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Model Tests of Shallow Foundations Stability Under Static and Cyclic Load

Bohdan Zadroga, Zygmunt Kurałowicz

Gdańsk Technical University, Environmental Engineering Faculty, Narutowicza 11, 80–952 Gdańsk, Poland

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Abstract

A bearing capacity and settlement analysis of shallow foundation subjected to axial and vertical static and cyclic loads is presented. Different formulae of calculations of settlement under cyclic load are reviewed. The scope, methodology and results of model tests in both plane-strain and axi-symmetrical states are analysed. The formula of ultimate bearing capacity under static load and method of settlement calculations of foundation under cyclic load are recommended.

1. Introduction

Bearing capacity and settlements of foundations subjected to static load have been studied extensively by many researchers for more than four decades. A review of different calculation methods based on various approaches can be found in Zadroga (1994).

During the last 20 years one could observe an essential increase of interest in stability problems of shallow foundations loaded cyclically. The problem is of great practical importance in many fields of civil and marine engineering such as foundations of silos and engines, roads, railways, various offshore structures. Gudehus and Hettler (1980, 1981) and Moore and Lokuratna (1987) show that for cyclic load the settlement of foundations resting on sandy soils may be several orders higher than due to static load. The rate and magnitude of final displacement of foundation depend on many factors such as: magnitude and frequency of cyclic load, number of cycles, shape and dimensions of foundation etc.

To analyse the influence of these factors series of model tests with strip and foot foundations rested on dry and wet sand, subjected to cyclic loading of low frequency were carried out. The foundations were loaded by cyclic vertical and axial forces the values of which varyied from very small up to the value corresponding to the ultimate static bearing capacity.

2. Stability of Shallow Foundations under Static Loads

A classical formula of the bearing capacity of the foundation resting on surface of non-cohesive soil and subjected to an axial and vertical static load can be written in the well-known form:

$$q_{us} = \frac{Q_{us}}{BL} = 0.5\gamma B N_B, \tag{1}$$

where:

- B and L – width and length of the foundation respectively,
- γ – unit weight of soil,
- N_B – bearing capacity factor.

The comparison of various formulae for N_B factor described in detail by Zadroga (1994) show essential differences. Also, bearing capacity model tests of shallow footings and strip foundations being carried out in geotechnical laboratories clearly show that model test results are, in general, much higher than those calculated by traditional methods. The comprehensive comparative analysis of both calculations and experimental results of bearing capacity of foundations for Polish, Finnish and Japanese model tests was made by Zadroga (1994). On the basis of statical elaboration of over 50 experiments for a wide range of angle of internal friction (26°–46°) the following formulae were recommended:

– for strip foundations

$$N_B = 0.657\exp[0.141\phi], \tag{2}$$

– for footings

$$N_B = 0.096\exp[0.188\phi], \tag{3}$$

where ϕ is given in degrees.

Experimental static failure load was determined from the load-settlement ($Q - s$) curves concerning the character of these curves. An example for static and cyclic loads is presented in Fig. 1.

In the interpretation of the results of settlement three main criterion were assumed in order to determine an experimental bearing capacity.

- the end of the clear rectilinear part of the $Q - s$ curve,
- settlement equal to 3–4% of the foundation's width or diameter,
- the moment of the first breaking of coloured strips of sand marked in the subsoil registered by video facilities.

The comparison of experimental and calculated ultimate bearing capacity in terms of formulae (2) and (3) is presented in Table 1.

Table 1. Comparison of experimental and calculated ultimate bearing capacity of shallow foundation under static load

Method	Series			
	A (dry sand) $L = 0.5$ m		B (dry sand) $D = 0.345$ m	C (wet sand) $D = 0.345$ m
Experimental q_{usexp} [kPa]	$B = 0.15$ m	60.0	123.1	80.3
	$B = 0.20$	70.0		
Calculated q_{uscal} [kPa]	$B = 0.15$ m	50.0	109.9	63.7
	$B = 0.20$	66.6		
$m = \frac{q_{usexp}}{q_{uscal}}$	$B = 0.15$ m	1.20	1.12	1.26
	$B = 0.20$ m	1.05		

where:

L, B length and width of rectangular foundation models respectively,
D diameter of circular foundation model.

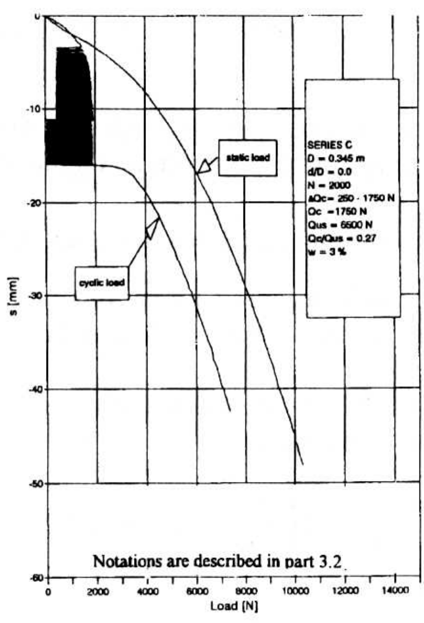


Fig. 1. Comparison of load-settlement $(q - s)$ curves for static and cyclic loads obtained from experiments in axi-symmetrical state

The results from Table 1 show good conformity between experimental and calculated values of ultimate bearing capacity of shallow foundation under static load. The difference between them is less than 20% which means that formulae (2) and (3) can predict reliable values of ultimate bearing capacity for non-cohesive soils.

To calculate the settlement of shallow foundation under static load the classical well known formulae were used.

3. Stability of Shallow Foundations under Cyclic Loads

The response of a subsoil under a foundation subjected to cyclic loads can be divided onto elastic and plastic parts. The latter imply a successive increase of permanent deformations. The aim of this analysis is an estimation of these deformations for practical engineering.

For the sake of brevity and simplicity, the following restrictions have been made:

- the subsoil is a uniform half space built of non cohesive dry or wet sand, so the pore pressure generation is excluded,
- there is a cyclic external load imposed, with low frequency (the inertia forces can be neglected),
- the amplitude of the cyclic load is low as compared with static ultimate bearing capacity.

This part of the paper will consist of the review of the literature, description of own model tests and comparison of experimental and calculated results.

3.1. Literature Review and Analyses

The review of some methods proposed by various authors aims at giving the possibility of better understanding of the problem and for multivariant calculation of settlement.

Hettler and Gudehus (1980) have proposed semi-empirical formula for calculation of settlement under cyclic loads:

$$\frac{s}{B} = A\left(\frac{p}{\gamma B}\right)^{\alpha}(1 + c_0 \ln N) \tag{4}$$

where:

B – width of foundation,
A – shape factor,
α, c_0 – constants determined from model tests,
p – external pressure,
N – number of cycles.

The above relationship has been determined on the basis of the results of model tests made on compacted dry sands with circular rigid foundations of diameters changing from 0.05 do 0.98 m.

Moore and Lokuratna (1987) carried out three series of model tests on circular footings 0.1 m and 0.15 m in diameter resting on dense dry sand in order to examine the effect of frequency, amplitude and level of cyclic load and compared the results with static bearing capacity and settlement behaviour of the footing.

On the basis of experimental results the following relationship between settlement and number of cycles was proposed:

$$\log\left(\frac{s}{D}\right) = m\log N + \log\left(\frac{s_1}{D}\right) \tag{5}$$

where:

- s – settlement under N load cycles,
- s_1 – settlement corresponding to $N = 1$,
- N – number of cycles,
- D – diameter of the footing,
- m – gradient of the $\log\left(\frac{s}{D}\right)$ vs. $\log N$ curve in model tests ($m = 0.3$ for surface footings).

For the 150 mm diameter surface footing and frequencies between 0.1 and 4 Hz the settlement can be described by the following formula:

$$\frac{s}{D} = cN^{0.3}\left(\frac{p_s}{p_{us}}\right)10^{\frac{kp_c}{p_{us}}} \tag{6}$$

where:

- c, k – constant values determined in a model test ($c = 0.0024$, $k = 4.0$ is recommended for 150 mm diameter surface footing),
- p_s – static pressure,
- p_{us} – static failure pressure,
- p_c – cyclic pressure amplitude,
- – remaining symbols as in equation (5).

Formulae (5) and (6) are valid for relations $p_s + p_c < p_{us}$ and $p_c < p_s$.

Ortigosa et al. (1985) assumed that in granular soils the settlement s of a shallow foundation is originate by volume changes of the skeleton, which, in turn, originated fundamentally by existence of a cyclic shear strain field. On the basis of cyclic tests on circular plates with diameters equal to 0.6 m and 0.9 m and for static pressure $p_s = 0.1$ do 1.0 MPa and cyclic pressure $p_c = (0.1 \div 0.45)p_s$ the settlement is given by:

$$s = m_c \frac{p_c}{p_s} S_s \qquad S_s = \frac{p_s(1-\nu)DA}{2G} \tag{7}$$

where:

- m_c – settlement coefficient (is a function of the number of cycles),
- S_s – settlement under static load p_s,
- A – classical shape factor,
- G – shear modulus of the soil for static loads,
- D – diameter of the footing.

Formula (7) is restricted to granular soils and to pure soil-structure interaction problems with dimensionless frequency of less than 0.25.

Van Impe (1980) carried out a large experimental programme of vertical cyclic loading on dry and saturated sand in a box of dimensions $1.06 \times 1.06 \times 0.55$ m in width, length and height respectively. Circular plates of 0.12 m in diameter were loaded cyclically with frequency 3 Hz and with different amplitudes.

On the basis of Van Impe test results the following empirical relationship was proposed:

$$s = \left(1.65\frac{p_c}{p_s} - 0.6\right) D \log \frac{N}{405 \ln \frac{p_c}{p_s} + 450} \tag{8}$$

where:

- p_c – cyclic pressure,
- p_s – permanent static pressure,
- D – diameter of the footing,
- N – number of cycles.

The relationship (8) is valid for values of $A < 35\%$, where A is given by:

$$A = \frac{p_s - p_c}{p_{us} - p_c} \tag{9}$$

for which pore water pressure in saturated sand was insignificant.

Raymond and El Komos (1977) prepared four series of model tests in plane strain state on shallow foundations with dimensions $B \times L = 0.075 \times 0.20$ m and 0.228×0.20 m. The experimental results were approximated by the hyperbolic relationship. Finally, the settlement after the first cycle in a cyclic load may be determined for any percentage of the static failure load F at any cycle using the following equation:

$$s = \frac{a}{\frac{1}{\log N} - b} \tag{10}$$

where:

N – number of cycles,
a, b – empirical parameters.

The values of parameters a and b were determined from model tests and the following formulae were proposed:

$$a = -0.15125 + 0.0000693(F + 6.09)B^{1.18} \tag{11}$$

$$b = 0.153579 + 0.0000363(F - 23.1)B^{0.821} \tag{12}$$

where:

B – width of foundation (in mm),
F – level of cyclic load to failure static load (in percent).

Formulae (10) to (12) are valid mainly for plane strain state conditions and for dry sands with characteristics similar to Ottawa 20–30 sands.

Sawicki and Świdziński elaborated an original method for settlement calculations of shallow foundation under cyclic loads. The work was carried out within the framework of grant No. 70488-91.01 supported by the Polish Research Committee (KBN) in Warsaw. In the method proposed it is assumed that the settlements are caused mainly by an oedometric compaction which takes place in a small zone beneath the foundation. The range of the compaction zone is estimated on the basis of some empirical observations. Settlements of the foundation are calculated on the basis of compaction theory. In the theory a compaction curve which results from the compaction law assumed can be described by the following relationship:

$$\varepsilon^p = C_1 \ln(1 + C_2\xi), \tag{13}$$

where:

ε^p – an irreversible volumetric strain due to compaction,
C_1 and C_2 – material constants determined from cyclic oedometric tests,

$$\xi = (1 - K_0)N\sigma_z \tag{14}$$

K_0 – earth pressure coefficient at rest,
N – number of loading cycles,
σ_z – cyclic vertical load.

In the case of sand used in the model tests of cyclically loaded foundations, the compaction curve takes the following form:

$$\varepsilon^p = 3.684 \ln\left[(1 + 3.494(1 - K_0)N\sigma_z\right]. \tag{15}$$

Finally, the settlement can be calculated by the integral:

$$s = \int_0^H \varepsilon^p dz, \tag{16}$$

where H is the depth of the active zone under a foundation of width B. The details of the method can be found in Sawicki et al. (1997).

3.2. Scope, Methodology and Results of Model Tests

Model tests on shallow foundations resting on a non-cohesive subsoil, subjected to a cyclic vertical axial load were performed in The Geotechnical Laboratory of the Hydro-Engineering faculty of Gdańsk Technical University in 1993–94. In order to receive comprehensive information the experiments were carried out in both plane strain and axi-symmetrical states on non-cohesive dry and wet sands. The foundations were subjected to different numbers of cyclic loadings and performed at different ratios of cyclic load Q_c to ultimate bearing capacity Q_{us}.

The 48 model tests comprised three main series A, B and C. Series A regarded the experiments performed in plane strain state in laboratory stand 2.65 m long, 0.50 m wide and 1.07 m high. Two rigid steel models of foundations of the dimensions $B \times L = 0.15 \times 0.50$ and 0.2×0.5 m respectively were placed on a homogeneous subsoil prepared by moving sand curtain method. The experiments of this series were performed on dry fine medium dense sand with the average unit weight $\gamma = 17.0$ kN/m^3 and $\phi = 29°$ with a standard deviation equal to 0.13 kN/m^3 for γ and 0.40° for ϕ. In series A 30 experiments for the ratio of $n = Q_c/Q_{us}$ ranging from 0.21 to 1.0 for maximum number N =1500 of loading cycles was carried out.

The tests of series B were performed in axi-symmetrical state in a box with dimensions of $2.40 \times 0.96 \times 1.0$ m, on a circular rigid foundation with diameter $D = 0.345$ m. The load was transmitted by a special joint. The subsoil was made of medium dense dry sand of $\gamma = 16.2$ kN/m^3 and $\phi = 32°$. Series B regarded 8 experiments for the ratio $n = 0.13$ to 0.60 and for the maximum number of loading cycles $N = 2000$.

Series C regarded the same state as in series B but experiments were performed in a special cylindrical tank 2.0 m in diameter and 2.5 m in height, filled with wet sand in terms of fluidisation method. The water content varied from $w = 3\%$, to 26%, $\gamma = 16.5$ kN/m^3 to 18.5 kN/m^3 and $\phi = 29°$.

The results of the experiments recorded by PC were the bases for both qualitative and quantitative analyses. Typical results of series A, B and C are presented in Figs. 2 to 4 respectively. The analysis has concerned mainly the influence of the following factors:

- number of loading cycles N,

- a ratio of cyclic load Q_c to static failure load Q_{us},
- shape of foundation.

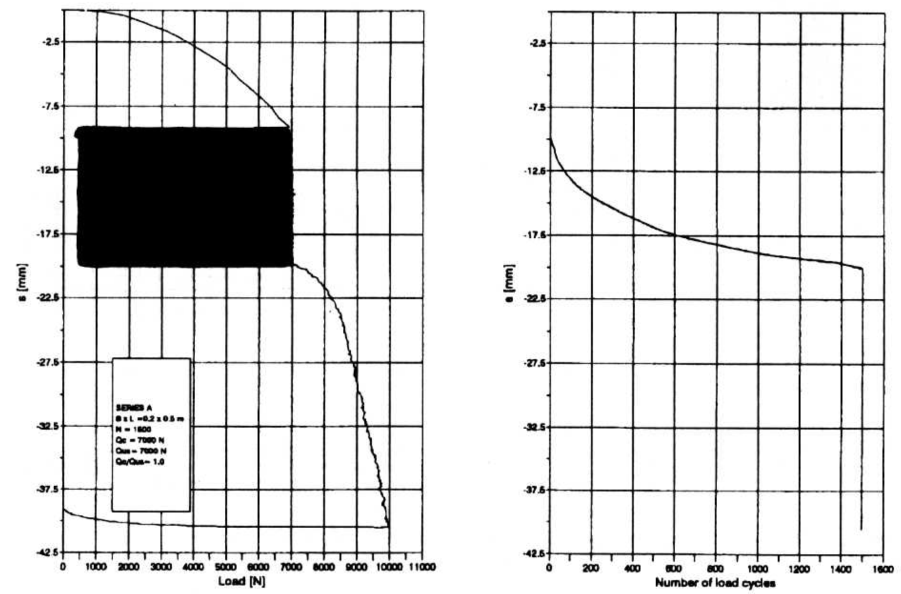


Fig. 2. Settlement vs. load and settlement vs. number of cycles. Experimental curves in plane-strain state. Series A on dry sand

The measured values of settlement of foundation excluding the first loading cycle as a function of number of loading cycles for various ratios of the cyclic to static failure loads $n = Q_c/Q_{us}$ are shown in Figs. 5 to 7. The results are presented into two scales: settlement in [mm] – number of loading cycles N and a relative settlement as a percentage ratio of a width (diameter) of foundation – number of loading cycles in normal logarithmic scale.

The detailed results of experiments are collected in Tables 2 and 3 for plane strain and axi-symmetrical states respectively.

A comparison of the settlement calculations for particular empirical methods analysed in chapter 3.1 shows that the best approximation of authors' model tests can be received by either Sawicki and Świdziński (1996) or Raymond and El Komos (1978) methods. The calculation results by the methods of other authors differed much more, significantly overestimating or underestimating model test results.

The following conclusions can be drawn on the basis of the results presented:

- settlement of foundation is influenced mainly by number of loading cycles and the level of cyclic to failure static loads,

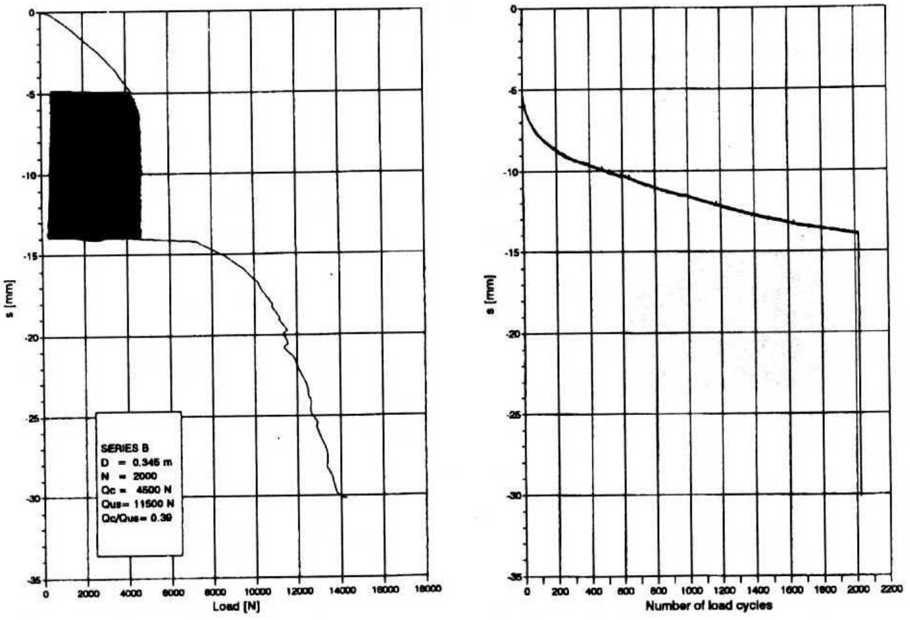


Fig. 3. Settlement vs. load and settlement vs. number of cycles. Experimental curves in plane-strain state. Series B on dry sand

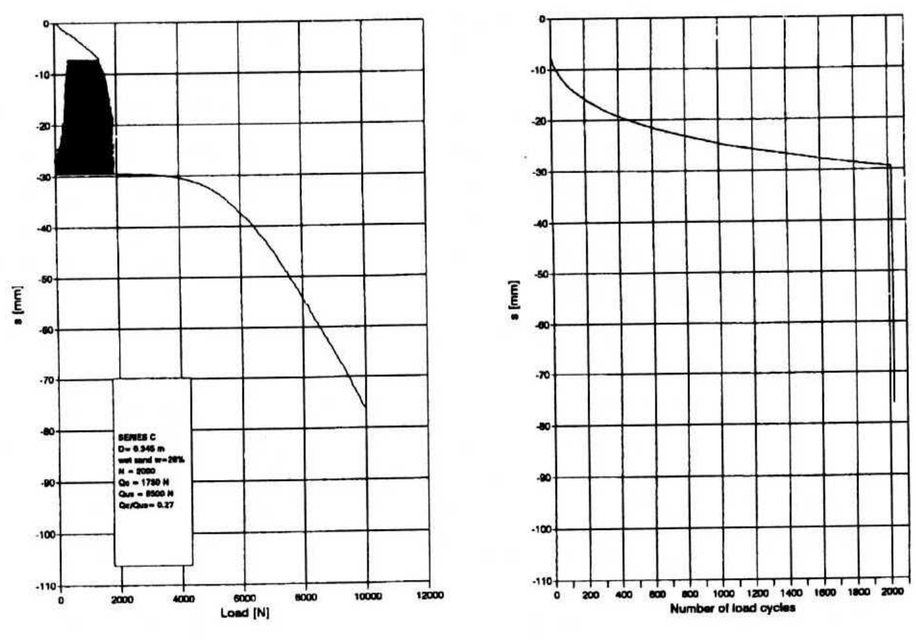


Fig. 4. Settlement vs. load and settlement vs. number of cycles. Experimental curves in axi-symmetrical state. Series C on wet sand

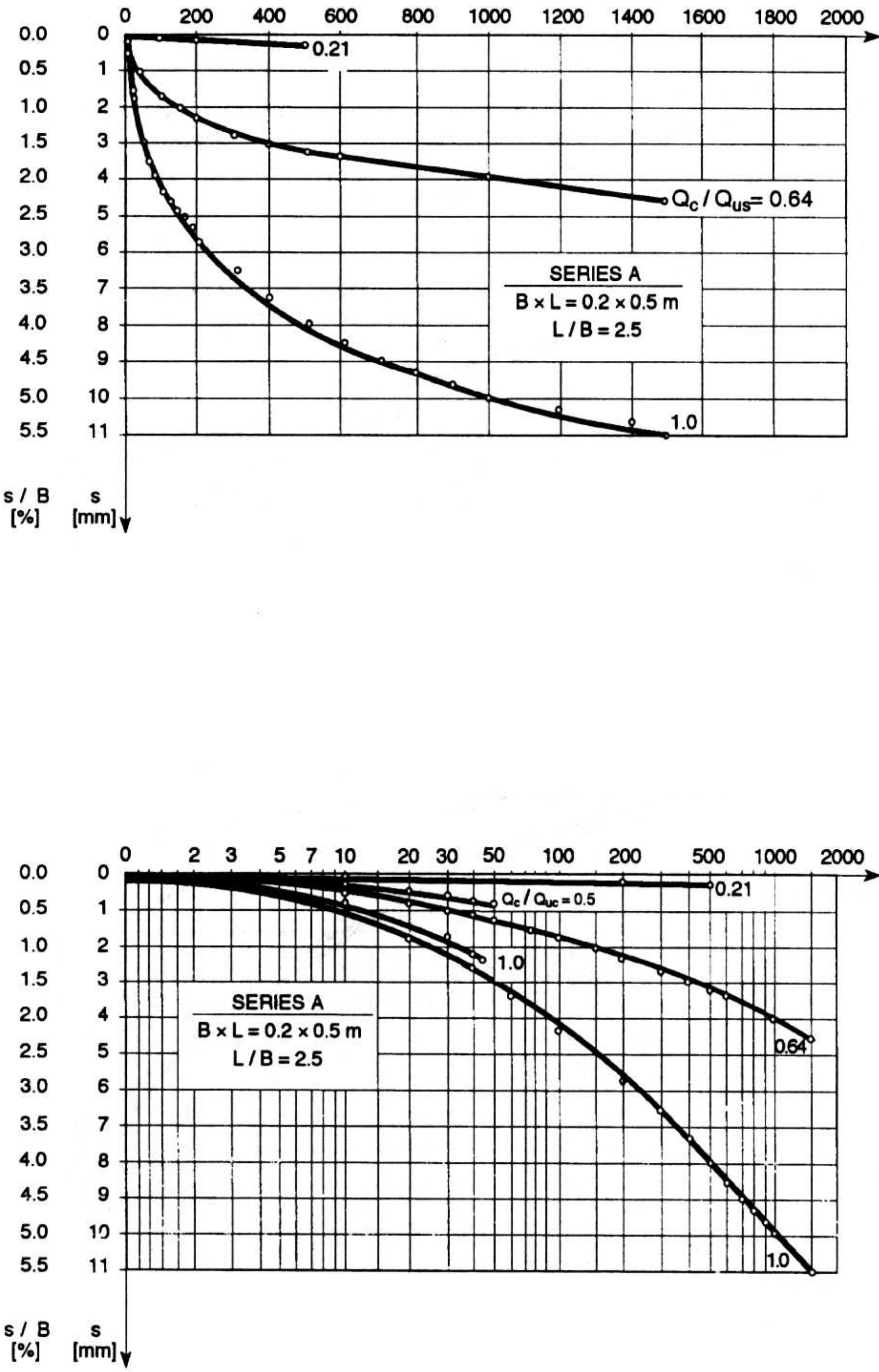


Fig. 5. Settlement vs. number of cycles. Experimental curves for various relative levels of cyclic loads to failure static load in series A

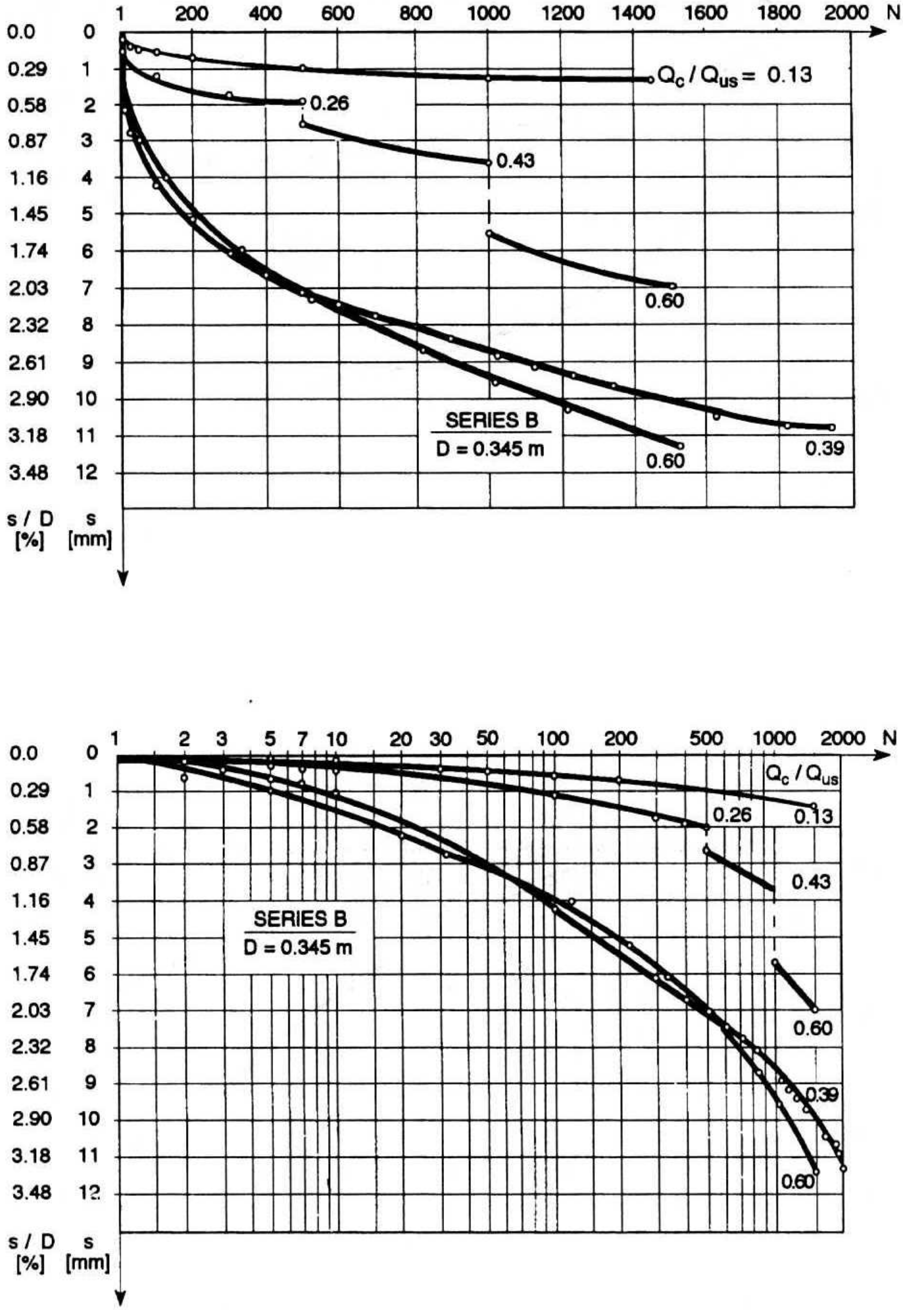
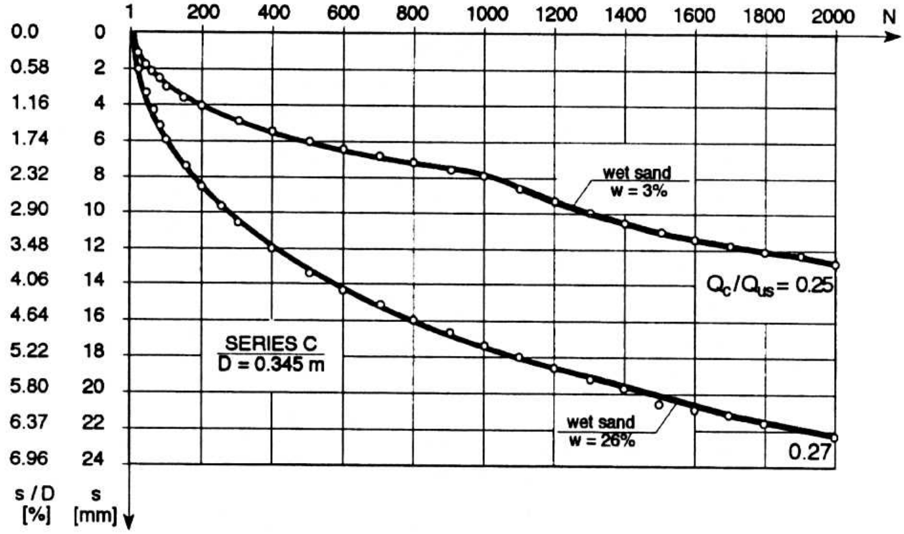


Fig. 6. Settlement vs. number of cycles. Experimental curves for various relative levels of cyclic loads to failure static load in series B



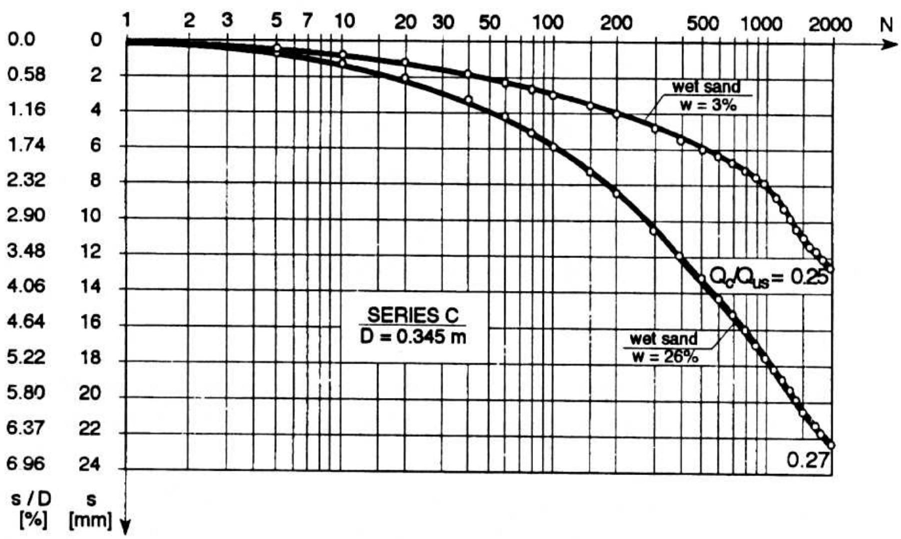


Fig. 7. Settlement vs. number of cycles. Experimental curves for various relative levels of cyclic loads to failure static load in series C

Table 2. Measured settlement of foundation for different ratios of $n = Q_c/Q_{us}$ in plane strain state. Series A

Number of cycles N	Settlement in [mm]				
	$n = 0.21$	$n = 0.50$	$n = 0.64$	$n = 1.0$	
50	0.12	0.81	1.25	3.00	3.23
100	0.17	–	1.70	4.30	–
500	0.29	–	3.15	8.00	–
1000	–	–	3.89	9.90	–
1500	–	–	4.53	11.0	–
Settlement after the first loading cycle	0.61	2.48	2.50	9.00	12.2

Table 3. Measured settlement of foundation for different ratios of $n = Q_c/Q_{us}$ in plane strain state. Series B

Number of cycles N	Settlement in [mm]			
	$n = 0.13$	$n = 0.26$	$n = 0.39$	$n = 0.60$
50	0.47	0.88	3.01	3.14
100	0.57	1.19	4.21	4.40
500	0.97	1.90	7.09	8.25
1000	1.17	–	8.70	10.71
1500	1.31	–	11.05	11.61
2000	–	–	11.26	–
Settlement after the first loading cycle	0.58	1.03	4.85	8.42

- settlement of foundation increases with an increase in number of loading cycles N and the rate of that increase decreases with N,
- settlement of foundation increases substantially with an increase of the Q_c/Q_{us} ratio.

Similar relations and conclusions may be drawn from the experimental results obtained by Raymond and El Komos (1978). Some of them are presented in Fig. 8.

A comparison of the experiment results and theoretical prediction proposed by Sawicki et al. (1996) is shown in Fig. 9 and Table 4. Additionally, in Table 4 the results are compared with those obtained by Raymond and El Komos empirical proposal. It can be seen that the Sawicki and Świdziński method produces quite satisfactory results especially for series A and B performed on the air dry sand.

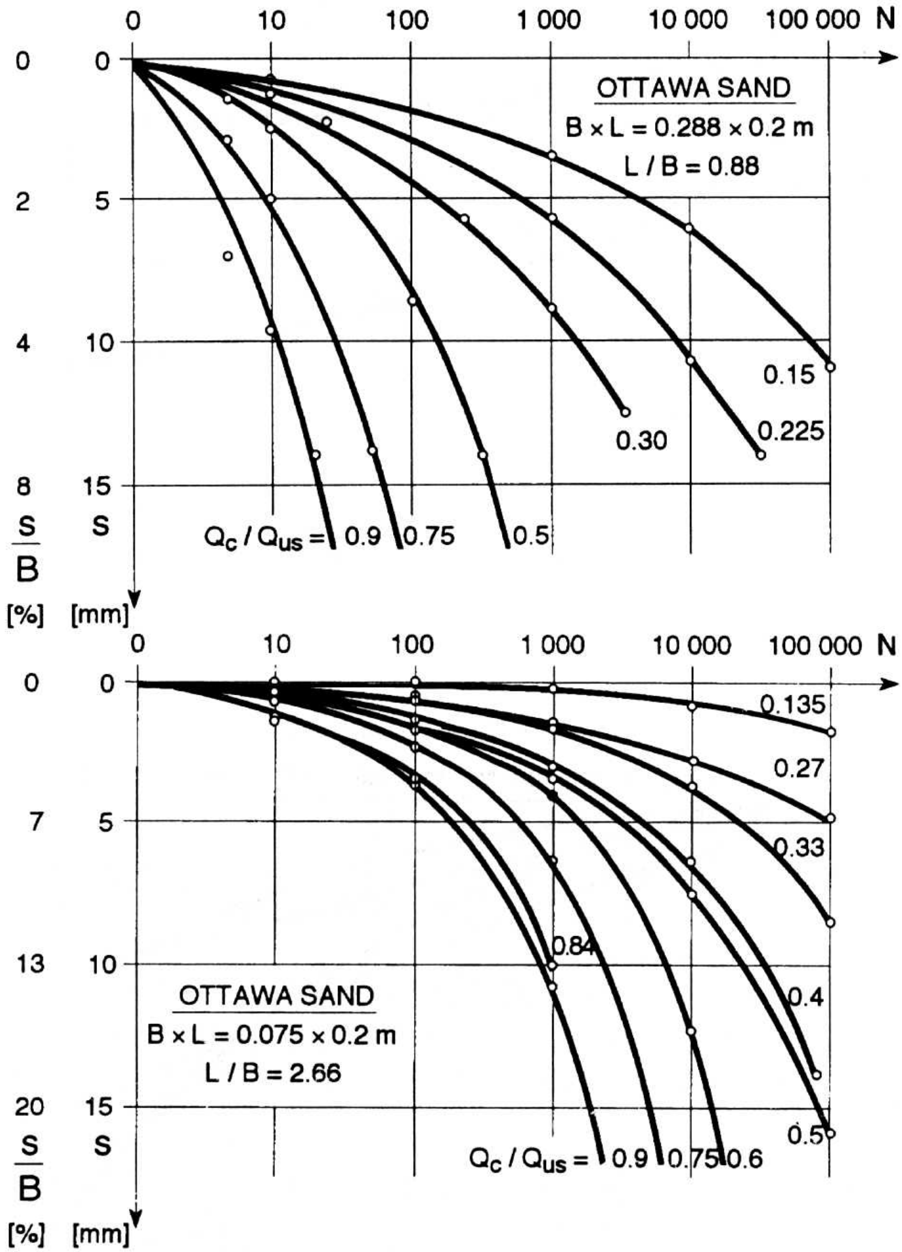
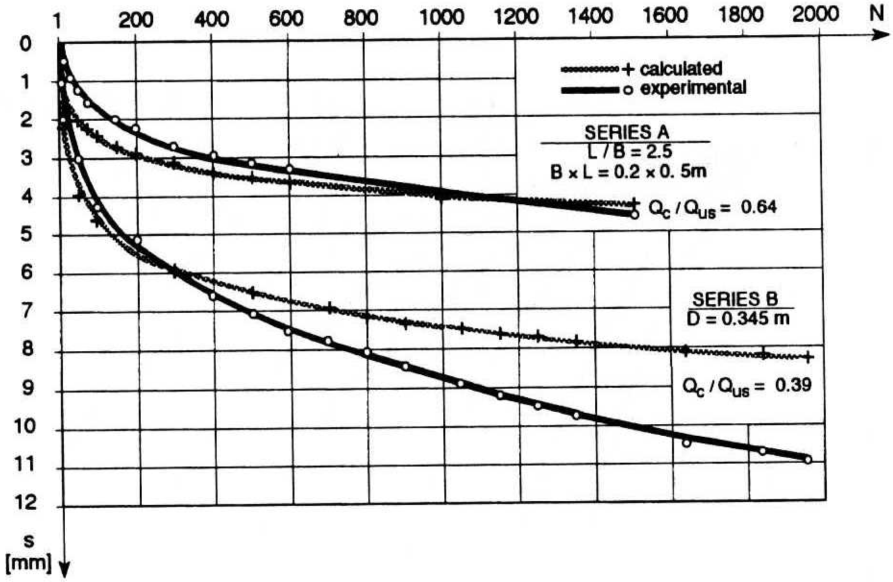


Fig. 8. Settlement vs. number of cycles. Experimental curves for various relative levels of cyclic loads to failure static load obtained by Raymond and El Komos (1978)



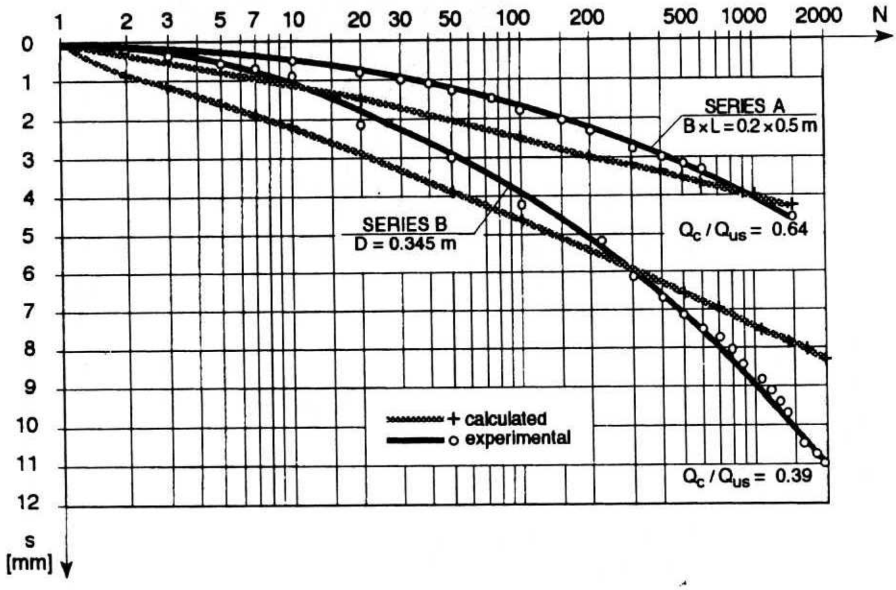


Fig. 9. Comparison of experimental and calculated settlement vs. number of cycles. Curves for chosen experiments in series A and B

Table 4. Measured and calculated settlements of foundation excluding the first loading cycle [in mm]

Number of cycles	SERIES A $n = 0.64$			SERIES B $n = 0.39$			SERIES C $n = 0.27$		
	Experimental	Calculated		Experimental	Calculated		Experimental	Calculated	
		Sawicki et al. (1997)	Raymond & El Kosmos (1978)		Sawicki et al. (1997)	Raymond & El Kosmos (1978)		Sawicki et al. (1997)	Raymond & El Kosmos (1978)
10	0.55	1.11	0.76	0.97	2.18	3.78	0.78	1.50	1.44
50	1.25	2.04	1.56	3.01	3.91	8.10	2.00	3.05	4.15
100	1.70	2.47	2.03	4.21	4.67	10.67	2.90	3.78	4.93
300	2.70	3.16	3.00	6.06	5.91	16.16	4.85	4.99	7.33
500	3.15	3.49	3.59	7.09	6.51	19.86	5.98	5.55	8.45
1000	3.89	3.93	4.59	8.80	7.46	27.23	7.83	6.32	10.20
1500	4.53	4.19	5.33	9.95	7.86	30.58	10.89	6.78	11.36
2000	–	–	–	11.26	8.21	36.93	12.58	7.10	12.27
	Plane strain state $B \times L = 2.0 \times 0.5$ m Air dry sand			Axi-symmetrical state $D = 0.345$ m Air dry sand			Axi-symmetrical state $D = 0.345$ m Wet sand		

Concluding the considerations regarding the experimental and theoretical results of settlement of foundations under static and cyclic loads presented in the paper the following can be stated:

- experimental and calculated results have similar qualitative and quantitative characters,
- an analysis of settlements plays an important role in stability analysis,
- a method proposed by Sawicki & Świdziński can be useful tool in prediction of foundation settlements caused by cyclic loading.

All model tests presented were performed on air dry or wet sand, which limits the validity of the results to granular soils only.

4. Summary

The main conclusions resulting from the model tests performed on foundations rested on sand and subjected to cyclic loading and theoretical prediction are as follows:

- stability of shallow foundations under static loading can be evaluated by empirical formulae (2) and (3) proposed by Zadroga (1994) for ultimate bearing capacity. The settlement of foundation can be calculated by classical formulae,
- stability of shallow foundations under cyclic loads depends mainly on settlement, which means that a proper and credible settlement calculation plays the most important role,

– settlement of foundation under cyclic load is influenced mainly by the number of loading cycles, the ratio od cyclic to failure static loads and shape of foundation,
– the prediction of settlements in terms of an approach proposed by Sawicki et al. (1996) conforms well with experimental observations and may be recommended for engineers to calculate the settlements of shallow foundations subjected to cyclic loading.

It must be clearly stated that the validity of calculation methods presented in this paper is restricted to non-cohesive soils and shallow foundations only. Further theoretical analyses and model tests of shallow foundations' stability will require the additional considerations of the following factors: pore pressure in a fully saturated subsoil in undrained conditions, the frequency of the cyclic load applied, the eccentricity and the inclination of the external cyclic load and scale effect.

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