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Hydraulic Consequences of Landfill-Liner Damage

Jerzy M. Sawicki
Gdańsk Technical University, Environmental Engineering Faculty, ul. Narutowicza 11/12, 80-952 Gdańsk, Poland

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Abstract

The body of each landfill or waste dump must be isolated from the subsoil. As the main element of the insulation a liner, made of synthetic foil, put on a clay layer, is very often applied. During the landfill exploitation the liner may be damaged. A puncture in this layer is a serious threat to groundwater quality. The paper deals with the hydraulic analysis of a situation after insulation damage. The velocity field of percolating water is described by the Laplace equation. The results of calculations have been compared with those of a laboratory scale experiment. After discussion of these results, some practical conclusions have been formulated.

1. Introduction

Each landfill, containing waste materials, both municipal or industrial, creates serious threats to the natural environment. One of them is the possibility of groundwater pollution, caused by residual fluming water and/or rain water, percolating through the landfill body. This water usually contains many harmful substances, dissolved during the contact with waste materials.

In order to avoid groundwater pollution, each landfill must be separated from the subsoil. For this purpose a special insulating layer under the landfill body is required. Such a layer usually consists of a base (made of clay, silt or equivalent material) and on top of it, a liner (usually a geosynthetic material).

Theoretically the insulating layer guarantees full leak tightness. However in practice one or more holes or gaps can appear in this layer. This may be the result of physical, chemical or/and biological factors, or unproper workmanship and exploitation.

The failure of the insulating layer is not easily detected. As a rule it reveals itself after a relatively long period of time, when pollutants from the stockyard body appear in the neighboring groundwater. The position of this puncture can be determined only approximately, and its repair is practically impossible (as for this purpose one has to remove whole the body of the functioning landfill, which is equivalent to the construction of a completely new object).

In this paper the hydraulic description of a failure of the insulating layer is presented.

2. Mathematical Description of the Problem

The mathematical description of the water percolation through the landfill body has quite simple form, when a construction without insulation is considered (Fig. 1a). A good approximation is given by a one-dimensional model of flow, with a constant value of the hydraulic conductivity. According to Darcy's law in this situation we can write:

$$u_f = -k\frac{\partial \varphi}{\partial z} = kH/L. \tag{1}$$

The total water discharge can be calculated by means of the continuity equation:

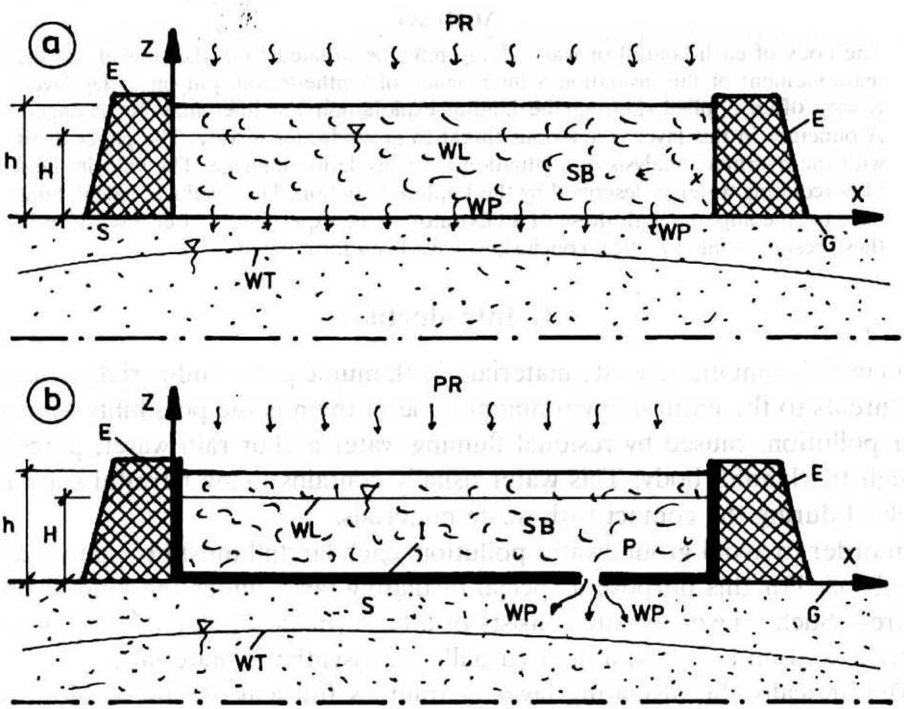

Fig. 1. Percolation of water through the stockyard (a – without insulation, b – with insulation; PR – precipitation, WT – water table, WL – water level in the stockyard, E – embankment, SB – stockyard body, G – ground level, I – insulation, P – puncture, WP – water percolation, S – subsoil)

$$Q = u_f S = kSH/L. \tag{2}$$

In the case of water flow through a hole (or several holes) in the insulation, the situation becomes more complex. The flow is of a three-dimensional character (or at the utmost – two-dimensional, vertical, when the damage to the insulation is in the form of straight slots). We have to make use of the full version of Darcy's law (Verruijt 1970, Zaradny 1990):

$$\vec{u}_f = -k\,\mathrm{grad}\varphi. \tag{3}$$

The total discharge of the hole must be determined by integrating the specific discharge:

$$Q = \int_{S_o} \vec{u}_f \vec{n} d S_o \tag{4}$$

whereas the hydraulic head is described by the Laplace equation:

$$\Delta\varphi = 0. \tag{5}$$

It is assumed in this paper that the hydraulic conductivity is constant (k=const.) and that dissolved substances do not influence the water velocity field. The first assumption is acceptable, as the stockyard body usually consists of waste materials of homogeneous properties (e.g. chemical waste products, cinders). Some reservations can arise as to the second assumption. If the dissolved substances influenced the velocity field, we would have to extend the system of governing relations by equations of dissolved substances transport, and in some cases even change the form of the Darcy law (IAHR 1972). In effect we would have to do with a complex and difficult system of equations. However in practical problems of environmental engineering especially important are such problems, when the concentration of dissolved substances are not very high (although these substances are very harmful).

The Eq. 5 must be completed by proper boundary conditions. On the free surface of water in the stockyard body we can write:

$$\varphi_s = H + p_a/\rho g. \tag{6}$$

Along the inside diameter of the hole (or several holes) in the insulation we have:

$$\varphi_p = \mathrm{const.} \tag{7}$$

whereas along the tight part of the insulating layer we can assume:

$$\frac{\partial\varphi_f}{\partial n} = 0. \tag{8}$$

3. Experimental Verification of the Problem Description

In order to evaluate the exactness of the model described above, a laboratory experiment was carried out.

A transparent glass cylinder with a diameter of $D = 2R = 0.192$ m (Fig. 2) was filled with coarse gravel (experimentally determined hydraulic conductivity: $k = 0.023$ m/s). The height of the filtration layer was $L = 0.46$ m. This layer was supported by a plastic partition (imitating the insulation), in the center of which a circular hole (imitating the puncture) with diameter of $d_o = 2r_o$ was made.

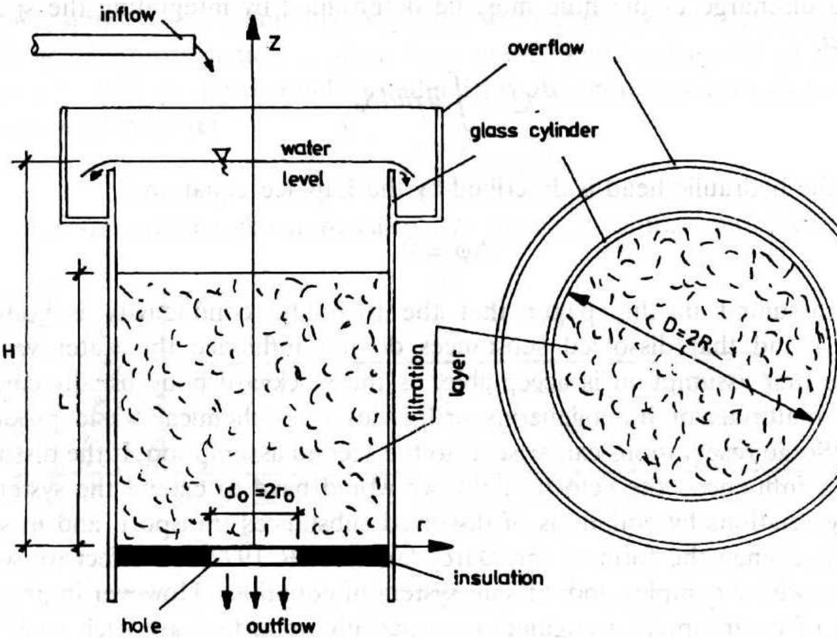

Fig. 2. The scheme of laboratory model

The column was supplied with fresh water from above. Water height was $H =$ 0.84 m. In the side-wall of the cylinder an overflow, which had to maintain the constant position of the water level was made. The water outflow, located from below, was free (into the atmosphere).

During the experiment the ordinate of water level was constant ($H = 0.84$ m) and the discharge of water percolating through the filtration layer was measured, for six different hole diameters (d_o = 1.0; 5.0; 9.0; 13.0; 17.0 and 19.2 cm). The results are shown in Fig. 3.

After the experiment, the problem was described mathematically, by the Laplace equation (Eq. 5). Taking into account the shape of the filtration column, the cylindrical coordinates (r, φ, z – Fig. 2) were applied. In such coordinates Eq. 5 has the following form:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \varphi}{\partial r}\right) + \frac{\partial^2 \varphi}{\partial z^2} = 0 \qquad (9)$$

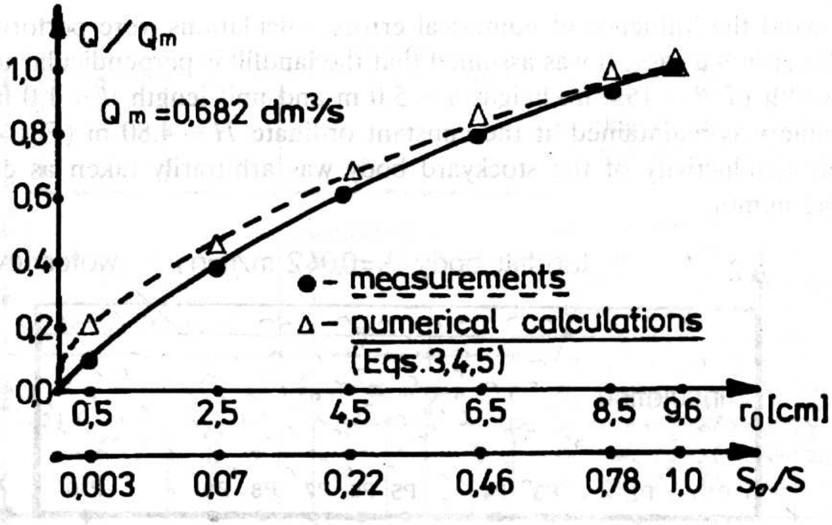

Fig. 3. Comparison of measured and calculated values of outflow discharge versus the hole radius r_0

together with boundary conditions:

- upper surface of the layer ($z = L, r = 0 \div R$) – Eq. 6,
- tight bottom and side walls – Eq. 8,
- hole in the foil ($z = 0, r = 0 \div r$):

$$\varphi_p = p_a / \rho g. \tag{10}$$

Eq. 9 was solved numerically, by means of the finite differences scheme and the successive over-relaxation method (Potter 1973).

The results of calculations, combined with experimental points, are shown in Fig. 3. The total discharge Q for each value of diameter d_o has been related to the maximum discharge Q_m (for completely opened bottom, i.e. when $d_o = D$).

Comparison of the results obtained leads to the conclusion, that the mathematical model proves quite acceptable conformity with experimental values. This conformity is full when the hole and column areas equal ($S_o = S$), as in this case the problem is reduced to a simple one-dimensional task. The level of conformity worsens with the decrease in hole diameter which has a negative meaning from the practical point of view (as in this case we are interested in relation $S_o \ll S$). But the main purpose of this paper is not to present a technical report, but a theoretical base of a practical concept.

4. Numerical Analysis of the Problem

After experimental verification of the proposed model, some numerical experiments have been carried out. In order to make these experiments more clear and to avoid the influence of numerical errors, calculations were performed for a two-dimensional case. It was assumed that the landfill is perpendicular in shape with a width of $B = 15.0$ m, height $h = 5.0$ m and unit length ($l = 1.0$ m). The water table was maintained at the constant ordinate $H = 4.80$ m (Fig. 4). The hydraulic conductivity of the stockyard body was arbitrarily taken as equal to $k = 0.042$ m/min.

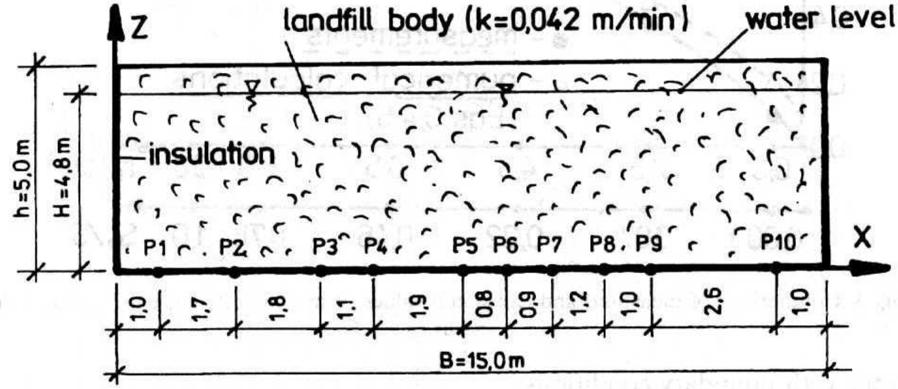

Fig. 4. The scheme of the landfill with punctures location (P1 ... P10)

It was assumed, that in the insulating layer under the stockyard body are some slots (from $n = 1$ to $n = 8$). Each slot has a width $b = 0.10$ m. Localization of these slots (from $P1$ to $P10$) is shown in Fig. 4.

During the calculations the two-dimensional Laplace equation:

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial z^2} = 0 \tag{11}$$

with boundary conditions (6, 8, 10) were solved. It was assumed that the subsoil is in the aeration zone. As a result we can treat the pressure in the pore space of the subsoil as equal to constant ($p = p_a$), which means that the hydraulic head along the slot is also constant ($\varphi_p = p_a/\rho g$).

As previously, the successive over-relaxation method was applied. After determination of the potential φ, the specific discharge u_f was computed using the Darcy law (Eq. 3), and then – the total discharge Q (Eq. 4).

Exemplary values of slot discharges q and total discharges Q for different combinations of slots are shown in Fig. 5. Each symbol of the experiment contains two numbers: n and m; the first denotes the number of slots in the insulation and the second – the number of slots combination. An example of a set of streamlines (for $n = 3$) has been shown in Fig. 6.

Analyzing results obtained one can observe certain regularities (Fig. 5). The discharge of one slot (q_m) decreases together with the number of slots. The discharge of several slots (Q) increases with their spacing, and decreases when the

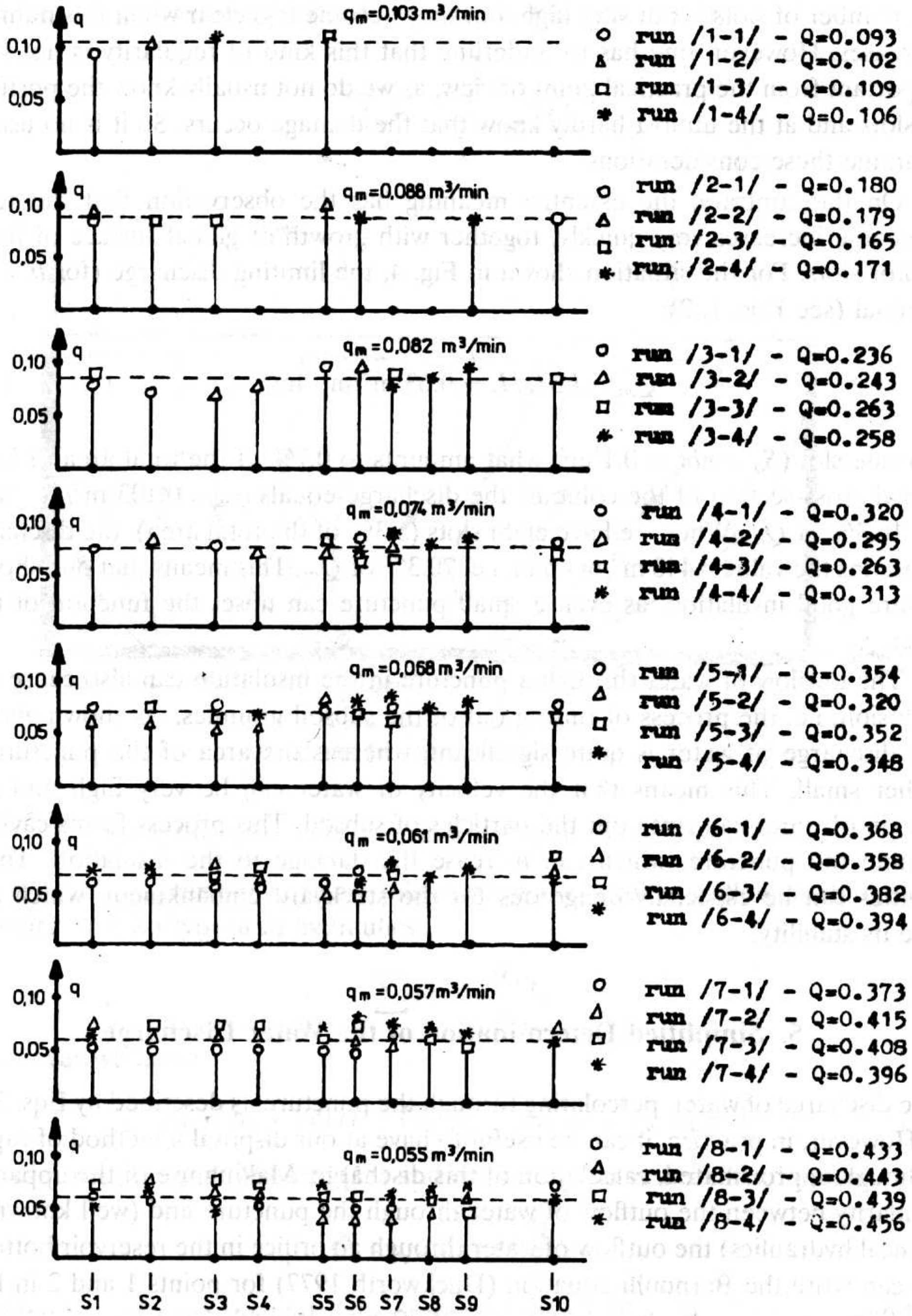

Fig. 5. Results of numerical calculations (discharges q_m and Q in [m^3/m·min])

slots are located closer to the wall. These regularities are more apparent when the number of slots is not very high, but they become less clear when this number increases. However, one has to underline that this kind of regularity is not very important from the practical point of view, as we do not usually know the position of slots and at the utmost hardly know that the damage occurs. So it is no use to continue these considerations.

On the contrary, the essential meaning has the observation that the total discharge increases very quickly, together with growth of global surface of holes and/or slots. For the situation shown in Fig. 4, the limiting discharge (for $b = B$) is equal (see Eqs. 1, 2):

$$Q_m = kSH/L = 0.63 \text{ m}^3/\text{m} \cdot \text{min}. \tag{12}$$

For one slot ($S_o = nbl = 0.1$ m^2, what amounts to 0.7% of the total area of horizontal cross-section of the column) the discharge equals $q_m = 0.103$ m^3/m · min, i.e. 16.3% of Q_m. When we have eight slots (5.3% of the total area), the discharge Q attains the value 0.443 m^3/m · min, i.e. 70.3% of Q_m. This means that one should ensure good insulation, as even a small puncture can upset the function of this layer.

The outflow of water through a puncture in the insulation can also cause the suffosion, i.e. the process of rinsing out of the subsoil granules. As shown above, the discharge of water is quite significant, whereas the area of the puncture is rather small. This means that the velocity of water can be very high and the stream of water can rinse out the particles of subsoil. This process forms cavities around the puncture, which can increase the damage to the insulation. These cavities can be especially dangerous for the stockyard embankment, which can lose its stability.

5. Simplified Determination of the Water Discharge

The discharge of water, percolating through the puncture, is described by Eqs. 3, 4, 5. However, in practice, it can be useful to have at our disposal a method of rapid, although approximated, calculation of this discharge. Making use of the apparent similarity between the outflow of water through the puncture and (well known in clasical hydraulics) the outflow of water through an orifice in the reservoir bottom, we can write the Bernoulli equation (Duckworth 1977) for points 1 and 2 in Fig. 1:

$$\frac{u_{f1}^2}{2g} + \frac{p_1}{\rho g} + z_1 = \frac{u_{f2}^2}{2g} + \frac{p_2}{\rho g} + z_2 + \sum h_l. \tag{13}$$

The loss of mechanical energy can be expressed according to the Darcy law by the following integral, calculated along the streamline (Fig. 6):

$$\sum h_l = \int_1^2 \frac{u(z)}{k} dz. \tag{14}$$

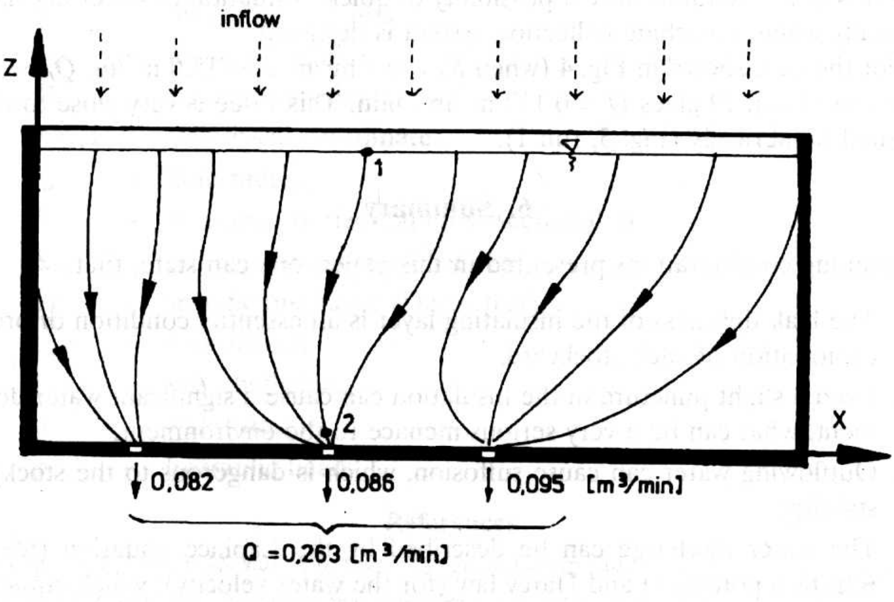

Fig. 6. Exemplary set of streamlines (for 3 punctures in the insulation)

It is very convenient to relate this value to the specific outflow discharge and to write relation typical in hydraulics:

$$\sum h_l = \xi \frac{u_o}{k} L. \tag{15}$$

Moreover we have:

$$p_1 = p_a + \rho g(H - L),\ p_2 = p_a,\ z_1 = L,\ z_2 = 0. \tag{16}$$

Substituting (15) and (16) into (13) and neglecting the velocity head we can write:

$$Q = u_{fo} S_o = \xi k \frac{H}{L} S_o. \tag{17}$$

Making use of experimental data presented above (Fig. 3), it is very useful to express the coefficient ξ as follows:

$$\xi = (S_o/S)^{-0.645}. \tag{18}$$

The value 0.645 has been obtained by the least square method. Substituting (18) to (17) and taking into account Eq. 12 we have:

$$Q_o = (S_o/S)^{0.355} Q_m. \tag{19}$$

This formula is valid for simple cases, when we have only one puncture in the installation. Of course, it is a very simplified relation, but for preliminary calculations it is quite useful to have a possibility of quick evaluation of water discharge, especially when a leachate collection system is designed.

For the case shown in Fig. 4 (when $S_o = 0.1$ m^2/m, $S = 15.0$ m^2/m, $Q_m = 0.63$ m^3/m · min) Eq. 19 gives $Q = 0.112$ m^3/m · min. This value is very close to those obtained numerically (Fig. 5, run 1).

6. Summary

Reassuming considerations presented in this paper, one can state, that:

1. The leak tightness of the insulating layer is an essential condition of proper exploitation of each stockyard.
2. Even a slight puncture in the insulation can cause a significant water decrement, what can be a very serious menace to the environment.
3. Outflowing water can cause suffosion, which is dangerous to the stockyard stability.
4. The water discharge can be described by the Laplace equation (for the filtration potential) and Darcy law (for the water velocity); which equations can be solved when the location (at least approximated) of a puncture is known.
5. In order to obtain an approximated evaluation of the water discharge of a single puncture, one can adopt Eq. 19.

Notation

B – landfill width,
b – slot width,
D – diameter of a cylinder,
d_o – hole diameter,
g – gravity acceleration,
H – water depth,
h – landfill height,
h_L – mechanical energy loss,
k – hydraulic conductivity,
l – landfill length,

L – distance of percolation,
$\vec{n}$ – unit vector, normal to flow area,
p – pressure,
p_a – atmospheric pressure,
Q – total discharge of percolating water,
Q_m – maximum discharge,
q – discharge of a hole,
q_m – mean value of q,
R – radius of a cylinder,
r, ψ, z – cylindrical coordinates,
r_o – hole radius,
S – stockyard horizontal cross-sectional area,
S_o – puncture cross-sectional area,
$\vec{u}_f$ – specific discharge (Darcy flux),
x, z – coordinates,
ξ – loss coefficient,
ρ – water density,
φ – hydraulic head.

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