-meters, determined by the response to different values of changes of fluid flux in time ($\Delta\dot{M}/\Delta t$) have been analysed.

In consideration of some editorial limitations only the final equations, being the mathematical models of flow-meters and at the same time, adequate literature data where their derivation and broad analysis can be found, are given here.

2. Mathematical Models of Flow-meters

Mathematical models of the considered types of flow-meters are presented in Table 1. These models connect the input quantity, which in each case is the fluid flux $\dot{M}$, variable in time, with the flow-meter indication, i. e., the pressure difference in the case of the orifice and rheometer, the circular frequency of the rotor of turbine flow-meter or the lift of rotameter float. The equations for constriction flow-meter and rotameter result from the Cauchy integral including energy losses registered for two control sections. In the case of the rotameter, apart from the fluid inertia between control sections, both float inertia and uplift pressure force, are taken into account. The mathematical model of turbine flow-meter is derived from the equation of the rotor motion in which its moment of inertia together with the fluid mass contained between its vanes, the moment of the force with which the fluid exerts its influence on the rotor and the sum of moments from the friction force are also taken into account. In case of the capillary rheometer, the relationship between unsteady mass flux and the pressure difference at the ends of the capillary tube are obtained from the solution of the equation of motion for laminar unsteady flow with the assigned form of variability in the pressure gradient. The solution of this equation for a specific impulse makes assigning the transition function of the rheometer possible. The simplified model of the dynamics of the rheometer results from the approximation of its characteristics of the inertia term (Zhao et al. 1987).

Table 1 gives the simplified equations for dimensionless quantities describing the dynamics of particular types of flow-meters, their time constants and the value of a dynamic error resulting from the response of the flow-meter to the change of flux.

Constriction flow-meter and rheometer are flow-meters the dynamics of which correspond to the dynamics of the sum of proportional and differential terms. The turbine flow-meter has the dynamics of an inertial term of the first order, whereas the dynamics of rheometer are those of the sum of a differential term and that one of inertial or oscillatory of the second order. The ultrasonic flow-meter is also a transmitter of the zero order. Apart from the rheometer, time constants of particular flow-meters depend not only on the geometry of flow-meter, the sort of material from which its moving elements were made, but also on the mass flux. It is, among other things, the cause of the extent of spread of the values of time constants of particular flow-meters. These constants in the function of the Reynolds number and those for the rheometer in the function of a coefficient of kinematics viscosity, alsoother quantities characterizing the dynamics of a given flow-meter, are presented in Fig. 1. Dependence of the time constant of the turbine flow-meter upon the values of mass flux results from nonlinearity of the input equation describing the dynamics of turbine flow-meter (Pospolia 1992). In case of constriction flow-meter nonlinearity of dependence between Δp and $\dot{M}$, with the defined input and output quantities as in Table 1, has also a direct influence upon dependence of the constant T on flow. Values of time constants were determined from the formulas given in Table 1. Data, referring to constriction flow-meters, were taken from the work by Mottram and Zarek (1967), whereas structural and material ones of Fischer-Porter rotameters were given on the basis of tables published in (Dijstelbergen 1964).

The range of variations in quantities characterizing the structure of turbine flow-meters has been assumed, including data published in the paper by Lee et al. (1975).

Table 1. Specification of mathematical models of the analysed types of flow-meters

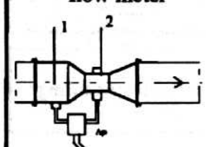
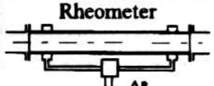
Flowmeter	Simplified equation of the flow-meter for dimensionless quantities	Time-constant	Model coefficients and notations	Value of dynamic error
1	2	3	4	5
Constriction flow-meter 	(Dobrowolski et al. 1991; Kabza, Pospolita 1989) $\Delta p^* = \dot{M}^{*2} + T\frac{d\dot{M}^*}{dt}$	$T = 2\alpha^2 \frac{l_e}{\overline{U_o}}$	$\dot{M}^* = \dot{M}/\overline{\dot{M}}$ – dimensionless mass flow rate $\overline{\dot{M}}$ – time mean value of $\dot{M}$ Δp_s – steady state value of Δp for the fluid flux $\overline{\dot{M}}$ $\alpha = \frac{\kappa\sqrt{\psi}}{\sqrt{c_2 + \xi - c_1 m^2 \kappa^2}}$ c_1, c_2 – Coriolis coefficients in sections 1 and 2 ξ – coefficient of loss between sections 1 and 2 $m = A_0/A_1$ – modulus of constriction flow-meter $\kappa = A_2/A_0$ – contraction coefficient $\psi = \frac{p_1 - p_2}{p_A - p_B}$ – coefficient taking into account pressure distribution in constriction flow-meter l_e – effective lenght (Dijstelbergen 1964) $\overline{U_0}$ – mean time and speace averaged velocity in section A_0	Ta where $a = \frac{\Delta\dot{M}^*}{\Delta t}$
Rheometer 	(Pospolita 1994) $\Delta p^* = \dot{M}^{*2} + T\frac{d\dot{M}^*}{dt}$	$T = \frac{R^2}{4\nu}$	R – radius of the capillary ν – kinematic coefficient of viscosity	Ta

Table 1 continued

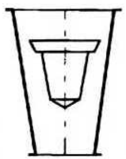
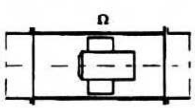
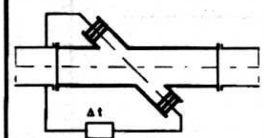
1	2	3	4	5
Rotameter 	(Pospolita 1994) $T_2^2 \frac{d^2x^*}{dt^2} + T_1 \frac{dx^*}{dt} + x^* = = T_3 \frac{d\dot{M}^*}{dt} + \dot{M}^*$	$T_1 = \frac{\overline{\overline{U_0}}\left(1+\frac{A_p}{A_0}\right)\overline{x_0}}{2l_z g\left(\frac{\rho_0}{\rho}-1\right)}$ $T_2 = \sqrt{\frac{A_0\frac{\rho_p}{\rho}+A_p}{2g\left(\frac{\rho_p}{\rho}-1\right)k_f}}$ $T_3 = \frac{2l_z}{\overline{\overline{U_0}}}$	$x^* = \frac{x}{\overline{x}}$ – dimensionless position of the float $\overline{x}$ – mean position of the float A_0 – minimum flow section area for $x = \overline{x}$ A_p – maximum area of the float section ρ, ρ_p – densities of fluid and float material l_z – substitute height of fluid column in the neighbourhood of the float $\overline{U_0}$ – mean time and space averaged velocity in section A_0 k_f – coefficient of proportionality between A_0 and x	$-T_1 a$ 1)
Turbine flow-meter 	(Pospolita 1992) $T\frac{d\omega^*}{dt} + \omega^* = \frac{b_1}{b_2}\dot{M}^*$	$T = \frac{J}{b_2 r_s^2 \overline{\overline{\dot{M}}}}$	$\omega^* = \frac{\omega r_s}{\overline{\overline{U_A}}}$ r – radius r_1 – inner radius of annular flow passage at rotor blade r_2 – outer radius of annular flow passage at rotor blade $b_1 = \frac{2\Pi}{r_s A}\int_{r_1}^{r_2}\frac{U(r)}{U_A}\left\|\frac{U(r)}{U_A}\right\| r^2 dr$ $b_2 = \frac{2\Pi}{r_s^2 A}\int_{r_1}^{r_2}\left\|\frac{U(r)}{U_A}\right\| r^3 dr$ $A = \Pi\left(r_2^2 - r_1^2\right)$, $r_s = \sqrt{r_1^2 + r_2^2}$ U_A – mean velocity in section A J – moment of rotor inertia	$-\frac{b_1}{b_2}Ta$ 2)

Table 1 continued

1	2	3	4	5
Ultrasonic flow-meter 	(Uri 1963) $\dot{M}^* = \frac{k_U}{k_{US}} U^*_R$	0	$U_R = \frac{1}{R}\int_0^R U dr$ $k_U = \frac{2}{R}\frac{\int_0^R U r dr}{\int_0^R U dr}$ – coefficient of sensitivity k_{US} – value of k_U for steady flow for a given Reynolds Number	3)

1) Maximum value of a dynamical error for $t >> T$

2) Maximum value (as for modulus) of a dynamical error when the rotameter is as inertial term, if the error component connected with time-constant T_3 is neglected

3) The dynamical error results from variations in the time function of velocity profile

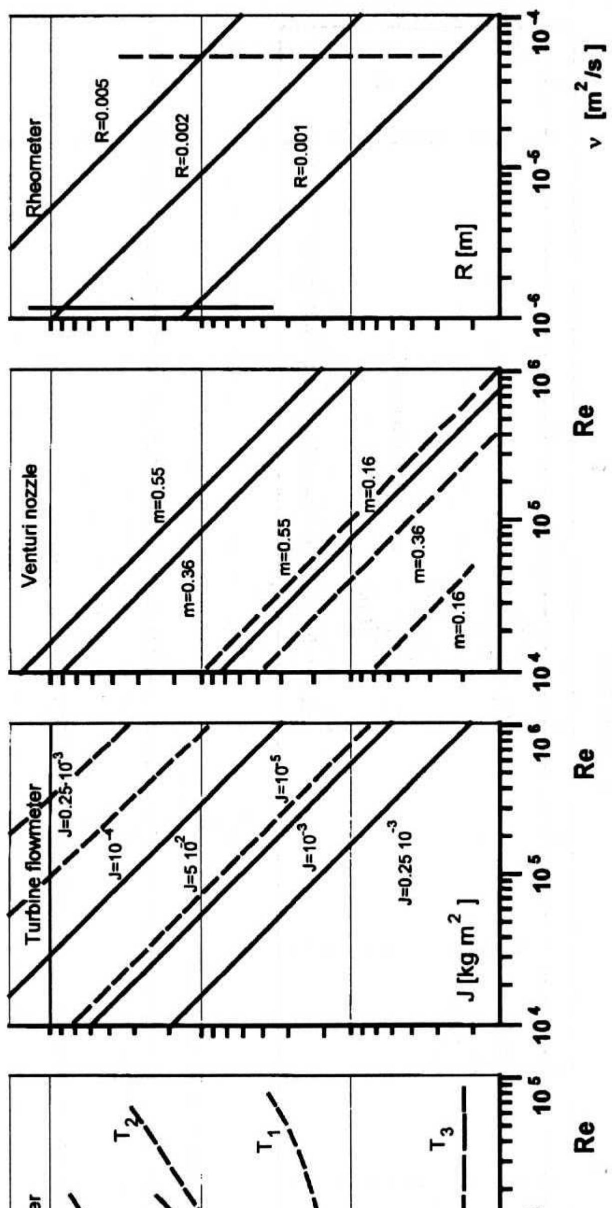


Fig. 1. Time-constants for the analysed types of flowmeters; - - - - - - for air flow; ——— for water flow

The results presented show that time constants of flow-meters, contingently upon the mean value of mass flux, the kind of fluid, and other quantities characterizing the flow-meter may be altered by three orders even in an extreme instance.

A dynamic error, in case of constriction flow-meter, rheometer and turbine flow-meter, is expressed by a simple relationship which is the product of a time-constant of flow-meter and the derivative $(d\dot{M}/dt)$. For rotameter this error, in a general case, is the complex function of time-constants. When the system has the characteristics of inertial terms of the second order, e. g., in the case of measurement of fluid it may be assumed that the maximum value of a dynamic error equals $-T_1a$. Then the influence of the constant T_3 upon α is neglected since $T_3 \ll T_1$.

The low values of time constants of flow-meters, in the limits of $0.001 \div 0.01$ s, make measurements of rapidly-changing fluxes possible. It is also possible to measure an instantaneous value of mass flux with a flow-meter of a higher time constant. This requires, however, to adapt a measuring system for determining the instantaneous value of mass flux on the basis of the analog and numerical solution of a differential equation describing the dynamics of the flow-meter. Literature on the subject gives examples of solutions of this type but they are measuring systems of a prototype character used under laboratory conditions.

3. Examples of Measuring Systems adapted to Measure Rapidly-changing Fluxes

Examples of measuring systems, in which determining the instantaneous value of mass flux is based on the solution of a differential equation connecting the flux with the indication of flow-meter, are presented, among other things, in (Bhatt, Wedekind 1985; Takahashi et al. 1982; Torigoe 1987; Torigoe 1987a). Bhatt and Wedekind (1985) presented the measuring system in which the instantaneous value of mass flux was determined after registering the pressure drop, variable in time, on the straight – axial section of pipeline, from the numerical solution of the one-dimensional differential equation of fluid motion. The calculated instantaneous values of mass flux were compared with the course of $\dot{M}(t)$, determined by the turbine flow-meter of a low time constant obtaining very good quantitative computability. Zhao et al. (1987) presented an analogous electronic system permitting the determining, in real time, of the instantaneous value of laminar, unsteady flow on the basis of the measured instantaneous values of the pressure drop on the straight – axial section of pipeline of a small diameter. The presented system accomplishes the transition function which approximates the compound $\dot{M} = f(\Delta p)$, for unsteady laminar flow in pipe (Locjanski 1973).

Takahashi et al. (1982) presented a mathematical model of laminar, pulsating flow through Venturi constriction flow-meter, based on the integrated equations of motion in the axial direction, in which some terms were omitted and others were approximated in compliance with the assumed geometry of the analysed flow system. As found in (Takahashi et al. 1982), this model enables calculation unsteady flow on the basis of the measured course of $\Delta p(t)$. Torigoe (1987) proposed a method for measuring of unsteady flow based on the conventional constriction flow-meter when the additional measurement of Δp on the pipeline section, carried out by means of a sensitive pressure-difference converter enables determination of the fluid acceleration and an appropriate algorithm of adaptation enables calculation of the instantaneous value of the unsteady term in the one-dimensional equation of motion. After substracting the signal, which corresponds to the value of this term, from the flow-meter indication, the registered signal is proportional to $\dot{M}^2(t)$. In (Torigoe 1987) are presented only some registered courses of $\dot{M}(t)$ without any metrological analysis of measuring accuracy. From other measuring systems of unsteady flows, presented in the literature, it is also possible to mention those (Torigoe 1987) in which unsteady mass flux was computed on the basis of the velocity in the time function in the symmetry axis of pipeline, measured with the Doppler laser anemometer.

In the measuring systems presented, the quasi-stationary values of coefficients in the equations, connecting mass flux with the flow-meter indications are assumed in computational algorithms.

In the publications cited and in some others there is a lack of analysis of the range of variation, in the time function, of coefficients of the applied mathematical models of flow-meters. The results of measurements presented in them, are of a fragmentary character and also short of the metrological analysis, among other things, subjecting the accuracy of the assigned values of $\dot{M}(t)$ to the parameters characterizing unsteady flow. This gives the reasons for the accomplishment of both theoretical and experimental investigations. The structure of proper systems, applied to measurements of rapidly-changing fluxes, should be preceded by the theoretical analysis of their dynamics and appreciation of their applicability.

4. Two-dimensional Mathematical Model of Unsteady Turbulent Fluid Flow

The measuring accuracy of the instantaneous value of mass flux is influenced not only by the time constant of flow-meter but also by the range of variations in the time function of model coefficients. The information on the values of these coefficients in the time function and in the function of parameters characterizing unsteady flow is, at the same time, the information on the range of applicability of a simplified one-dimensional model.

In order to estimate the range of changes in these coefficients, non-stationary flows in the analysed systems were examined numerically.

4.1. Equations of Motions and Turbulence Model

A two-dimensional, time-dependent, turbulent flow of an incompressible viscous fluid through analysed flow-systems is considered. The schemes of the hydraulic systems are shown in Fig. 2. Axial symmetry of the stream is assumed. Average parameters of the stream can be described by the Reynolds equation, complemented by the semi-empirical model of turbulence (Launder, Spalding 1971; Launder et al. 1975). In the cylindrical coordinate set, the general form of equations of transport is as follows:

$$\frac{\partial \phi}{\partial t} + \frac{\partial}{\partial z}(U\phi) + \frac{1}{r}\frac{\partial}{\partial r}(r\,V\phi) = \frac{\partial}{\partial z}\left(\Gamma_\phi \frac{\partial \phi}{\partial z}\right) + \frac{1}{r}\frac{\partial}{\partial r}\left(r\Gamma_\phi \frac{\partial \phi}{\partial z}\right) + S_\phi \tag{1}$$

where by the symbol ϕ the following quantities are denoted in turn: 1 (equation of continuity), axial U and radial V components of the velocity vector, kinetic energy of turbulence k, and rate of dissipation of kinetic energy of turbulence ε. Coefficients of the set of equation (1) are collected in Table 2. Magnitudes c_μ, c_1, c_2, σ_k, σ_ε are empirical constants, ν is the kinematic coefficient of viscosity.

Table 2. Coefficients of the equation (1)

Φ	Γ_ϕ	S_ϕ
U	ν_{ef}	$\frac{\partial}{\partial z}\left(\nu_{ef}\frac{\partial U}{\partial z}\right) + \frac{\partial}{r\,\partial r}\left(r\,\nu_{ef}\frac{\partial V}{\partial z}\right) - \frac{\partial p}{\rho\,\partial z}$
V	ν_{ef}	$\frac{\partial}{\partial z}\left(\nu_{ef}\frac{\partial U}{\partial z}\right) + \frac{\partial}{r\,\partial r}\left(r\,\nu_{ef}\frac{\partial V}{\partial r}\right) - \frac{2\nu_{ef}V}{r^2}\frac{\partial p}{\rho\,\partial z}$
k	$\frac{\nu_{ef}}{\sigma_k}$	$G-\varepsilon$
ε	$\frac{\nu_{ef}}{\sigma_\varepsilon}$	$\frac{\varepsilon}{k}(c_1 G - c_2\varepsilon)$

where:

$$G = \nu_{ef}\left\{2\left[\left(\frac{\partial U}{\partial z}\right)^2 + \left(\frac{\partial V}{\partial r}\right)^2 + \left(\frac{V}{r}\right)^2\right] + \left(\frac{\partial U}{\partial r} + \frac{\partial V}{\partial z}\right)^2\right\}, \quad \nu_{ef} = \nu + \nu_t,$$

$$\nu_t = c_\mu \frac{k^2}{\varepsilon},\ c_\mu = 0.09,\ c_1 = 1.02,\ c_2 = 1.92,\ \sigma_k = 1,\ \sigma_\varepsilon = 1.3$$

4.2. Boundary Conditions

It is necessary to formulate the boundary conditions for all the dependent variables of the set (1). The analysed flow systems are presented in Fig. 2.

In the inlet section (γ_1) in all systems the fully developed unsteady flow in angular channel is assumed. In this case all variables are only dependent on r and

t. Then the $V = 0$, $\partial\phi/\partial z = 0$ equation (1) is reduced to

$$\frac{\partial\phi}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\Gamma_\phi\frac{\partial\phi}{\partial r}\right) + S_\phi \tag{2}$$

where ϕ denotes U, k, ε in turn. Coefficients of the set (2) are collected in the Table 3.

Table 3. Coefficients of the equation (2)

Φ	Γ_ϕ	S_ϕ
U	ν_{ef}	$-\frac{1}{\rho}\frac{\partial}{\partial z}$
k	$\frac{\nu_{ef}}{\sigma_k}$	$\nu_{ef}\left(\frac{\partial U}{\partial r}\right)^2 - \varepsilon$
ε	$\frac{\nu_{ef}}{\sigma_\varepsilon}$	$\frac{\varepsilon}{k}\left(c_1\nu_{ef}\left(\frac{\partial U}{\partial r}\right)^2 - c_2\varepsilon\right)$

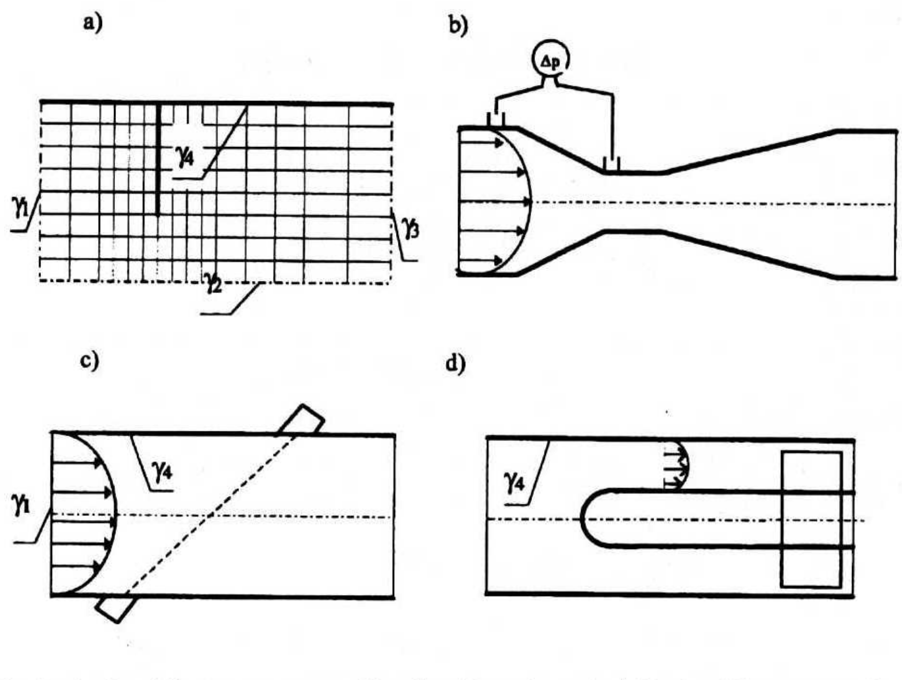


Fig. 3. Analysed flow-system: a – orifice, b – Venturi nozzle (with the difference mesh), c – ultrasonic flow-meter, d – turbine flow-meter

The set (2) is solved under the assumption that the pressure gradient $\partial p/\partial z$ corresponds with the assumed course of mass flux in time $\dot{M}(t)$. The value of $\partial p/\partial z = f(t)$ is obtained by solving the equation

$$\frac{\partial p}{\partial z} = A\dot{M}^2(t) + B\frac{d\dot{M}(t)}{dt} \tag{3}$$

Constants A and B are chosen in such a manner that after solving system (2), flow, resulting from the integration along the section of velocity profile with the assumed accuracy, should correspond with the course $\dot{M}(t)$. In the case of pulsating flow, the pressure gradient $\partial p/\partial z$ is assumed as the periodic function with the constant component (Boś et al. 1996; Dobrowolski, Pospolita 1987).

On the symmetry axis in the analysed flow systems, $V = 0$ and $\partial\phi/\partial r = 0$ are assumed. In the nozzle cross-section (γ_3) the boundary conditions are formulated as follows:

$$V = 0, \quad \frac{\partial \phi}{\partial z} = 0 \tag{4}$$

In the nodes of the difference mesh which are adjacent to the walls of the orifice, nozzle and the pipe (γ_4) boundary conditions are formulated according to the model $k - \varepsilon$ for large Reynolds numbers (Launder, Spalding 1971). If it is assumed that the point P is in the region of developed turbulence, then the component U_p of the velocity vector, parallel to the wall, is described by the logarithmic formula

$$\frac{U_p}{(\tau_w/\rho)^{\frac{1}{2}}} = \frac{1}{\chi}\ln\left(\frac{Ey_p\,(\tau_w\rho)^{\frac{1}{2}}}{\nu}\right) \tag{5}$$

where y_p is the distance of point P from the wall, $\chi = 0.42$ and $E = 9.7$ are empirical constants, μ is the coefficient of dynamic viscosity. Assumption that the shear stress τ is constant along the segment connecting point W on the wall and point P in the boundary layer, implies

$$\tau_p = \tau_w = \rho c_\mu^{\frac{1}{2}} k_p \tag{6}$$

Compiling the above formula we obtain the identity

$$\tau_p = \frac{\rho\chi c_\mu^{\frac{1}{4}} k_p^{\frac{1}{2}} U_p}{\ln\left(Ey_p c_\mu^{\frac{1}{4}} \rho k^{\frac{1}{2}}/\mu\right)} \tag{7}$$

which relates the shear stress on the wall to the kinetic energy of turbulence and the component of velocity vector parallel to the wall.

Value of dissipation rate ε_p in the boundary point can be found from the relation (Dobrowolski, Pospolita 1987)

$$\varepsilon_p = \frac{c_\mu^{\frac{3}{4}} k_p^{\frac{3}{2}}}{\chi y_p} \tag{8}$$

The value of kinetic energy of turbulence k_p at the point adjacent to the wall is found from the relation general equation of balance with diffusion neglected. The component representing dissipation $\rho\varepsilon$ in the equation of transport is estimated as follows:

$$\rho\varepsilon = c_\mu^{\frac{3}{4}} \frac{k_p^{\frac{3}{2}}}{\chi} \ln\left\{\frac{E y_p c_\mu^{\frac{1}{4}} k_p^{\frac{1}{2}}}{\nu}\right\} \tag{9}$$

The boundary value problem obtained for the set of five non-linear differential equations was solved by means of the finite difference method. Distributions of dependent variables corresponding to the steady flow were used as the initial conditions. Being obtained by solving equations (1) with $\partial p/\partial z = \text{const}$ in the time interval which was much larger than the time of flow disturbance.

4.3. Finite-difference Equations and Computation Algorithm

Discretization of the differential equations of transport was based upon integrating the components of these equations on the difference mesh (Fig. 2a), the fragment of which is shown in Fig. 3 (Dobrowolski, Pospolita 1987).

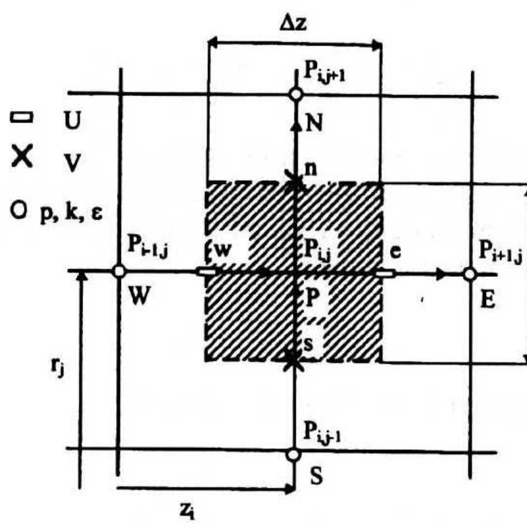


Fig. 4. Fragment of the difference mesh with denotations

As result of integration, the following finite-difference equation was obtained

$$\begin{aligned} &\tfrac{r_P}{\Delta t} A_P^\phi \left(\phi_P^{n+1} - \phi_P^n\right) + \left(r\,U\phi - r\,\Gamma_\phi \tfrac{\partial\phi}{\partial z}\right)_e^{n+1} A_e^\phi - \\ &+ \left(r\,U\phi - r\,\Gamma_\phi \tfrac{\partial\phi}{\partial z}\right)_w^{n+1} A_w^\phi + \left(r\,V\phi - r\,\Gamma_\phi \tfrac{\partial\phi}{\partial r}\right)_n^{n+1} A_n^\phi + \\ &- \left(r\,V\phi - r\,\Gamma_\phi \tfrac{\partial\phi}{\partial r}\right)_s^{n+1} As_e^\phi = \left(S_P^\phi \phi_P + S_U^\phi\right)_P^{n+1} r_P A_P^\phi \end{aligned} \tag{10}$$

where the indices e, w, n and s denote the nodes lying half-way from the central point P to one of the adjacent main nodes E, W, N and S. Quantities A^{ϕ} are related to the areas of surfaces formed by the edges of a control surface, when it is rotated by a unit angle, and $A_P^{\phi} = \Delta z \cdot \Delta r$. Components S_P^{ϕ} and S_U^{ϕ} are concerned with the linearization of source terms of the set (1).

Since in the general difference equation the unknowns in the main nodes of difference mesh must appear, the estimation of shares of the convection and diffusion fluxes through the control surfaces is required. It was demonstrated in the paper (Patankar 1980) that the stability of the algorithm is ensured by a hybrid scheme, which is the combination of a central and a "stream oriented" scheme. Considering the surface A_w^{ϕ} we obtain

$$\left(U\phi - \Gamma_{\phi}\frac{\partial \phi}{\partial z}\right)_w r_w A_w^{\phi} = \tag{11}$$
$$= \begin{cases} U_w r_w A_w^{\phi}(\phi_w + \phi_P) - \Gamma_{\phi} r_w A_w^{\phi}(\phi_P - \phi_w)\delta z & \text{for } |\mathrm{Re}^{\delta}| < 2 \\ U_w r_w A_w^{\phi}\phi_w & \text{for } |\mathrm{Re}^{\delta}| \geq 2 \\ U_w r_w A_w^{\phi}\phi_P & \text{for } |\mathrm{Re}^{\delta}| \leq -2 \end{cases}$$

where $\mathrm{Re}^{\delta} = \varphi U_w \Delta z / \left(\Gamma_{\phi}\right)_w$ is a mesh Reynolds number. After applying the hybrid scheme to the remaining elementary surfaces we obtain the difference equation in a general form

$$a_P^{\phi}\phi_P^{n+1} = a_e^{\phi}\phi_e^{n+1} + a_w^{\phi}\phi_w^{n+1} + a_n^{\phi}\phi_n^{n+1} + a_s^{\phi}\phi_s^{n+1} + \Gamma_P A_P^{n+1} a_P^{\phi} S_U^{\phi} + M_P\phi_P^n \tag{12}$$

where

$$a_P^{\phi} = a_e^{\phi} + a_w^{\phi} + a_n^{\phi} + a_s^{\phi} - r_P A_P^{\phi} S_U^{\phi} + M_P\phi_P^n \tag{13}$$

$$M_P = \frac{r_P}{\Delta t} A_P^{\phi} \tag{14}$$

and the upper index ϕ is related to the variable ϕ. The difference equation (12) is valid in every interior point of a difference mesh. It relates the value of variable ϕ in central point P of the mesh to the corresponding values in four neighbouring main points. Coefficients a^{ϕ} refer to the joint share of convection and diffusion estimated according to the scheme (11). As the difference scheme for variable t is implicit, the sets of difference equations of type (12) are solved iteratively, for every time step $n + 1$. The solution of the set of difference equations is based on the SIMPLE method (Patankar 1980). A single interior iteration cycle consists of the solution of the set of equations of motion

$$a_P^{v} V_P^{*} = \sum_j a_i^{v} V_j^{*} + S_U^{v} + A_s^{v}\left(p_s - p_P\right) \tag{15}$$

$$a_P^{U} V_P^{*} = \sum_j a_i^{v} U_j^{*} + S_U^{U} + A_s^{U}\left(p_w - p_P\right) \tag{16}$$

where quantities denoted ( )* are the approximate values and solution of a Poisson equation for pressure correction p'

$$a_P^p p'_P = \sum_j a_j^v p'_j + S_U^v + S_U^p \tag{17}$$

to satisfy the continuity equation the correction for velocity field

$$U_p = U_p^* + D^U \left(p_w - p'_p\right) \tag{18}$$

$$V_p = V_p^* + D^V \left(p_s - p'_p\right) \tag{19}$$

where

$$D^U = \frac{r_j}{\Delta z a_p^u}$$

$$D^V = \frac{r_j - 1/2}{\Delta r_j a_p^v}$$

is carried out for the current values of pressure correction p'. Component S_U^p depend sources of mass resulting from the fact the approximate velocity field does not satisfy on the equation of continuity. Equations of the turbulence model are solved simultaneously with those of motion.

The internal iteration process is continued until the convergence criterion is fulfilled

$$\max \{\mathrm{Re}\, s(\phi)\} \leq \lambda \tag{20}$$

where

$$\mathrm{Re}\, s(\phi) = \max_{i,j} \left\{ a_p^\phi \phi_p = \sum_{N,S,E,W} a_j^\phi \phi_j - S_U^\phi \right\} \tag{21}$$

When condition (20) is satisfied, the calculations concerning the next time step Δt are undertaken. The value of Δt was chosen experimentally. No limitation on the value of Δt concerned with the stability of algorithm was encountered.

5. The Analysis of the Influence of Unsteadiness of Fluid Motion on the Range of Changes in Model Coefficients

The influence of changes of flow upon the range of variations of coefficients in models of flow-meters has been analysed. In case of orifice and Venturi nozzle the number of flow α was determined on the basis of the formula presented in Table 1. Such quantities, occurring in it, as Coriolis coefficients in cross-sections 1 and 2, loss factor ξ and the remaining were determined from their definitional equations on the basis of a velocity field and pressures, determined numerically, within the limits of the nozzle. Details concerning the manner of determining the variability of the flow number α in the time function, are presented in (Dobrowolski et al.

1991) and (Dobrowolski, Pospolita 1987; Pospolita 1994). In the case of a turbine flow-meter, the coefficients b_1 and b_2 were determined from the equations given in Table 1. Velocity distributions, variable in time, were obtained on the basis of the numerical solution of system (2) for unsteady flow in the annular channel. Similarly the coefficient of sensitivity k_U of the ultrasonic flow-meter was determined from the equation given in Table 1. As above, velocity distributions variable in time were obtained on the basis of the numerical solution of system (2) for unsteady pipe flow.

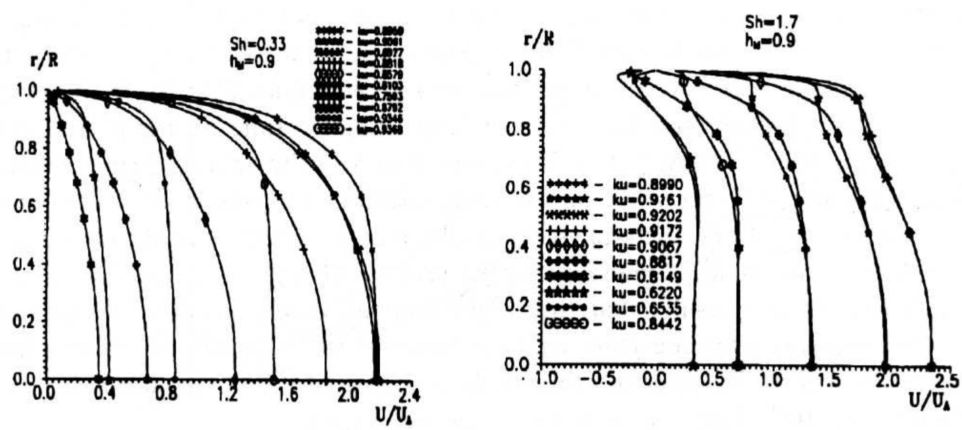


Fig. 5. Velocity profiles and coefficient k_U during stream pulsation; $\bar{k}_A$, $\bar{k}_U$ – quantities averaged at time, h_M – relative amplitude of mass stream pulsation

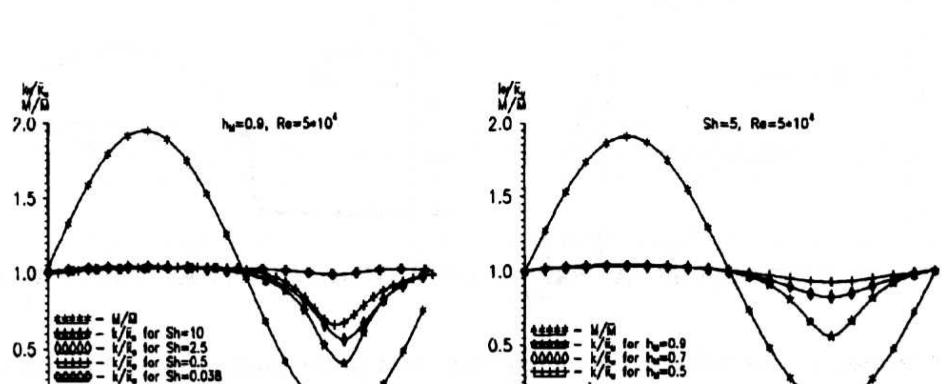


Fig. 6. Influence of the Strouhal number and amplitude of mass stream pulsation in the range of changes of k_U while pulsation

By way of example Fig. 4 presents velocity profiles, determined numerically, for pulsating flow by ultrasonic flow-meter. In Fig. 5, corresponding with the flows, the values of the coefficient of sensitivity k_U with regard to the values k_U for stationary flow (Boś et al. 1996) are presented. It may be found that in the case of pulsating flow, with a relatively considerable amplitude of stream pulsation, in that part of a pulsation period when flow lessens, the changes of coefficient k_U are considerable. This results from the big changes of velocity profile in that part of a pulsation period, together with the possibility of occurring reverse flow in the vicinity of the wall. The ranges of changes of velocity profiles and the coefficient k_U are smaller in case of changes connected with the increase in the diameter of stream velocity (accelerated flow) in particular, when the changes take place for flows characterized by Reynolds numbers greater than 10^4. Fig. 6 shows the changes in k_U versus time for a velocity leap. Relative time on the X-axis from 1 to 2 corresponds to the steady flow, from 2 to 3 to the velocity leap. Quantity of $\bar{k}_U$ corresponds to the value of sensitivity coefficient for the steady flow before the velocity leap. From the analysis it results that for short times of such a leap the relative value of $k_U/\bar{k}_U$ undergoes the greatest changes, i.e. about 6%. From calculations it also results that the largest change in the coefficient value can be observed just after the leap, next k_U tends to its value for the steady flow, proper for a given Re after the velocity leap. In Fig. 6 the Strouhal number (22) characterizes turbulence in the form of a velocity leap.

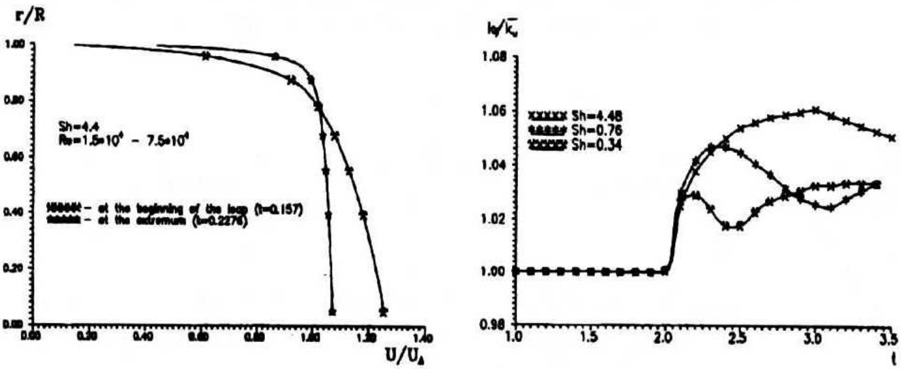


Fig. 7. Velocity profiles before the stream acceleration and for the extremum of k_U. Values of k_U for the stream acceleration ($\bar{k}_U$ – directly before acceleration)

In (Boś et al. 1996; Dobrowolski, Pospolita 1987; Pospolita 1992) are presented the results of experimental and numerical investigations concerning pulsating flows through constriction, turbine and ultrasonic flow-meters and the analysis of the influence of pulsation upon the metrological properties of these flow-meters.

Flows accelerated from the steady state in the range of numbers Re = $10^4 \div 10^5$, were also examined. In numerical investigations the flow disturbance was treated in the form of a flux acceleration $d\dot{M}/dt$.

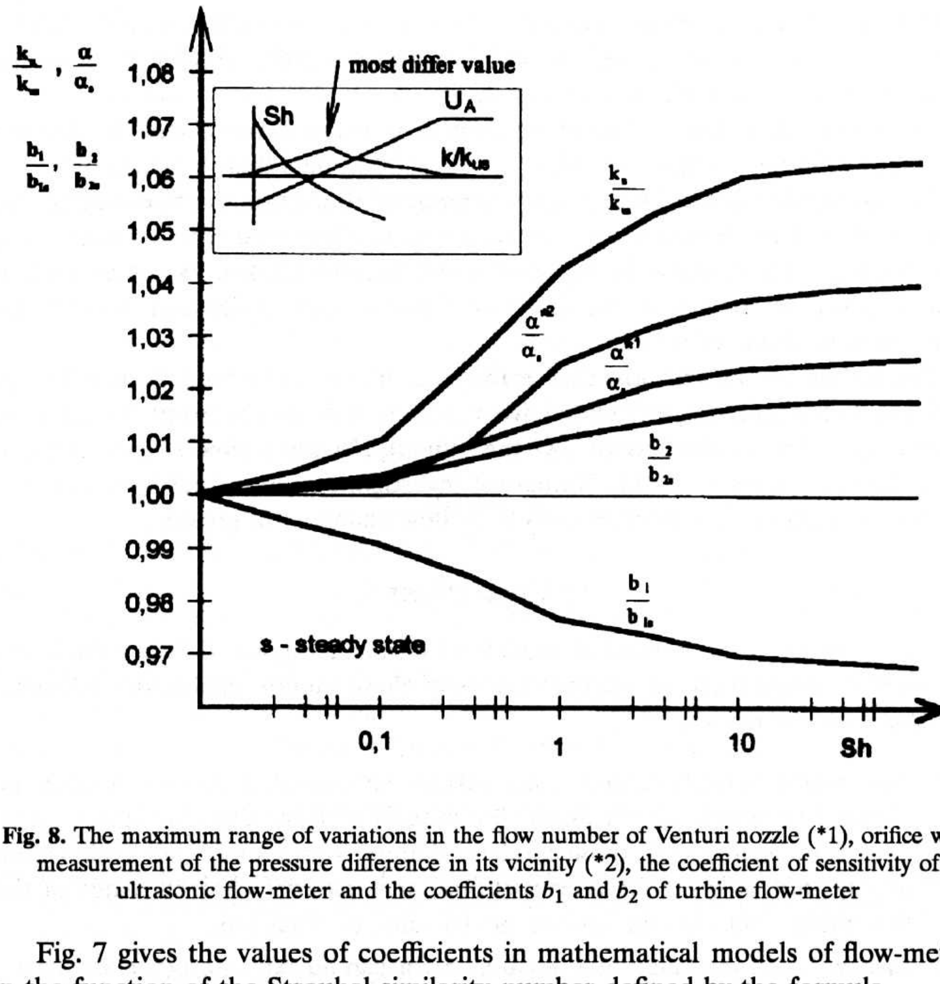


Fig. 8. The maximum range of variations in the flow number of Venturi nozzle (*1), orifice with measurement of the pressure difference in its vicinity (*2), the coefficient of sensitivity of ultrasonic flow-meter and the coefficients b_1 and b_2 of turbine flow-meter

Fig. 7 gives the values of coefficients in mathematical models of flow-meters in the function of the Strouhal similarity number defined by the formula

$$Sh = \frac{\frac{dU_A}{dt} D}{U_A^2} \tag{22}$$

where U_A is the velocity averaged in the section of flow-meter, D – diameter of pipeline.

These are the values which within the change U_A differ most from the values corresponding to steady flow. The Strouhal numbers, given on the axis of abscissae, are the maximum ones in a given range of flux change.

It can be found that the change in profile, associated with the change of flux, influences, the coefficient k_U of the ultrasonic flow-meter to the highest degree, increasing its value about $4 \div 5\%$ for $Sh > 1$. Similarly, in the case of accelerated laminar flow, the equation describing the velocity profile points to the oblateness of the profile (Dorden 1979), in comparison with the Poiseuille flow. This is also associated with the several percent rise in the values of the coefficient of sensitivity. The range of changes in coefficients of turbine flow-meter, brought about by the change of flux, is comparatively the least.

The variations in the coefficient b2 are little whereas the coefficient b_1 changes its value within the compass of 1% for Strouhal numbers less than 0.1.

In case of constriction flow-meters the range of changes in the flow number for a given value of the Strouhal number, depends mostly on the type of constriction flow-meter and its modulus. In Fig. 7 are presented the relative values of the flow number in the function of Sh for the Venturi nozzle with modulus of $m = 0.6$ and orifice with modulus of 0.25.

The results presented show that in the case of the Venturi nozzle with a large modulus, the number α, calculated for unsteady flow strays from the value α_s characteristic for steady flow of less than about 1% when flow is characterized by the Strouhal number < 0.1. Numerical investigations show that in the case of orifices, changes in flow number caused by flow change, are greater.

6. Conclusions

The comparison of mathematical models of particular types of flow-meters and the analysis of variations in the coefficients of these models enable the following conclusions to be drawn:

1. The mathematical models with certain simplified assumptions, can be brought to equations which are the models of basic dynamic terms. Time constants occurring in these models, change considerably and their values depend not only on the constructional and material characteristics of the flow-meter, but also the instantaneous value of mass flux.
2. In most cases the value of a dynamic error can be expressed by a simple relationship, being the product of the time constant of flow-meter and the derivative $d\dot{M}/dt$. This relationship, useful from the point of view of measuring practice, enables estimation of the usefulness of a given type of flow-meter for measuring a given class of non-stationary flows.
3. The numerical analysis of the range of changes in model coefficients was carried out with turbulent flow in the form of the change of flux. It showed that the range of variations in the coefficients b_1 and b_2 of the equation of turbine flow-meter is the least. The changes occur within the range of 1% of the values of stationary coefficients in the

entire range of the values of the *Sh* number analysed. The changes the sensitivity coefficient of ultrasonic flow-meter are relatively substantial and contained within the range of $5 \div 6\%$ when $Sh > 1$. In the case of constriction flow-meters the range of variations in the flow number depends on the type of constriction flow-meter and its modulus. In the case of Venturi nozzle the range of changes is less. The problem requires further experimental and theoretical investigations.

4. Low values of the time constants of flow-meters within the range of $0.001 \div 0.015$ make, measurement of rapidly-changing fluxes possible in many cases. However, initially it is necessary to evaluate the character of flux transitions considering the value of a dynamic error and the range of changes in the model coefficients. When it is possible to assume that the range of variations in the coefficients is so small that it does not cause any additional measuring error, it is still possible to measure the instantaneous value of mass flux by means of flow-meter with a greater time constant. This requires, however, the adaptation of the measuring system for determining the instantaneous or mean values of mass flux on the basis of the analog and numerical solutions of the equation connecting mass flux with the indications of flow-meter.

References

Baldin A. A., Boszniak L.L., Solowskij W. M. (1983) *Rotamery*, Leningrad, Maszinostrojenie.

Bhatt B. L., Wedekind G. L. (1985) Measurements of Flowrates under Transient Conditions using a Differential Pressure Transient *Proceeding of the 16th Southeastern Seminar*, Springer-Verlag, 93–102.

Boś K., Kabza Z., Pospolita J. (1996) Mathematical modeling in measurements of unsteadly flows with ultasonic flowmeters, *Mechanika Teoretyczna i Stosowana*, No. 3, 591–603.

Dijstelbergen H. H. (1964) Rotameter dynamics, *Chemical Engeneering Science*, Vol. 19, 856–865.

Dobrowolski B., Kabza Z., Pospolita J. (1981) Numerical investigations of the influence of pressure pulsation on the errors of mass flux measurement (in Polish), *Archiwum Hydrotechniki*, Vol. 28, No. 3, 377–386.

Dobrowolski B., Kabza Z., Pospolita J. (1983) Theoretical estimation of the influence of pulsation on the error of measurement of the mean value of mass flux (in Polish), *Archiwum Hydrotechniki*, Vol. 30, No. 2, 165–174.

Dobrowolski B., Kabza Z., Pospolita J. (1985) *The Analysis of Meterology performance of Constriction Flow-Meters under Pulsating Flow Condition*, IMEKO, X World Congress, Praha.

Dobrowolski B., Kabza Z., Pospolita J. (1991) *Theoretische und experimentelle Untersuchungen des Einflusses der Pulsationsstromung auf die Charakteristiken der Drosselgerate*, Fortschritt-Berichte VDI Reihe 7: Stromungstechnik nr 193, Dusseldorf.

Dobrowolski B., Pospolita J. (1987) Theretical Estimation of the Applicability Range of the Differential Pressure Type Flowmeters in Presence of Pulsation of the Mass Flux, *Arch. Mech.*, Vol. 39, No. 6, 571–588.

Dorden J. (1979) Charakteristiki izmerenij stacjonarnowo i niestacjonarnowo raschoda i ekspluatacjonnyje charakteristiki ultrazwukowo razchodomiera, *IX Kongres IMEKO*, Moskwa.

Hamidulin W. K. (1989) *Ultrazwukowe kontrolno-izmeritelnyje ustrojstwa i sistemy*, Leningrad.

Kabza Z., Pospolita J. (1989) Analysis of dynamics of constriction flowmeters (in Polish), *Metrologia i Systemy Pomiarowe*, No. 2, 23–39.

Kremlewskij P. P. (1989) *Raschodomiery i sczotcziki koliczestwa*, Maszinostrojenie, Leningrad.

Kuzniecov J. N., Prewzner W. I., Kalinin E. T. (1981) Ob izmereni raschoda odnofaznoj sredy suzajuszczim ustrojstwam w niestacjonarnych procesach, *Teploenergetika*, No. 12, 57–59.

Launder B. E., Spalding D. B. (1971) *Mathematical Models of Turbulence*, New York, Academic Press.

Launder B. E., Reece G. B., Rodi W. (1975) Progress in the Development of a Reynolds Stress Turbulence Closure, *J. Fluid Mech.*, Vol. 68, No. 3, 537–566.

Lee W. F. Z., Kirk M. J., Bonner J. A. (1975) Gas Flow-meter Measurement of pulsating Flow, *Journal of Engineering Power*, Oct., 531–539.

Locjanski L. G. (1973) *Mechanika zidkosti i gaza*, Izd. Nauka, Moskwa.

Maki H., Ikeda Y. (1981) Measurement of pulsating Flow Rate by means of Float-area-type flowmeters, *Bull. of the JSME*, Vol. 24, No. 189, 555–561.

Mottram R. C., Zarek J. M. (1967) The Behaviour of Orifice Plate and Venturi Nozzle Meters under Pulsating Air Flow Condition, *The Engineer*, Dec., 25–37.

Pospolita J. (1992) Theoretical evaluation of metrological properties of turbine flow-meters for measurement of pulsating flow, *Archiwum Hydrotechniki*, Vol. 39, No. 4, 3–17.

Pospolita J. (1994) *Estimation of usability of some selected flowmeters for measurements of unsteady flows*, Wyższa Szkoła Inżynierska w Opolu, Studia i monografie, z. 75, Opole.

Takahashi K., Yamashita Y., Yamagichi Y. (1982) The Measurement of Laminar Pulsating Instantaneous Flow Rate with a Venturi Meter, *Trans. Soc. Instrum. and Control Eng.*, Vol. 18, No. 6.

Torigoe I. (1987) A new Method for Measuring Flow Acceleration and its Application to Pulsating Flow Measurement, *Trans. Soc. Instrum. Control Eng.*, Vol. 23, No. 8, 794–799.

Torigoe I. (1987a) Mass Flowmeter Utilizing Small Pulsation, *Trans. Soc. Instrum. Control Eng.*, Vol. 23, No. 7, 671–676.

Uri J. E. (1963) The Pulsation Error of Rotameters, *Isr. Journ. of Technol.*, Vol. 1 No. 1, 27–29.

Zhao T., Kitagawa A., Kagawa T., Takenaka T. (1987) A Real Time Method of Measuring Unsteady Flow Rate and Velocity Employing Differential Pressure in a Pipe, *JSME International Journal*, Vol. 30, No. 260, 263–270.

Patankar S. V. (1980) *Numerical heat transfer and fluid flow*, Hemisphere Publ. Corp., New York.