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Energy Dissipation in a Saturated Porous Half-Space Under a Vibrating Rigid Block

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Abstract

The paper deals with the problem of time harmonic vibrations of a rigid block on a water-saturated sand subsoil. The main interest is focused on the phenomenon of mechanical energy dissipation in the two-phase medium due to the flow of the viscid fluid through the pores of the solid. The behaviour of the saturated poro-elastic medium is studied on the basis of Biot's dynamic theory of consolidation. The problem is solved in a discrete way by using the finite element method. In order to confine the analysis to a finite domain, artificial boundaries are introduced, on which approximate absorbing conditions for Biot's medium are applied. Numerical results, obtained for three types of the block vibrations, illustrate the distribution of energy dissipation density in the subsoil. The influence of the frequency of oscillations and the sand permeability on the energy dissipation is investigated. Some results related to the problem of wave energy transmission are also presented.

1. Introduction

The problem of vibrations of a rigid block on a half-space of a fluid-filled porous medium may be regarded as typical in hydro-engineering. This theoretical model is commonly used when analysing dynamic behaviour of such structures as: breakwaters, foundations under drilling platforms and oil storage tanks, earth dams, and other large-scale off-shore objects. In the problems of structure vibrations, apart from the necessity for determination of displacement and stress fields in the subsoil under a structure, in a number of cases also the evaluation of damping of vibrations can be of importance. Particularly in the cases, when the phenomenon of resonance may occur – either due to the dynamic interaction of a structure considered with the underlying subsoil itself, or – due to transmission of waves in the subsoil – with other objects founded in the neighbourhood of a vibrating structure. The phenomenon of damping in soil is connected with two mechanisms. The first consists in transmission of energy from a source of disturbances into the far field by progressive waves, and takes place in any open-type dynamic problem. The second mechanism is caused by losses (dissipation) of mechanical energy in a fluid-saturated soil due to a number of micro-scale phenomena occurring in the medium. From among these phenomena, the following two are the most significant: irreversible (inelastic) changes in the internal structure (granular rearrangements) of the soil skeleton, mainly due to its shearing, and viscous damping, caused by the flow of viscid fluid through the pores of the medium. The irreversible structural changes in non-cohesive soils lead to its compaction (pore volume decrease), and in the case of a water-filled sand may give rise to the phenomenon of pore pressure generation and, in consequence, to its liquefaction. The theoretical description of the latter phenomena is a complex task (Morland 1993, Morland and Sawicki 1985) and, as yet, the applications of already developed theories are, as a rule, confined to one-dimensional wave problems, for instance transverse wave propagation (Sawicki and Morland 1985) or longitudinal wave propagation (Morland and Staroszczyk 1995, 1996). The solution of two-dimensional dynamic problems is possible only after some simplifying assumptions have been introduced – an example is the paper by Sawicki and Staroszczyk (1995), dealing with the problem of sand liquefaction due to the propagation of Rayleigh surface waves. In order to simplify the analysis of the problems, in which irreversible changes occur, some attempts to apply soil plasticity theories (Zienkiewicz 1982) were undertaken, but they led to ill-posed boundary value problems, particularly in the case of wave propagation.

In the light of the above it seems that the description of the mechanism of viscous damping due to relative motion of the two constituents of the soil is a simpler task. This phenomenon can be analysed on the basis of already well established theories, such as the theory of mixtures (Bowen 1982), or the dynamic theory of consolidation (Biot 1956). The latter theory, despite a number of significant simplifications, such as that of a constant porosity of the two-phase medium (which makes impossible the analysis of the soil densification phenomena), enables the solving of a number of problems of both engineering and theoretical importance. In Biot's model the mechanism of damping is connected with the phenomenon of friction between the pore fluid, treated as a viscid liquid, and the soil skeleton in their relative motion. With such a description of damping, the amount of dissipated mechanical energy depends on a certain damping parameter, being a function of the soil permeability and an angular frequency of vibrations, and the relative velocity of both constituents of the medium (i.e. the loss of mechanical energy does not occur when the pore fluid and skeleton move in unison).

In the present paper, Biot's model has been applied to analyse the plane problem of harmonic in time vibrations of a rigid block on a liquid-saturated sand half-space. Since the problem leads to a mixed boundary-value problem, which cannot be solved analytically, it is necessary to resort to one of discrete methods. In this case, however, we have to confine considerations to a finite domain, of only a limited number of discrete points. In the open-type problems, in which wave propagation phenomena occur, such an approach gives rise to additional difficulties, related to the necessity of ensuring undisturbed transmission of energy from the excited zone to infinity, without introducing disturbances due to reflection of waves at imaginary boundaries enclosing the region of interest. To this aim special conditions at the artificial boundaries have to be applied. In the case of steady-state variations it proved effective to employ the idea of Lysmer and Kuhlemeyer (1969), who developed approximate absorbing conditions for purely elastic solids. Extending this idea, the author formulated analogous boundary conditions for Biot's two-phase medium (Staroszczyk 1992). Their usefulness in discrete problems was confirmed by solving Lamb's steady-state problem and comparing the results obtained with those evaluated analytically (Staroszczyk 1993, 1992b). The present work is a continuation of the above papers, but now the interest is focused on the phenomena related to the loss (dissipation) of energy accompanying wave propagation in a saturated subsoil due to harmonic motions of a block. On the basis of a discrete solution to the problem, constructed by the use of the finite element method, a numerical analysis has been carried out. For three types of the block vibrations (vertical, horizontal and rocking) the subsoil domains, in which the greatest losses of mechanical energy occur, have been determined. Next, the dependence of the energy dissipation on the frequency of oscillations and the soil permeability has been investigated. In addition, the fluxes of wave energy transmitted from the vibrating block in two directions: horizontal (along the surface of the subsoil) and vertical (deep into the half-space) have been calculated. These quantities, in particular the amount of energy transmitted horizontally and being associated with the propagation of the Rayleigh wave, provide some information useful in evaluating the influence of a vibrating structure on other object in its neighbourhood.

2. Formulation of the Problem

The plane problem of motion of a porous elastic medium saturated by a fluid, occupying the half space $z \geq 0$ (Fig. 1), is considered. The motion of the medium is excited by time-harmonic motions of a rigid and impermeable block. Apart from the region under the block base ($-a \leq x \leq a$), the free surface of the half--space is assumed to be free of stresses. Our objective is to determine dissipation of mechanical energy in the saturated subsoil that accompanies propagation of waves in the half-space.

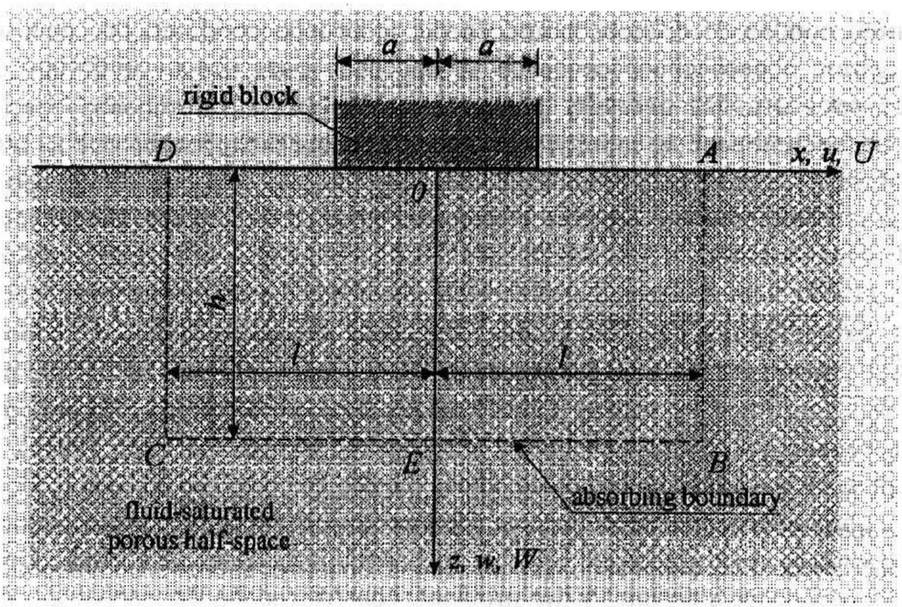

Fig. 1. Rigid block on a water-saturated porous half-space

In the plane Cartesian co-ordinate system Oxz Biot's (1956) equations of motion of the two-phase medium may be written as follows:

$$\begin{aligned}
\nabla^2(P\mathrm{div}\mathbf{u} + Q\mathrm{div}\mathbf{U}) &= \frac{\partial^2}{\partial t^2}(\rho_{11}\mathrm{div}\mathbf{u} + \rho_{12}\mathrm{div}\mathbf{U}) + bF(\kappa)\frac{\partial}{\partial t}\mathrm{div}(\mathbf{u} - \mathbf{U}),\\
\nabla^2(Q\mathrm{div}\mathbf{u} + R\mathrm{div}\mathbf{U}) &= \frac{\partial^2}{\partial t^2}(\rho_{12}\mathrm{div}\mathbf{u} + \rho_{22}\mathrm{div}\mathbf{U}) - bF(\kappa)\frac{\partial}{\partial t}\mathrm{div}(\mathbf{u} - \mathbf{U}),\\
N\nabla^2\mathrm{rot}\mathbf{u} &= \frac{\partial^2}{\partial t^2}(\rho_{11}\mathrm{rot}\mathbf{u} + \rho_{12}\mathrm{rot}\mathbf{U}) + bF(\kappa)\frac{\partial}{\partial t}\mathrm{rot}(\mathbf{u} - \mathbf{u}),\\
0 &= \frac{\partial^2}{\partial t^2}(\rho_{12}\mathrm{rot}\mathbf{u} + \rho_{22}\mathrm{rot}\mathbf{U}) - bF(\kappa)\frac{\partial}{\partial t}\mathrm{rot}(\mathbf{u} - \mathbf{U}),
\end{aligned} \tag{1}$$

where $P = 2N + C, t$ denotes time and ∇^2 is the Laplace differential operator. In the equations $\mathbf{u} = [u, 0, w]^T$ and $\mathbf{U} = [U, 0, W]^T$ denote the displacement vectors of the skeleton and the pore fluid, respectively. N, C, Q and R are material constants of the Biot's medium. The constitutive relations have the form

$$\begin{aligned}
\sigma_{ij} &= 2Ne_{ij} + \delta_{ij}(C\mathrm{div}\mathbf{u} + Q\mathrm{div}\mathbf{U}),\\
s &= Q\mathrm{div}\mathbf{u} + R\mathrm{div}\mathbf{U},
\end{aligned} \tag{2}$$

with components of the strain tensor in the soil skeleton defined by

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}). \tag{3}$$

In equations (2) σ_{ij} denotes the partial stress tensor in the skeleton, s – the partial stress in the pore liquid (related to the intrinsic pore fluid pressure p and the volume porosity n by $s = -np$), δ_{ij} is the Kronecker symbol, and i and j take the values 1, 2, 3.

Four material constants of Biot's model N, C, Q and R are functions of intrinsic elastic moduli of the skeleton, pore water and the medium porosity (Biot and Willis 1957). Under the assumption, widely applied in soil mechanics, that the skeleton grains are incompressible, Biot's moduli can be related to the constants commonly used in the theory of elasticity as folows:

$$\begin{aligned} N &= G \qquad C = E_w \frac{(1-n)^2}{n} + \frac{2G\ni}{1-2\ni}, \\ Q &= E_w(1-n), \quad R = E_w n, \end{aligned} \tag{4}$$

where G is the soil skeleton shear modulus, $\ni$ is the Poisson ratio, and E_w denotes the bulk compressibility modulus of water.

The parameters ρ_{11}, ρ_{12} and ρ_{22}, appearing in the motion equations (1), denote partial densities, which are related to the intrinsic densities ρ_s and ρ_w of the solid skeleton and pore fluid, respectively, by means of the formulae

$$\begin{aligned} \rho_{11} &= (1-n)\rho_s + \rho_a, \\ \rho_{12} &= -\rho_a, \\ \rho_{22} &= n\rho_w + \rho_a. \end{aligned} \tag{5}$$

In equation (5) so-called apparent mass coefficient ρ_a appears. This parameter was introduced by Biot to express the effect of internal coupling of the motion of both components of the saturated solid. Applying the interpretation given by Kowalski (1983), this coefficient may be defined by

$$\rho_a = n\rho_w \frac{\beta}{1-\beta}, \tag{6}$$

with β being the ratio of the volume of fluid imprisoned in the skeleton (i.e. moving together with it) to the total volume of the pore fluid. As the results of papers by Kubik and Kaczmarek (1988) and Staroszczyk (1992c) showed, the influence of the internal coupling on the wave propagation is negligibly small within the range of frequencies encountered in hydroengineering. Thus, in order to simplify the analysis, we assume $\beta = 0 (\rho_a = 0)$.

The term accounting for the energy dissipation in the saturated porous medium is written in equation (1) as a product of the damping coefficient b and the function $F(\kappa)$. The parameter b can be expressed in terms of the commonly used filtration coefficient k_f [m/s] as follows

$$b = \frac{n^2 \rho_w g}{k_f}, \tag{7}$$

where g denotes Earth's gravitational acceleration. The function $F(\kappa)$ describes the dependence of internal friction between the skeleton and the pore fluid on the angular frequency of vibrations ω [rad/s], and may be written in the following way (Biot 1956):

$$F(\kappa) = \frac{1}{4} \frac{\kappa^2 T(\kappa)}{\kappa + 2i T(\kappa)}, \tag{8}$$

with

$$T(\kappa) = \frac{-i^{3/2} J_1(i^{3/2}\kappa)}{J_0(i^{3/2}\kappa)}, \tag{9}$$

where J_0 and J_1 are the Bessel functions of the first kind, and i denotes the imaginary unit. The dimensionless frequency parameter κ, following Biot (1956), is defined as

$$\kappa = \delta(\omega k_f / ng)^{1/2}, \tag{10}$$

where δ is a certain dimensionless parameter, depending on the internal geometry of the pores.

The equation of motion (1) must be supplemented by the conditions at the boundaries enclosing the finite region. At the free surface $z = 0$ the boundary conditions are expressed in terms of prescribed displacements under the block base and zero stresses beyond the block. These mixed conditions are written separately for three types of block oscillations:

(a) vertical vibrations with the amplitude w_0:

$$\begin{aligned}
&w(x,0,t) = w_0 \exp(i\omega t), && |x| \le a,\\
&W(x,0,t) = w_0 \exp(i\omega t), && |x| \le a,\\
&\sigma_{zz}(x,0,t) = 0, && |x| > a,\\
&\sigma_{xz}(x,0,t) = 0, && |x| > a,\\
&s(x,0,t) = 0, && |x| > a;
\end{aligned} \tag{11}$$

(b) horizontal vibrations with the amplitude u_0:

$$\begin{aligned}
&u(x,0,t) = u_0 \exp(i\omega t), && |x| \le a,\\
&\sigma_{zz}(x,0,t) = 0, && |x| > a,\\
&\sigma_{xz}(x,0,t) = 0, && |x| > a,\\
&s(x,0,t) = 0, && |x| > a;
\end{aligned} \tag{12}$$

(c) rocking vibrations with the angular amplitude α_0:

$$\begin{aligned}
&w(x,0,t) = \alpha_0 x \exp(i\omega t), && |x| \le a,\\
&W(x,0,t) = \alpha_0 x \exp(i\omega t), && |x| \le a,\\
&\sigma_{zz}(x,0,t) = 0, && |x| > a,\\
&\sigma_{xz}(x,0,t) = 0, && |x| > a,\\
&s(x,0,t) = 0, && |x| > a.
\end{aligned} \tag{13}$$

Note that for the case of horizontal vibrations we do not require the coupling between the displacements of the block base and the pore fluid, assuming that there is no relative friction (the fluid is treated as a non-viscid one when considering its interaction with the block).

In order to construct a numerical model of the wave propagation problem in a semi-infinite domain, artificial boundaries (rectangle ABCD in Fig. 1), enclosing all irregular features of the system, are introduced. At these boundaries special radiation conditions are to be applied to ensure undisturbed flow of wave energy to infinity. Unfortunately, the construction of perfect transmission conditions is possible only for a few plane problems. e.g. for the Love shear wave propagation problem (Lysmer and Waas 1972). In other cases only approximate radiation conditions can be derived. In the present paper we adopt approximate viscous boundary conditions, which were firstly formulated by Lysmer and Kuhlemeyer (1969) for a purely elastic medium. The approach of these authors has been extended to the fluid-saturated poro-elastic media by Staroszczyk (1992a). The idea of the viscous boundary conditions consists in supporting the finite region on a system of infinitesimal viscous dashpots, whose aim is to absorb incoming wave energy in such a way that the artificial reflection of waves is minimised. Since we deal with the two-phase medium – two separate sets of dashpots have to be established at the viscous boundary. The parameters of dashpots depend on the material properties of the medium, frequency of oscillations and the type of waves propagating in the subsoil and their effectiveness increases with increasing distance from the source of disturbances. These parameters can be estimated by solving the problem of reflection of body waves at a plane viscous boundary at various angles of incidence, and carrying out averaging procedure in order to find their optimal values (for details cf. Staroszczyk 1992a). Only in the case of Rayleigh waves arriving at a vertical boundary is it possible to completely absorb the energy of incoming waves. As the results presented by Staroszczyk (1993) showed, such an approach usually leads to errors not exceeding 5%, and thus is fully applicable for purposes of engineering practice. In accordance with the results of Lysmer and Kuhlemeyer (1969) and Staroszczyk (1992a) different absorbing conditions are assumed at the horizontal and vertical region under consideration. Namely, on the horizontal plane $z = h$ we adopt the conditions which ensure ideal absorption of one distorsional and two dilatational (fast and slow) waves, propagating in unbounded saturated porous solid. In turn, at the vertical boundaries $|x| = l$ the conditions construced for absorbing Rayleigh-type waves are assumed. The afore-said conditions are written in the following form:

(a) at the horizontal boundary $z = h$, $|x| \le l$

$$\begin{aligned} \sigma_{zz}(x,h,t) &= -\lambda_1(\omega)\dot{w}(x,h,t), \\ s(x,h,t) &= -\lambda_2(\omega)\dot{W}(x,h,t), \\ \sigma_{xz}(x,h,t) &= -\lambda_3(\omega)\dot{u}(x,h,t); \end{aligned} \tag{14}$$

(b) at the vertical boundaries $|x| = l, z \le h$

$$\begin{aligned} \sigma_{xx}(\pm l,z,t) &= -\lambda_1^R(\omega)\dot{u}(\pm l,z,t), \\ s(\pm l,z,t) &= -\lambda_2^R(\omega)\dot{U}(\pm l,z,t), \\ \sigma_{xz}(\pm l,z,t) &= -\lambda_3^R(\omega)\dot{w}(\pm l,z,t); \end{aligned} \tag{15}$$

where the dot denotes differentiation with respect to time. In equations (14) the quantities λ_1, λ_2 and λ_3 are the parameters of viscous dashpots that perfectly absorb plane body waves arriving normally to the artificial viscous boundary, while the parameters λ_1^R, λ_2^R and λ_3^R, appearing in equations (15), define viscous dashpos which absorb the Rayleigh surface waves, propagating in the half-space.

3. Energy Dissipation and Transmission

3.1. Instantaneous Energy Dissipation

In Biot's dynamic theory of consolidation it is assumed that the flow of the viscid pore fluid relative to the porous skeleton is of the Poiseuille type i.e. we deal with small Reynolds numbers. Under such an assumption, the microscopic flow pattern is uniquely determined by the velocity fields of both constituents of the medium: **v** – the soil skeleton, and **V** – the pore liquid. The relative motion of both soil components with the velocity $\mathbf{v} - \mathbf{V}$ gives rise to friction forces, whose work results in the loss of mechanical energy in the medium. The components of the friction forces, acting on the two constituents in a unit volume of the medium, are defined by means of so-called dissipation function D, defined by Biot (1956) as follows:

$$q_i^s = \frac{\partial D}{\partial v_i}, \quad q_i^w = \frac{\partial D}{\partial V_i}, \quad i = 1, 2, 3, \tag{16}$$

where q_i^s and q_i^w denote the components of friction forces exerted on the soil skeleton and the pore fluid, respectively. In the Cartesian co-ordinate system the dissipation function is written as a homogeneous quadratic form of the skeleton and fluid velocities:

$$D = \frac{\bar{b}}{2}(\mathbf{v} - \mathbf{V})(\mathbf{v} - \mathbf{V}), \tag{17}$$

where $\bar{b} = bF(\kappa)$. Calculation of the mechanical power of the friction forces (16) yields the relation describing the power dissipation in a unit volume of the soil in terms of the velocity components as follows:

$$E_D = \bar{b}[(\dot{u} - \dot{U})^2 + (\dot{w} - \dot{W})^2]. \tag{18}$$

3.2. Instantaneous Energy Transmission

The flux of mechanical power per unit area of a single-phase medium is defined by the following scalar product (Achenbach 1973):

$$E_T = \mathbf{f}\mathbf{v}, \tag{19}$$

where **f** is the surface traction vector, **v** is the particle velocity vector, and real parts of respective quantities have to be used in calculations. In the case of the two-phase medium we deal with separate stress and velocity fields related to the skeleton and the pore liquid, so the total flux of power across a unit area is equal to the sum of powers transmitted by the both phases of the medium, which is written by

$$E_T = \mathbf{f}^s\ \dot{\mathbf{u}} + \mathbf{f}^w\ \dot{\mathbf{U}}, \tag{20}$$

where $\mathbf{f}^s$ denotes the stress acting in the soil skeleton, and $\mathbf{f}^w$ – that in the pore fluid. Taking advantage of the Cauchy stress formula, the components of the vectors $\mathbf{f}^s$ and $\mathbf{f}^w$ can be expressed by means of

$$\begin{aligned} f^s_m &= \sigma_{km} n_k, \\ f^s_m &= s n_m, \qquad k, m = 1, 2, 3, \end{aligned} \tag{21}$$

where n_k and n_m denote the components of a unit vector **n**, normal to the surface element. By substituting (21) to (20) we obtain the expression

$$E_T = (\sigma_{ij}\dot{u}_i + s\dot{U}_j)n_j. \tag{22}$$

In the present analysis we confine attention to calculating the power fluxes across vertical and horizontal boundaries, for which formula (22) yields the relations:

(a) for a vertical boundary – transmission of energy in the horizontal direction, along the ground free surface

$$E_{TH} = \sigma_{xx}\dot{u} + \sigma_{xz}\dot{w} + s\dot{U}, \tag{23}$$

(b) for a horizontal boundary – transmission of energy in the vertical direction

$$E_{TV} = \sigma_{zz}\dot{w} + \sigma_{xz}\dot{u} + s\dot{W}. \tag{24}$$

The total power of external forces, which is transmitted from the vibrating block across the contact region ($z = 0, -a \le x \le a$) into the half-space, is defined by the relation

$$E_0 = (F_V w_0 + F_H u_0 + M\alpha_0)i\omega\exp(i\omega t), \tag{25}$$

where F_V, F_H and M denote the forces exerted by the block on the half-space: F_V and F_H are vertical and horizontal reactions, respectively, and M is the turning moment.

3.3. Time-averaged Energy Dissipation and Transmission

All parameters related to the power dissipation and transmission, described by equations (18) and (23) – (25), are instantaneous quantities, changing cyclically with the angular frequency ω. In order to compare the amount of energy either dissipated in the region or transmitted through its boundaries, with the energy put into the system, it is convenient to deal with quantities averaged over one period of oscillations T, i.e. to calculate the mean values of respective integrals:

$$\langle E \rangle = \frac{1}{T} \int_{t}^{t+T} E dt, \tag{26}$$

where E is one of the parameters from among E_D, E_{TH}, E_{TV} and E_0, defined by (18), (23), (24) and (25), respectively, and $\langle . \rangle$ denotes the quantity averaged over the period T. It can be seen that all quantities related to the mechanical power have the form of sums of products of real parts of two complex-valued parameters, each of them varying cyclically with the frequency ω (i.e. they are multiplied by the same factor $e^{i\omega t}$). Denoting the first and the second factor of these products by X and Y, where

$$X = X_0 \exp(i\omega t), \qquad Y = Y_0 \exp(i\omega t), \tag{27}$$

with X_0 and Y_0 being complex amplitudes of respective parameters, one can easily derive the following formula, useful when calculating time-averaged power fluxes:

$$\langle E \rangle = \langle \mathrm{Re}(X) \cdot \mathrm{Re}(Y) \rangle = \frac{1}{2}\mathrm{Re}(XY^*) = \frac{1}{2}\mathrm{Re}(X_0 Y_0^*), \tag{28}$$

where Re(.) denotes the real part of a complex number, and $(.)^*$ stands for the complex conjugate. By making use of (28), we can express the time-averaged quantities, defined by equations (18) and (23) – (25), as follows:

– the mean power dissipation density

$$\langle E_D \rangle = \frac{1}{2}\bar{b}\left(|\dot{u} - \dot{U}|^2 + |\dot{w} - \dot{W}|^2\right), \tag{29}$$

– the mean power transmission density in the horizontal direction

$$\langle E_{TH} \rangle = \frac{1}{2}\mathrm{Re}(\sigma_{xx}\dot{u}^* + \sigma_{xz}\dot{w}^* + s\dot{U}^*), \tag{30}$$

– the mean power transmission density in the vertical direction

$$\langle E_{TV} \rangle = \frac{1}{2}\mathrm{Re}(\sigma_{zz}\dot{w}^* + \sigma_{xz}\dot{u}^* + s\dot{W}^*), \tag{31}$$

– the mean total exciting power (due to external forces) put into the half-space

$$\langle E_0 \rangle = \frac{1}{2}\omega \mathrm{Im}(F_V w_0 + F_H u_0 + M\alpha_0), \tag{32}$$

with Im (.) denoting the imaginary part of a complex number.

Having determined the time-averaged power densities (29) – (31), we can now evaluate the total power dissipated inside the region of interest as well as that transmitted across its boundaries. To this aim we calculate surface (for power dissipation) or line (for power transmission) integrals of respective functions. By comparing quantities so obtained with the mean total power (32) put into the subsoil, we can define some dimensionless parameters, which describe concisely the features of the dynamic system considered:

– energy dissipation ratio in a finite domain Ω

$$C_D = \frac{\langle \int_\Omega E_D d\Omega \rangle}{\langle E_0 \rangle}, \tag{33}$$

– energy transmission ratio in the horizontal direction (i.e. across the boundaries AB and CD in Fig. 1)

$$C_{TH} = \frac{\langle \int_S E_{TH} dS \rangle}{\langle E_0 \rangle}, \tag{34}$$

– energy transmission ratio in the vertical direction (i.e. across the boundary BC in Fig. 1)

$$C_{TV} = \frac{\langle \int_S E_{TV} dS \rangle}{\langle E_0 \rangle}. \tag{35}$$

In an idealised case of a steady-state problem (in which both kinetic and elastic energies averaged over a period of oscillations T do not change) the following relation, expressing the energy conservation principle, should hold for any rectangular domain $|x| > a, z > 0$:

$$C_D + C_{TH} + C_{TV} = 1. \tag{36}$$

However, due to the imperfect character of adopted absorbing conditions, admitting some, though very limited, reflection of waves at the artificial boundaries, some discrepancies from this idealised situation may occur in the numerical model of the problem. As the results of calculations have shown, this "gap" in energy balance does not usually exceed 2% of the total energy put into the vibrating system, and can be reduced to some extent by increasing the dimensions of the finite domain taken for numerical computations.

4. Discrete Solution to the Problem

The mixed boundary-value problem, defined by the equations of motion (1) and the boundary conditions (11) to (15), is solved approximately by applying the finite element method. The rectangular continuous domain $ABCD$ (see Fig. 1) is replaced by a discrete system consisting of rectangular finite elements; the elements of four nodes, with four degrees of freedom per node (Fig. 2), are used.

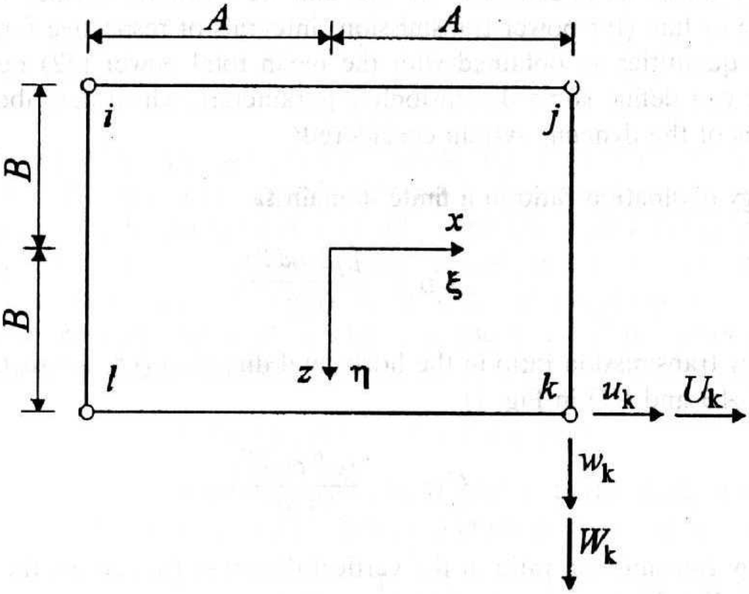

Fig. 2. Rectangular finite element

The discretization process transforms the problem to the solution of the set of linear equations in the form:

$$(-\omega^2\mathbf{M} + i\omega\mathbf{C} + \mathbf{K})\mathbf{q} = \mathbf{F}, \tag{37}$$

where **M**, **C** and **K** are, respectively, the system mass, damping and stiffness matrices, **q** is the vector of nodal displacement amplidutes and **F** is the vector of nodal external forces. The system matrices and vectors can be assembled from the respective element matrices and vectors in a standard typical for the FEM, manner. The element matrices can be derived by applying either the principle of virtual work or the Galerkin (residual weighted) method (cf. Zienkiewicz and Taylor 1989). As regards Biot's medium, details can be found in the paper (Staroszczyk 1993); here, for reference, we quote in brief the most important results, relevant to the present analysis.

Assuming the linear variation of the displacements u, w, U and W within the finite elements, interpolation (shape) functions can be expressed as follows:

$$N_r = \frac{1}{4}(1+\xi_r\xi)(1+\eta_r\eta), \qquad r = i, j, k, l, \tag{38}$$

where, for convenience, normalised co-ordinates ξ and η are introduced

$$\xi = x/A, \qquad \eta = z/B, \tag{39}$$

and $(\xi_r, \eta_r) = (\pm 1, \pm 1)$ are the co-ordinates of the element nodes in the local co-ordinate system $0\xi\eta$. The stiffness, damping and mass matrices of the finite element considered are 16 by 16 matrices, and each of them is made up of sixteen 4 by 4 submatrices. The element stiffness matrix $\mathbf{k}^e$ has the form:

$$\mathbf{k}^e = \begin{bmatrix} \mathbf{k}_{ii} & \mathbf{k}_{ij} & \mathbf{k}_{ik} & \mathbf{k}_{il} \\ \mathbf{k}_{ji} & \mathbf{k}_{jj} & \mathbf{k}_{jk} & \mathbf{k}_{jl} \\ \mathbf{k}_{ki} & \mathbf{k}_{kj} & \mathbf{k}_{kk} & \mathbf{k}_{kl} \\ \mathbf{k}_{li} & \mathbf{k}_{lj} & \mathbf{k}_{lk} & \mathbf{k}_{ll} \end{bmatrix}, \tag{40}$$

where each of the component submatrices is defined by

$$\mathbf{k}_{rs} = \begin{bmatrix} PI_{11}^{rs} + GI_{22}^{rs} & CI_{12}^{rs} + GI_{21}^{rs} & QI_{11}^{rs} & QI_{12}^{rs} \\ CI_{21}^{rs} + GI_{12}^{rs} & PI_{22}^{rs} + GI_{11}^{rs} & QI_{21}^{rs} & QI_{22}^{rs} \\ QI_{11}^{rs} & QI_{12}^{rs} & RI_{11}^{rs} & RI_{12}^{rs} \\ QI_{21}^{rs} & QI_{22}^{rs} & RI_{21}^{rs} & RI_{22}^{rs} \end{bmatrix}, \tag{41}$$

with

$$\begin{aligned} I_{11}^{rs} &= \frac{\xi_r\xi_s}{4}\frac{B}{A}\left(1+\frac{\eta_r\eta_s}{3}\right), \quad I_{12}^{rs} = \frac{\xi_r\eta_s}{4}, \\ I_{21}^{rs} &= \frac{\eta_r\xi_s}{4}, \quad I_{22}^{rs} = \frac{\eta_r\eta_s}{4}\frac{A}{B}\left(1+\frac{\xi_r\xi_s}{3}\right), \quad r, s = i, j, k, l. \end{aligned} \tag{42}$$

The element damping matrix $\mathbf{c}^e$ consists of sixteen submatrices given by

$$\mathbf{c}_{rs} = \bar{b} I_0^{rs}\mathbf{H}, \quad r, s = i, j, k, l, \tag{43}$$

where

$$I_0^{rs} = \frac{AB}{4}\left(1+\frac{\xi_r\xi_s}{3}\right)\left(1+\frac{\eta_r\eta_s}{3}\right) \tag{44}$$

and

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}. \tag{45}$$

And finally, the element mass matrix $\mathbf{m}^e$ is composed of 4 by 4 submatrices of the form

$$\mathbf{m}_{rs} = I_0^{rs}\mathbf{r}, \quad r,s = i,j,k,l, \tag{46}$$

where $\mathbf{r}$ is the matrix of partial densities

$$\mathbf{r} = \begin{bmatrix} \rho_{11} & 0 & \rho_{12} & 0 \\ 0 & \rho_{11} & 0 & \rho_{12} \\ \rho_{12} & 0 & \rho_{22} & 0 \\ 0 & \rho_{12} & 0 & \rho_{22} \end{bmatrix}. \tag{47}$$

5. Numerical Examples

In order to decrease a number of discrete points of the model we take advantage of the feature of symmetry or asymmetry of the problem (depending on the type of block vibrations) with respect to z-axis, which allows to confine considerations to the region $OABE$ in Fig. 1. The basic model used in numerical computations consisted of 900 rectangular elements (30 elements in both horizontal and vertical directions) and had 3844 degrees of freedom. For each frequency of vibrations the absorbing boundaries were assumed to be at a distance of one Rayleigh wave length from the origin of the co-ordinate system, i.e. $h = l = L$, the maximum finite element side not exceeding 1/10 of the surface wave length L.

The values of material parameters used in numerical analysis are listed in Table 1. These data pertain to a water-saturated coarse sand and correspond to the following Biot's model parameters: $N = 3.75 \times 10^8$ Pa, $C = 2.82 \times 10^9$ Pa, $Q = 1.38 \times 10^9$ Pa, $R = 9.2 \times 10^8$ Pa, $\rho_{11} = 1.59 \times 10^3$ kg/m^3, $\rho_{12} = 0$, $\rho_{22} = 4 \times 10^2$ kg/m^3, $b = 1.57 \times 10^5$ Ns/m^4, and $\delta^2 = 8$ in equation (10).

Table 1. Values of physical parameters used in numerical calculations

parameter	notation	magnitude
skeleton density	ρ_s	2.65×10^3 kg/m^3
water density	ρ_w	10^3 kg/m^3
porosity	n	0.4
skeleton shear modulus	G	3.75×10^8 Pa
water bulk compressibility modulus	E_w	2.3×10^9 Pa
Poisson's ratio	$\ni$	0.333
filtration coefficient	k_f	10^{-2} m/s

In the computations the block base half-width is $a = 10$ m, and a dimensionless frequency parameter $\widetilde{a}$ has been used:

$$\widetilde{a} = ka, \tag{48}$$

where $k = \omega / V_r$ is the wave number of a distorsional wave, propagating with the phase velocity V_r (for the data used in calculations $V_r = 434.1$ m/s).

As the first example, in Figs 3 to 5 we present the distribution of the power dissipation densities in the subsoil in the vicinity of the vibrating block. For three kinds of vibrations (vertical, horizontal and rocking ones) at the dimensionless frequency $ka = 1.0$ ($\omega = 43.41$ rad/s) the contour plots of the time-averaged power dissipation densities within the rectangular domain $0 \leq x \leq 3a$, $0 \leq z \leq 2a$ are shown. For illustrative purposes, the quantities discussed are presented in the normalised form:

$$\widetilde{E}_D = \frac{\langle E_D \rangle}{\langle E_0 \rangle} L^2, \tag{49}$$

where E_D is defined by equation (29). As can be seen from the figures, the qualitative pictures of the energy dissipation in the subsoil due to vertical (Fig. 3) and horizontal (Fig. 4) vibrations of the block exhibit considerable similarity. The greatest losses of mechanical energy occur just under the free surface of the half-space, in the neighbourhood of the block edge (for $x \approx 1.35a$). In this domain the relative velocities between the soil skeleton and the pore water reach the greatest magnitudes. Directly under the block base these relative velocities are much smaller, particularly for vertical oscillations – in this case the motion of a rigid and impermeable block compels the soil skeleton and the pore water to move together (cf. equation (11)). In the case of horizontal vibrations such an influence of the block on the underlying subsoil is smaller – only the soil skeleton is forced to move together with the block base at the contact zone, which results in a relatively greater energy dissipation directly under the structure for this type of oscillations. A different energy dissipation pattern can be observed in the case of rocking vibrations (Fig. 5). Now two regions of high energy dissipation occur. The first is situated similarly to the foregoing cases of vertical and horizontal oscillations, i.e. under the subsoil free surface near the block edge, but this time the power dissipation density is much higher than in the two previous cases. The second domain of high energy dissipation occurs directly under the block base, and is due to high horizontal velocities of the pore water in this region. The existence of two domains of high energy dissipation in the latter case is the reason why of the three kinds of block vibrations the highest energy loss in the subsoil (expressed in terms of the dimensionless coefficient c_D) occurs in the case of rocking vibrations.

The results displayed in Figs 3 to 5 concern the region $0 \leq x \leq 3a, 0 \leq z \leq 2a$. It may be of interest to investigate how the amount of energy dissipated in a region and the relations between the energy transmitted across its boundaries depend on the domain dimensions. The results of such an analysis, carried out for three kinds of the block oscillations at frequency $ka = 1.0$, are shown in Fig. 6. In the analysis it has been assumed that the region considered has the shape of a square (i.e. $h = l$ – see Fig. 1), and for various ratios of the square side h to the Rayleigh wave length L dimensionless energy dissipation and transmission ratios have been evaluated. As follows from the results plotted, the highest energy loss accompanies the rocking vibrations of the block, and the least – the horizontal ones. Depending on the type of vibrations, the relation between the amounts of energy transmitted in both directions changes significantly. In the case of vertical and rocking oscillations over 60% of the total energy, introduced by a vibrating block, is radiated away from the region $L \times L$ in the horizontal direction by the surface waves, while the respective quantities concerning the vertical transmission do not exceed 20% for the vertical oscillations and 6% for the rocking ones. In the case of the block horizontal motions we deal with quite a different situation, in which the majority of energy (almost 60%) is radiated in the vertical direction – deep into the half--space, and only about 20% is transmitted by the waves propagating along the free surface of the subsoil. This means that in the case of vertical motions of the block the influence of the object on its neighbourhood is much smaller than in the cases of the two remaining kinds of block oscillations.

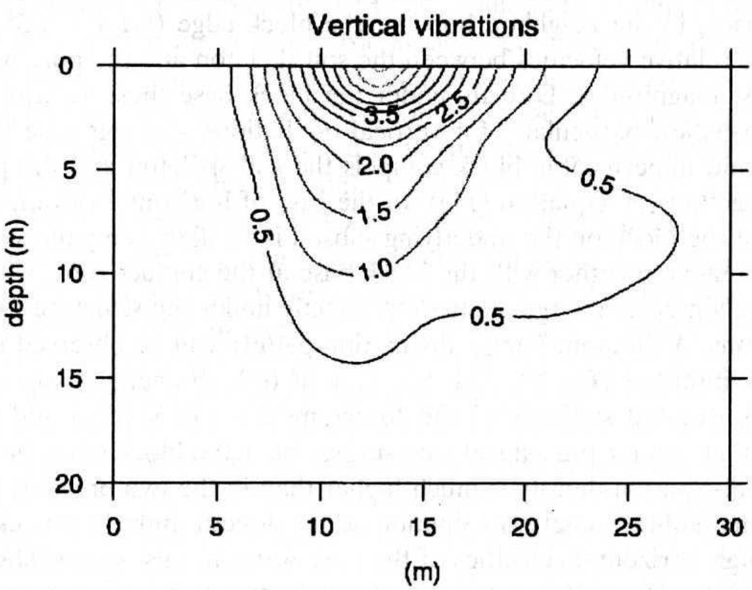

Fig. 3. Power dissipation density distribution in the subsoil – vertical vibrations of the block

In Fig. 7 the influence of the block vibrations frequency on the parameters describing the energy dissipation and transmission is illustrated. For a wide range of dimensionless frequencies $0.1 \le ka \le 2.0$ the values of the coefficients c_D, c_{TH} and c_{TV} for the square domain $L \times L(|x| \le L, |z| \le L, L$ is the Rayleigh wave length) have been evaluated. It follows from the plots that the greatest frequency sensitivity of the ratios considered c_D, c_{TH} and c_{TV} are exhibited in the case of rocking motions of the block, particularly for low frequencies. For this type of oscillations the energy dissipation decreases and the horizontal energy transmission

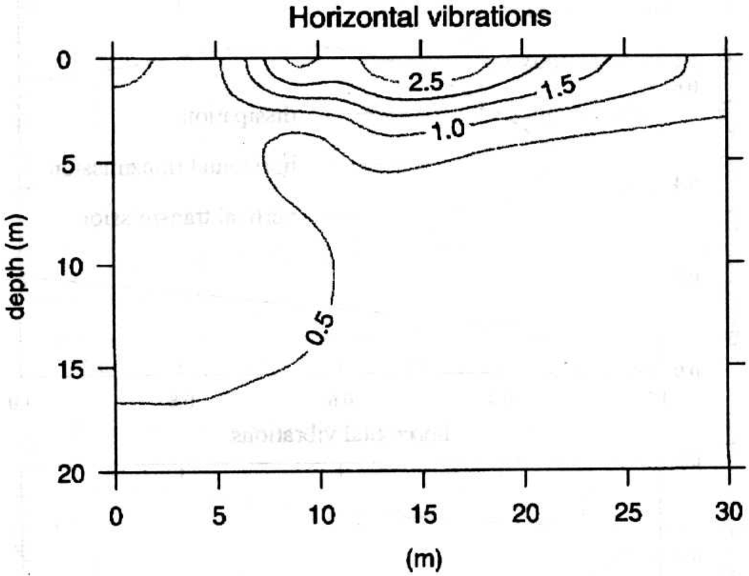

Fig. 4. Power dissipation density distribution in the subsoil – horizontal vibrations of the block

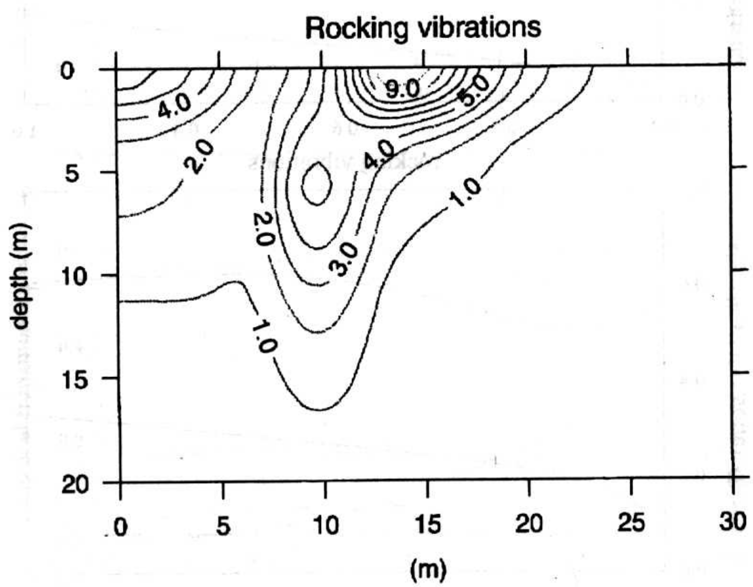

Fig. 5. Power dissipation density distribution in the subsoil – rocking vibrations of the block

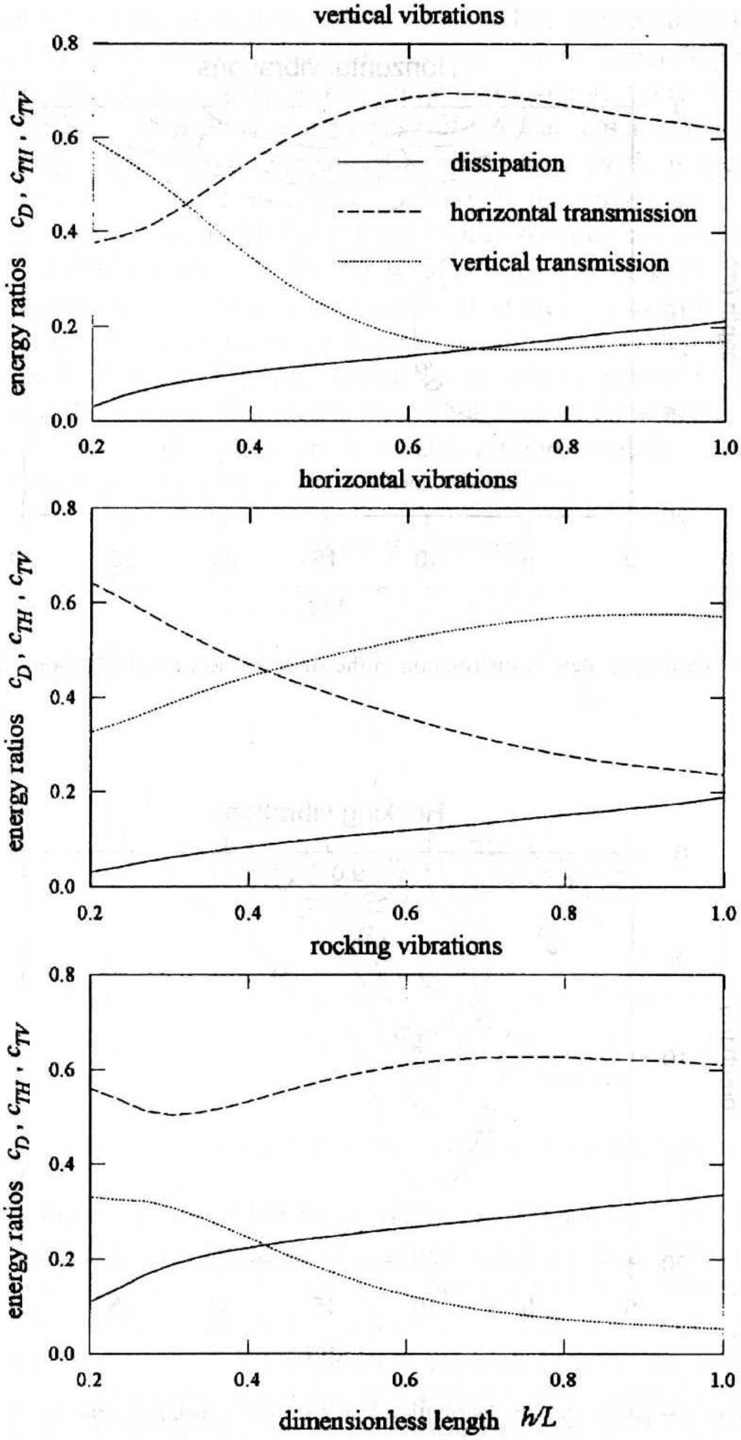

Fig. 6. Energy dissipation and transmission ratios as functions of a square domain dimensions

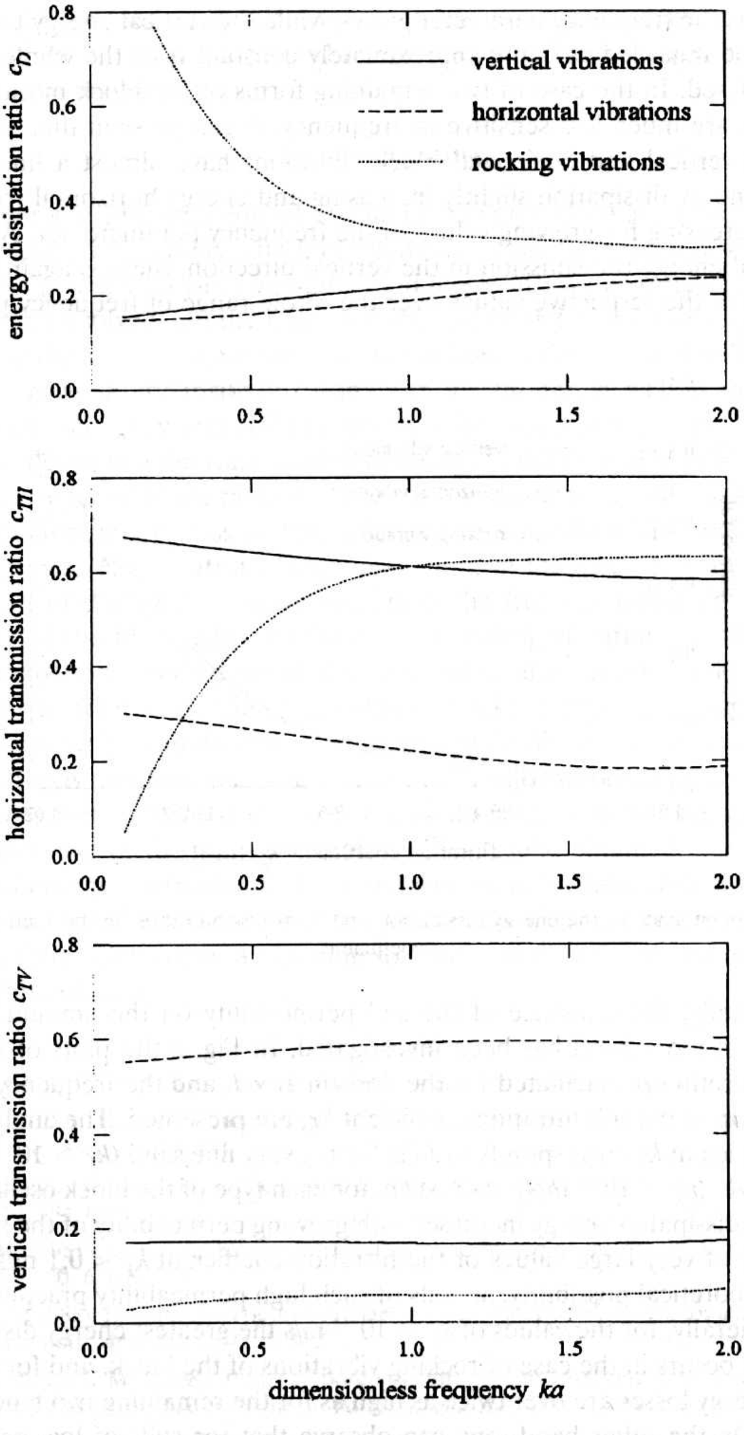

Fig. 7. Dependence of the energy dissipation and transmission ratios on the frequency of the block motions

increases as the frequency parameter grows, while the vertical energy transmission ratio can be regarded as being approximately constant over the whole frequency range analysed. In the case of two remaining forms of the block motions the energy ratios are much less sensitive to frequency. It can be seen that all the plots related to vertical and horizontal block vibrations have almost a linear character, with energy dissipation slightly increasing and energy horizontal transmission slightly decreasing for growing values of the frequency parameter ka. As concerns the ratio of energy transmission in the vertical direction, there is again practically no change in the respective values over the whole range of frequency parameters considered.

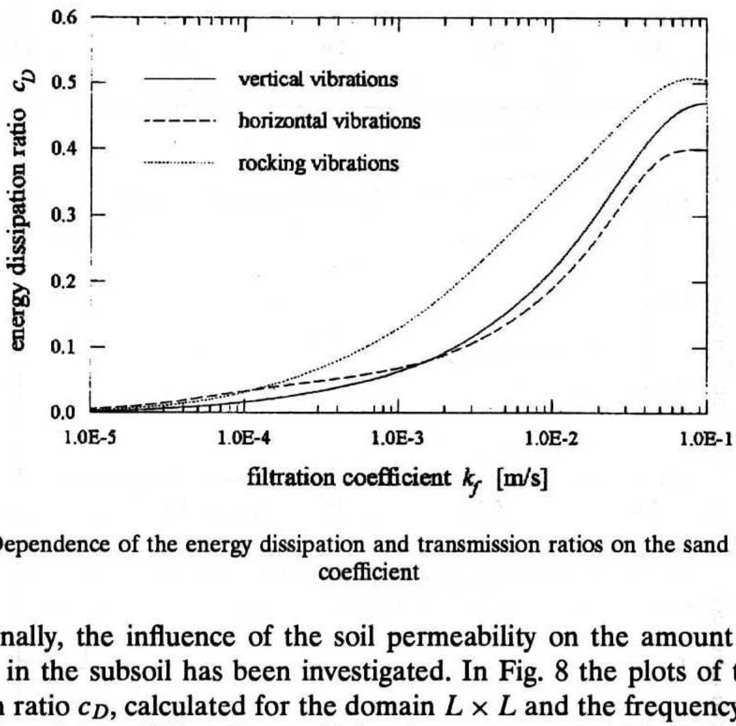

Fig. 8. Dependence of the energy dissipation and transmission ratios on the sand filtration coefficient

And finally, the influence of the soil permeability on the amount of energy dissipated in the subsoil has been investigated. In Fig. 8 the plots of the energy dissipation ratio c_D, calculated for the domain $L \times L$ and the frequency $ka = 1.0$, as a function of the soil filtration coefficient k_f, are presented. The analysed range of the coefficient k_f corresponds to soils from a very fine sand ($k_f \approx 10^{-5}$ m/s) to a coarse gravel ($k_f \approx 10^{-1}$ m/s). As is seen, for each type of the block oscillations the amount of dissipated energy increases with growing permeability of the soil, except in the case of very large values of the filtration coefficient $k_f \approx 0.1$ m/s (which is rather a theoretical possibility, as soils of such high permeability practically do not exist). Generally, for the values of $k_f > 10^{-4}$ m/s the greatest energy dissipation in the subsoil occurs in the case of rocking vibrations of the block, and for $k_f \approx 10^{-3}$ m/s the energy losses are over twice as high as for the remaining two types of block motions. On the other hand, one can observe that for soils of low permeability the energy dissipation ratio c_D rapidly approaches zero: e.g. for $k_f = 10^{-5}$ m/s only about 0.5% of the mechanical energy is dissipated in the domain $L \times L$. This means that in wave propagation problems low permeable sands can be treated as non-dissipative media.

6. Conclusions

In the paper the phenomenon of mechanical energy dissipation, accompanying wave propagation phenomena in fluid-filled porous media, has been studied. With this aim, the problem of vibrations of a rigid block on a water-saturated sand half--space has been solved. On the basis of a discrete model of the problem, constructed by using the finite element method, an analysis aiming at the determination of the energy dissipation density distribution in the subsoil, as well as the influence of vibrations frequency and soil permeability has been carried out. In addition, parameters describing the transmission of energy in the subsoil have been determined. Three types of block motions have been considered and it has been found that the greatest energy loss accompanies the rocking vibrations, and the least – horizontal ones. The greatest energy dissipation in the saturated subsoil occurs, independent of the kind of vibrations, under the free surface of the medium in the vicinity of the block edge. In the case of rocking vibrations another domain of high dissipation, situated under the block base, also occurs. Next, it has been found that the greatest frequency sensitiveness takes place in the case of rocking vibrations, particularly in the low frequency range, while in the case of vertical and horizontal motion of the block the influence of frequency is relatively small. And finally, it has been observed that the energy dissipation in the subsoil increases with the increase of the soil permeability. The results obtained, concerning the mechanical energy dissipation and transmission in water-saturated sands, provide some information, both theoretical and practical, which can be useful in the assessment of the influence of a vibrating structure on the subsoil and other objects in its neighbourhood.

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