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Approximate Description of Stokes' Type Waves Generated in Fluid of Constant Depth[1]

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Abstract

In the paper, the generation of non-linear waves with moderate amplitude in water of finite depth is investigated. The surface waves are described by means of the Stokes' approximation up to the second order of expansion. The solution discussed is based on the description model given by Wilde and Romanczyk (1989) in which, the random Stokes' type wave with two components has slow varying phase and amplitude in time. The problem considered is solved in two steps. In the first, the linear component of the wave is obtained as the output of the linear time invariant system induced by a sample function of the stochastic process describing the wavemaker velocity. Then, in the second step, a supplementary solution is created. The latter corresponds to the double frequency of the wavemaker and expresses the second component of the Stokes' wave. Comparison of the theoretical solution obtained with experimental data shows that the model proposed leads to results of practically acceptable accuracy.

1. Introduction

In the analysis of water waves a basic solution is obtained within the linear theory of potential motion supplied with linear boundary conditions at the free surface of the fluid. An example of such a case is the linear solution to the problem of harmonic generation of long waves of small amplitude in a hydraulic flume. The linear theory applied for this case provides results which fit experimental data fairly well. There are also regions of our interest however, where the waves generated are of relatively considerable heights and thus a more advanced theory is needed for their description. A useful method of solving the latter problem is, to some extent, the perturbation method in which the solution is expressed in the form of a series in powers of small parameter. Such a method of solution of the two-dimensional problem of generation of water waves in a hydraulic flume was given by Hudspeth and Sulisz (1991). These authors developed a complete

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second-order solution to the aforementioned problem. In cases where the free surface elevation is measured far enough from the wavemaker it is possible to describe the surface wave by means of the Stokes' approximation in which the wave profile consists of harmonic components corresponding to multiple of the wave basic frequency. The amplitudes of the successive components of the Stokes' wave are monotone decreasing and thus, in practical applications, it is often reasonable to confine our attention to some of the lowest components of the wave. Since the Stokes' wave is a periodic wave propagating in an infinite fluid domain, it does not satisfy the boundary condition at the wavemaker in general. Therefore, when applying the Stokes' approximation to the description of waves generated in a flume, additional free waves, which are not components of the Stokes wave, should be taken into account. This problem was discussed by Bendykowska (1980) and Massel (1982). For harmonic generation of water waves in a flume, the authors assumed three components in the description of the free surface elevation. The lowest two components corresponding to the wavemaker frequency and its doubling propagate with the same velocity. The third component, associated with the double frequency of the wavemaker, propagates with its own velocity which differs from the previous ones. The approximate description discussed, enables us to find the Stokes' type wave (its two components) in measurements in a hydraulic flume and then to calculate the additional free wave.

Another approach to the problem of description of waves generated in a hydraulic flume was presented by Wilde and Romanczyk (1989). These authors assumed that the generated wave is random in nature and may be described by the random Stokes' type wave with phase and amplitude slowly varying in time. The main goal of the paper was to decompose the measurements into the Stokes' type wave and the free wave propagating in the flume. To save space we shall refer to this paper using the abbreviation W-R.

The aim of the present paper is to construct an approximate solution to the problem of generation of random waves with moderate amplitudes in water of constant depth. In the method considered, the waves are generated by a piston-type wavemaker placed at the beginning of a semi-infinite layer of fluid. The surface waves are measured at points a considerable distance from the wavemaker and thus, only progressive waves are taken into account. Following the mathematical model developed in W-R (1989), the problem in question is reduced to the transformation of a narrow-band stochastic process describing the generator motion into the stochastic process of the free surface elevation. Generally speaking this transformation is a non-linear one. In our approach however, it is assumed that the generated wave may be properly described by means of a two component Stokes' type wave. The first linear component of the wave is obtained by means of direct linear transformation of the generator motion process. The second, non-linear component of the solution, is obtained indirectly, with the help of the first component given. The linear transformation of the stochastic processes considered is based on the impulse response function for the generator-fluid system. With this function, the free surface elevation is calculated as the output of the linear generator-fluid system forced by the sample of the generator motion process assumed. The main goal of the investigations is to check accuracy of the model considered by means of comparison of its results with data obtained in experiments.

2. The Stokes' Type Wave with Two Components

Let us consider an infinite layer of fluid of depth h and the rectangular co-ordinate system $0xz$ with $-\infty < x < \infty$, $-h \leq z \leq \eta$. It is assumed that a Stokes' wave of rigid profile propagates through the layer from the left, into the right side of the layer (in the direction of positive values of x). To the second order of approximation, the free surface elevation of this wave is expressed as follows (Druet 1978):

$$\eta(x,t) = Re\left[\frac{H}{2}e^{i(kx-\sigma t)} + \frac{1}{4}\frac{H^2}{h}\gamma e^{2i(kx-\sigma t)}\right], \tag{1}$$

where H is the wave height, σ is the dominant frequency of the wave, k is the wave number, i is the imaginary unit, and

$$\gamma = \frac{kh}{2}\,\mathrm{ctgh}(kh)\left[1 + \frac{3}{2\sinh^2(kh)}\right]. \tag{2}$$

The dispersion relation for the wave is the same as in the case of linear solution and is given by the formula:

$$\sigma^2 = kg\,\mathrm{tgh}(kh) \tag{3}$$

where g is the gravitational acceleration.

According to the Stokes' perturbation method, the second, non-linear term in H which enters the solution (1), must be small compared to the first-linear term. In laboratory experiments, the free surface elevation was measured at a chosen point and thus, without loss of generality, we may assume $x = 0$, and write:

$$\eta(t) = Re\left[\frac{H}{2}e^{-i\sigma t} + \frac{1}{4}\frac{H^2}{h}\gamma e^{-2i\sigma t}\right]. \tag{4}$$

Following the generalization given by W-R (1989), it is assumed that the Stokes' wave has an amplitude and phase shift slowly varying in time and thus, instead of Eq. (4) we will consider the expression:

$$\eta(t) = Re\left[C(t)e^{-i\sigma t} + \frac{\gamma}{h}C^2(t)e^{-2i\sigma t}\right] \tag{5}$$

where:

$$C(t) = A(t) + i\,B(t) \tag{6}$$

is a complex function.

From this substitution it can be seen that the absolute value of $C(t)$ describes the amplitude of the basic wave generated. As regards the description given in W-R (1989), functions $A(t)$ and $B(t)$ in the equation are assumed to be two independent stationary gaussian processes without dominant frequencies. Moreover, the processes are assumed to have zero means and the same correlation functions. All details associated with processes $A(t)$ and $B(t)$ i.e., their derivation, properties, differentiability etc., may be found in the book by Wilde and Kozakiewicz (1993) hereinafter referred to as W-K. For further purposes it is convenient to introduce the following notation:

$$Z(t) = C(t)e^{-i\sigma t} = X(t) + i\,Y(t) \tag{7}$$

where:

$$\begin{aligned} X(t) &= A(t)\cos(\sigma t) + B(t)\sin(\sigma t), \\ Y(t) &= -A(t)\sin(\sigma t) + B(t)\cos(\sigma t). \end{aligned} \tag{8}$$

According to the characteristic features of processes $A(t)$ and $B(t)$, the resulting processes $X(t)$ and $Y(t)$ are also stationary gaussian processes with zero means.

Substituting the latter equations into Eq. (5) we arrive at the following formula:

$$\eta(t) = X(t) + \frac{\gamma}{h}[X^2(t) - Y^2(t)]. \tag{9}$$

The formulation of the problem considered enables us to decompose the measurements in a hydraulic flume into components corresponding to the dominant frequency and its doubling, respectively. This will be performed by means of the Kalman filter method (see W-K (1993)) and the related system of computer programs prepared by Wilde (1992).

3. Laboratory Experiments

In order to investigate the accuracy of the approximate description presented so far, laboratory experiments in a hydraulic flume were carried out. These experiments consisted in the generation of random waves with moderate heights in the flume by a piston-type wavemaker. The scheme of the wave generation system is shown in Fig. 1.

The experiments were performed at the Polish Academy of Sciences Institute of Hydro-Engineering in Gdańsk. The 0.50 m wide hydraulic flume was filled with water up to 0.60 m. The water waves were generated by a programmable piston-type wavemaker. In the experiments, the motion of the generator (the horizontal

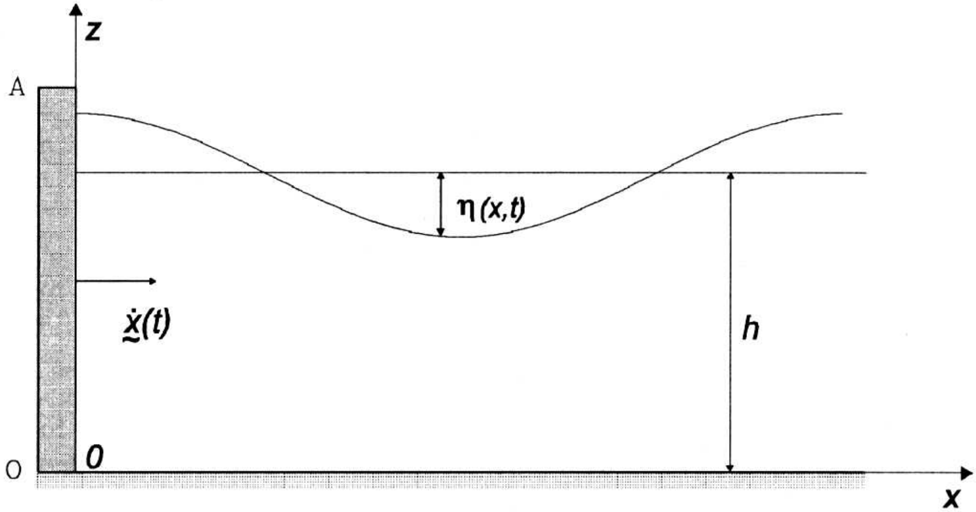

Fig. 1. Definition sketch for the generator-fluid system

displacements of the rigid wall OA in Fig. 1) was imposed in the form of the stationary gaussian process:

$$X_g(t) = A_g(t)\cos(\sigma t) + B_g(t)\sin(\sigma t) \tag{10}$$

where, as in the previous section, $A_g(t)$ and $B_g(t)$ were stationary gaussian processes with zero means and the same correlation functions. The dominant frequency σ of the generator was equal to 2π. All the stochastic processes considered were substituted by discrete parameter processes i.e., by sequences of random numbers corresponding to the discrete time steps: $t_n = n\Delta t, n = 0, 1, 2, \ldots, N$. In this way a sample function of the proces (10) was expressed in the form of a finite sequence of numbers. The latter were calculated before experiments by means of methods outlined in W-K (1993).

The random sequence generated was then used as the displacement history input for the steering system of the wavemaker. Besides, the displacements of the generator were measured by an additional inductive gauge and recorded by the computer system. With the help of the displacement record it was a simple task to calculate the time history of the wavemaker velocity. A sample of this velocity record is shown in Fig. 2.

The surface wave generated in this way was measured at a distance of $x = 4.78$ m from the wavemaker by means of a wave gauge. The record of the free surface elevation corresponding to the excitation history shown in Fig. 2. is presented in Fig. 3 (A). The latter was then processed by means of a Kalman filter providing components corresponding to the dominant frequency and its doubling, respectively.

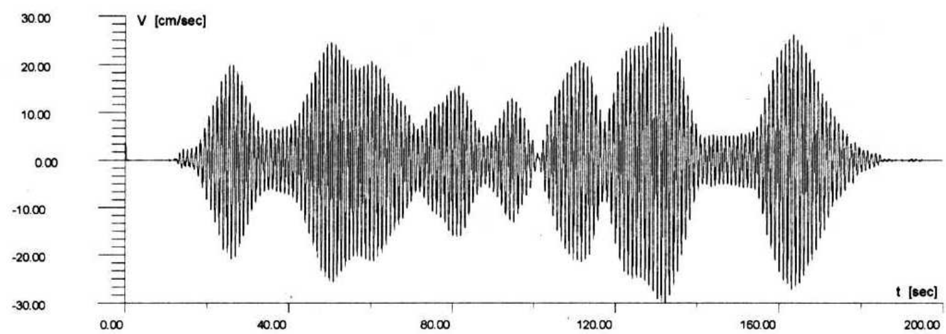

Fig. 2. Sample function of the generator motion process

In light of the above, the components in Figs. 3 (B, C) correspond to the relevant components of the Stokes' wave. It can be seen from the plots that the amplitude of the second component of the wave is, as it should be, smaller than the amplitude of the first component. To get a better insight into the results of experiments, part of the free surface elevation together with its two components is shown in Fig. 3 (E). From the plots it is seen that there is no phase shift between the first and the second component of the wave. A similar result was obtained for the wave measured at a distance of 10.83 m from the wavemaker. Therefore, for the case of random wave generated in the hydraulic flume discussed no free wave was detected in the experiments performed.

4. Approximate Solution to the Problem of Generation of the Stokes' Type Wave

Knowing the sample function describing the generator velocity shown in Fig. 2, it is possible to calculate the first linear component of the surface wave (8). This may be done by means of the impulse response function $h(t)$ of the system mentioned. Thus, let us consider now the linear problem of potential motion of incompressible inviscid fluid. For the assumed unit impulse of velocity of the wall OA (Fig. 1), the relevant free surface elevation is given by the formula (for details see Szmidt 1993, 1994):

$$h(t) = \frac{1}{\pi}\int_0^\infty \frac{\operatorname{tgh}(sh)}{s}\cos(sx - rt)\,ds + \frac{4}{\pi}\int_0^\infty \frac{\sin(\sigma t)}{\sigma}\sum_{j=1}^{\infty}\frac{\sin^2\beta_j e^{-k_j x}}{2\beta_j + \sin(2\beta_j)}\,d\sigma, \tag{11}$$

where

$$\sigma^2 = r^2 = gs\operatorname{tgh}(sh), \quad \beta_j = k_j h, \quad \frac{\sigma^2 h}{g} = -\beta_j \operatorname{tg}(\beta_j), \quad j = 1, 2, \ldots. \tag{12}$$

The first term of the solution (11) describes the propagating wave and the second one corresponds to a standing wave which dies out when going to infinity

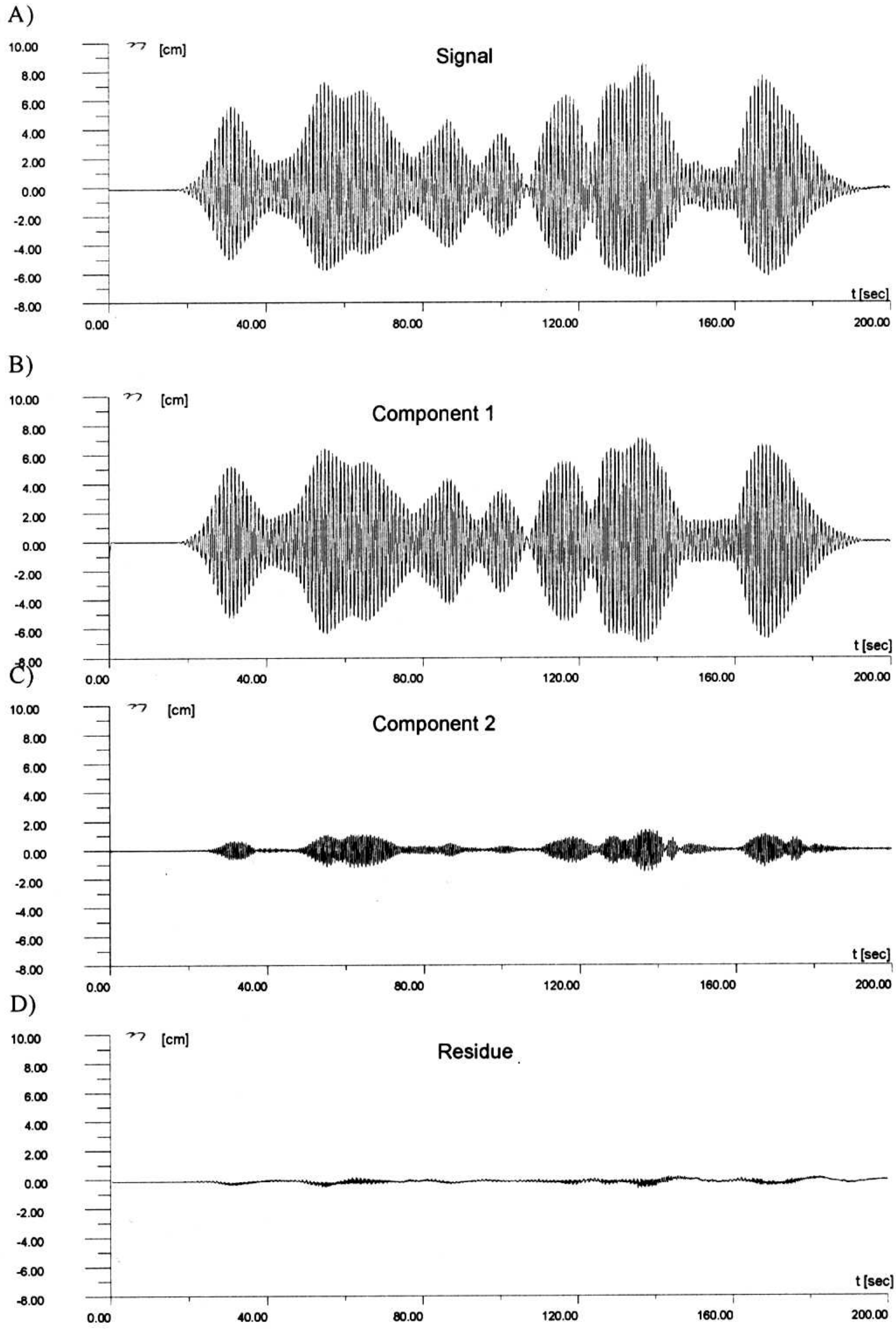

Fig. 3. Free surface elevation (A) and its components (B, C, D) registered in laboratory experiments

E)

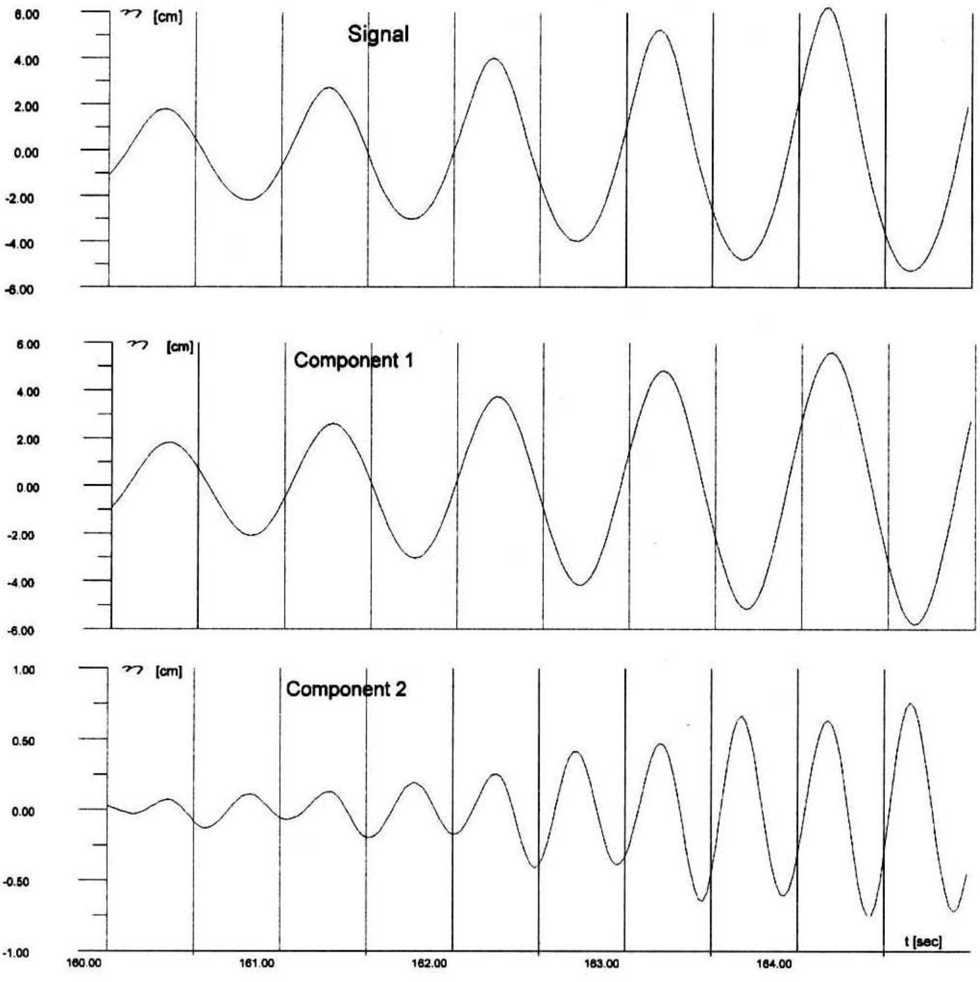

Fig. 3. continued

$(x \to \infty)$. From numerical analysis performed in Szmidt (1994) it follows that within the limit $t \to 0^+$, $x > 0$, the following relation holds:

$$\lim_{t\to 0^+} \frac{4}{\pi} \int_0^\infty \frac{\sin \sigma t}{\sigma} \left(\sum_{j=1}^{\infty} \frac{\sin^2 \beta_j e^{-k_j x}}{2\beta_j + \sin 2\beta_j} \right) d\sigma =$$

$$= \frac{1}{\pi} \int_0^\infty \frac{\operatorname{tgh} sh}{s} \cos sx ds = \frac{1}{\pi} \ln \left[\operatorname{ctgh} \left(\frac{\pi x}{4h} \right) \right]. \quad (13)$$

Besides, for large values of x $\left(\frac{x}{h} > 2\right)$ and sufficiently large t (t means elapse of time measured from the starting point), the influence of the second integral in Eq. (11) on the final result is negligible, and thus the response function may be assumed in the form:

$$h(t) \cong \frac{1}{\pi} \int_0^\infty \frac{\operatorname{tgh}(sh)}{s} \cos(sx - rt) ds. \quad (14)$$

The function (11) is expressed in the form of improper integrals. In order to find values of the function for a chosen sequence of the time steps ($x > 0, t = n\Delta t, n = 0, 1, 2, \ldots$) it is necessary to resort to approximate numerical integration. The impulse response function obtained in this way is shown in Fig. 4.

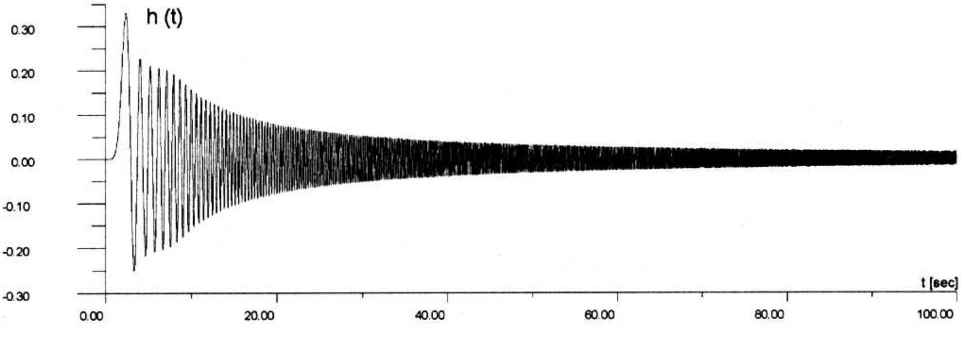

Fig. 4. Impulse response function for the generator-fluid system

It is seen that the function rapidly oscillates and diminishes in absolute value with the passage of time. The frequency of the oscillations grows with the passage of time. To investigate behaviour of the function for large values of time, the method of stationary phase was applied (Achenbach 1973). One can show, that for large time, the impulse response function may be approximated by the formula:

$$h(t) \cong \sqrt{\frac{2}{\pi}} \sqrt{\frac{4x}{gt^2}} \sin \left(\frac{gt^2}{4x} + \frac{\pi}{4} \right), \; t \gg 0. \quad (15)$$

By virtue of this result, the amplitude of the function decreases as $1/t$ for $t \to \infty$, while the angular frequency of vibrations increases with time:

$$\sigma_h \cong \frac{gt}{2x}, \; x \gg 0, \; t \to \infty. \tag{16}$$

Knowledge of the time history of the wavemaker velocity and of the impulse response function, enables us to find the linear part of the free surface elevation:

$$\eta(x = \text{const}, t) = X(t) = \int_0^t \underset{\sim}{\dot{x}} (t - \tau) h(\tau) d\tau, \tag{17}$$

where for $t \leq 0$, the generator-fluid system is at rest.

In laboratory experiments, the velocity of the generator, as well as the free surface elevation measured, are both expressed in the form of finite sequences of numbers corresponding to the chosen time step $\Delta t > 0$. Similarly, in numerical computations, the impulse response function is expressed in the form of a sequence of numbers with its own time step. Therefore, in order to calculate the latter integral, it is necessary to use approximate numerical integration. The assumed time step of the integration must satisfy the Nyquist condition (Otnes and Enochson 1979):

$$\Delta t \leq \frac{4}{5} \frac{\pi}{\sigma_c} \tag{18}$$

where σ_c is the maximum frequency observable.

The last condition is especially important for large values of time ($t \gg 0$) when the frequency σ_h of the impulse response function may exceed the Nyquist frequency, i.e.:

$$\sigma_h > \sigma_N = \frac{\pi}{\Delta t}. \tag{19}$$

On inserting Eq. (16) into condition (18) and making simple manipulations we arrive at the following inequality:

$$t_{\max} \leq \frac{8\pi x}{5g\,\Delta t} \tag{20}$$

which is stronger than the previous one.

The last formula derived defines the maximum value of time which is allowed to be used in numerical computations. In cases when the observation time (the length of records describing the time history of the generator motion and the free surface elevation) is greater than the value (20), we have to consider some additional modifications in calculating the integral (17). The first possibility is to choose a smaller time step Δt in such a way that the condition (20) should be satisfied. In this case one may expect a longer time of computer calculations.

Besides, it may happen that the length of experimental records (the number of points in one record) must be limited for technical reasons. Therefore, it is more promising to change the impulse response function so that the integration (17) may be performed in a finite range of time not depending directly on the length of the records. In other words, an appreciable contribution to the integral comes from the interval $(0 \div t)$, where $t \leq t_{\max}$. The latter approach to the problem rests on the observation that for large t the impulse response function oscillates very rapidly with a self-cancelling effect on the integral. Accordingly, instead of the original function $h(t)$ we may use weighted values of it, which are obtained by means of digital filtering (see Otnes and Enochson 1979). As regards the above, let us consider now the recursive low-pass digital filter of the second order:

$$y_i = 2\alpha \cdot y_{i-1} - \alpha^2 \cdot y_{i-2} + (1-\alpha)^2 \cdot x_i \tag{21}$$

where $x_i = h(i\,\Delta t)$ and $0 \leq \alpha < 1$.

For the sequence of numbers $x_i (i = 0, 1, 2\ldots)$ representing the impulse response function $h(t)$ and chosen value of α it is a simple task to calculate the new sequence of numbers $y_i (i = 0, 1, 2\ldots)$. The latter represents the new filtered function $h(t)$ which differs from the previous one. Since the filter (21) is the low-pass one, the amplitudes of the impulse response function $h(t)$ for large values of time are much smaller than those of the original function. In this way, amplitudes of the tail of $h(t)$ may be compressed to arbitrary small values. On the other hand, it is desirable for the new function to be as close as possible to the original function within the range time: $0 \leq t \leq t_{\max}$. To learn more about the filter mentioned, let us calculate its complex frequency response function. Simple manipulations give:

$$H(\sigma) = \frac{(1-\alpha)^2}{1 - 2\alpha \exp(i\sigma\Delta t) + \alpha^2 \exp(2i\sigma\Delta t)}. \tag{22}$$

The absolute value of Eq. (22) calculated at $\sigma = 0$ is:

$$H(\sigma)H^*(\sigma) = \frac{(1-\alpha)^4}{(1-\alpha)^4} = 1 \tag{23}$$

where H^* is the complex conjugate of H.

In the limit $\sigma \to \sigma_N$, the following relation holds:

$$\lim_{\sigma \to \sigma_N} H(\sigma)H^*(\sigma) = \left(\frac{1-\alpha}{1+\alpha}\right)^4. \tag{24}$$

The formula obtained allows us to assume a desirable level of "damping":

$$\beta = H(\sigma_N)H^*(\sigma_N) \ll 1 \tag{25}$$

and then to calculate the relevant value of

$$\alpha = \frac{1 - \sqrt[4]{\beta}}{1 + \sqrt[4]{\beta}}. \tag{26}$$

In order to illustrate the considerations, numerical computations were made. The computations were performed for the assumed $\beta = 0.0001$. Some of the numerical results obtained are shown in Fig. 5 where the plots of $h(t)$ and $h^*(t)$ are depicted.

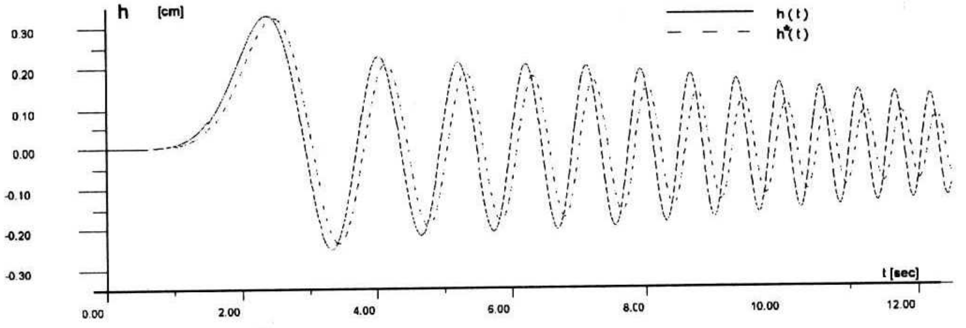

Fig. 5. Impulse response function $h(t)$ and its picture $h^*(t)$ after filtration

It can be seen that the filter applied has changed both the amplitude and phase of $h(t)$. The phase shift between $h(t)$ and $h^*(t)$ is relatively small and may be assumed as independent in time. For the discussed case of laboratory experiments ($x = 4.78$ m and $\Delta t = 0.025$ s) condition (20) gives:

$$t_{\max} \leq 97.969 \text{ s.} \tag{27}$$

The range of time (27) is large enough and thus, with respect to the behaviour of $h(t)$ for $t > t_{\max}$ (the function is small in absolute value and rapidly oscillating), it is justified to use the ideal low-pass rectangular filter:

$$h^*(t) = \begin{cases} h(t) & \text{for} \quad 0 \leq t \leq t_{\max}, \\ 0 & \text{for} \quad t > t_{\max}. \end{cases} \tag{28}$$

Accordingly, in calculating the integral (17) the shift in time of the integrand functions does not exceed the maximum time allowed by (27). Therefore, in light of the substitution (28), the maximum time allowed is called the memory time of the system mentioned. The last assumption may be supported by observation of a physical situation of generation of water waves in a hydraulic flume. In reality, the dissipation of energy due to viscous forces occurs of necessity and thus, the duration of fluid vibration forced by impulse of the generator velocity may be assumed to be finite. From theoretical point of view the memory time is infinite, but the explanations presented above justify the assumptions introduced, which in turn, allow us to use the aforementioned ideal low-pass rectangular filter.

5. Comparison Between Theory and Experiment

For the purpose of verifying the computational model presented above, numerical calculations were made. The calculations correspond directly to the experiments performed in the hydraulic flume, therefore the wavemaker velocity entering the integrand in Eq. (17) was taken from the experiments (see Fig. 2). The numerical calculations of the integral (17) were performed for the impulse response function filtered with the help of the ideal low-pass rectangular filter. The result of the computation is the linear solution which corresponds to the first component of the free surface elevation obtained in experiments and shown in Fig. 3 (B). For comparison, the results of the numerical solution together with results obtained in experiments are depicted in Fig. 6.

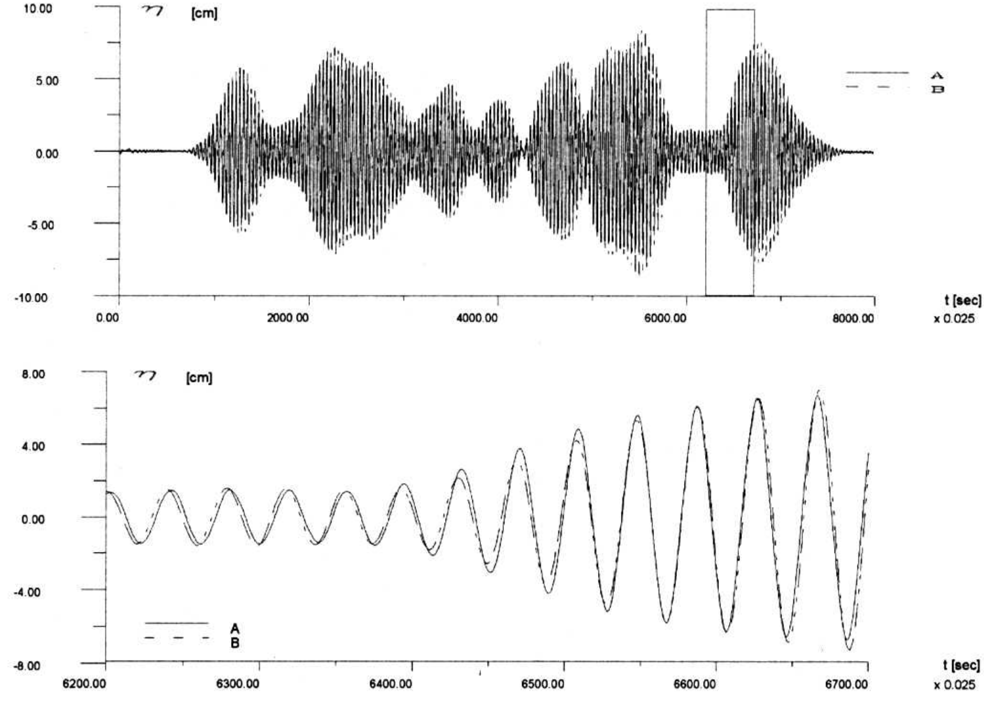

Fig. 6. Linear part of the free surface elevation measured in the hydraulic flume (A) and resulting from theoretical linear transformation (B)

It can be seen from the plots that the results of the theoretical model considered, fit the first component obtained in experiment quite well. With assumption that the stochastic processes discussed herein are ergodic it is possible to calculate the relevant correlation coefficient of these plots. For the case in Fig. 6, this coefficient is equal to 0.96 which indicates that the linear transformation mentioned leads to an accurate description of the linear part of the surface wave generated.

On the basis of the linear solution represented by the sequence $X(t)$ (Eq. 8 and Fig. 6) and, with the help of computer programs (Wilde 1992), it was possible to calculate the second, non-linear component of the solution discussed. In view of the previous considerations, the latter component corresponds double the wavemaker basic frequency. The additional solution obtained in this way is shown in Fig. 7 where, for comparison, the second component of the Stokes' wave obtained in experiment is also presented.

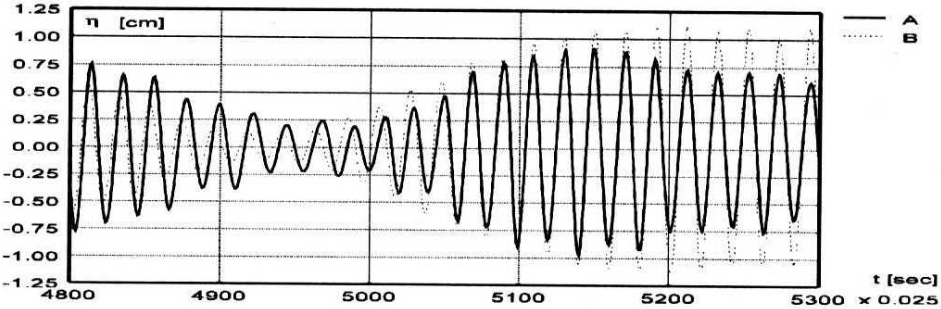

Fig. 7. Additional non-linear component of the surface wave obtained in experiments (A) and calculated by means of the theoretical model (B)

The differences between the plots in Fig. 7 are greater than those of the first component of the wave, but they are still so small that they may be ignored in practical applications. The correlation coefficient for this case is equal to 0.87. Knowing that the amplitude of the second component of the wave (8) does not exceed 20% of the amplitude of the first component, the results obtained may be considered as being sufficiently accurate.

6. Concluding Remarks

A relatively simple analytical solution to the problem of generation of surface waves of moderate amplitude in fluid of constant depth has been presented. The solution obtained is based on the mathematical model of the description of such waves given by Wilde and Romańczyk (1989) and on the impulse response function for the generator fluid system. The numerical results of the theoretical model developed in this paper were compared with experimental data. Within to the Stokes' second order of expansion, the model discussed leads to accurate results in the stochastic sense. The theoretical solution to the problem on hand was derived in two steps. In the first step, the linear solution was constructed by means of the linear transformation of the generator motion process into the free surface elevation process. Then, in the second step, the linear component of the surface wave was used in calculating the additional, non-linear component of the wave. In this way, the Stokes' type wave with two components was created which, in another formulation, would need a solution to the non-linear transformation of the stochastic processes involved. From the records of the surface wave shown in Fig. 3 (E) it can be seen that the crests of the wave components propagate with the same phase and thus are solely components of the Stokes' wave. The first component of the solution discussed (Eq. 17) was obtained by means of the linear solution to the initial value problem of fluid motion starting from rest. The assumed time of observation (Eq. 27) was so long, that transients associated with sudden motion of the wave maker at $t = 0$ died out due to a dissipation mechanism in the system. It was therefore possible to apply the Stokes' expansion in the description of the waves generated as in the case of steady-state harmonic generation of the waves. In our case however, the latter was described by means of a narrow-band stochastic process and thus, correlation functions were used in estimating the accuracy of the theoretical model proposed.

References

Achenbach J. D. (1973): *Wave propagation in elastic solids,* North Holland.

Bendykowska G. (1980): Transformacja fali spowodowana przejściem nad prostokątnym zatopionym progiem, *Archiwum Hydrotechniki,* Vol. XXVII, No. 2, pp. 171–187.

Druet Cz. (1978): *Hydrodynamika morskich budowli i akwenów portowych,* Wyd. Morskie, Gdańsk.

Hudspeth R. T., Sulisz W. (1991): Stokes' drift in two-dimensional waves flumes, *J. Fluid Mech.,* Vol. 230, pp. 209–229.

Massel S. (1992): Mechaniczna generacja falowania wodnego w kanale hydraulicznym w świetle teorii nieliniowej, *Archiwum Hydrotechniki,* Vol. XXIX, No. 3.

Otnes R. K., Enochson L. (1979): *Analiza numeryczna szeregów czasowych,* Wyd. Nauk.-Techn, Warsaw.

Szmidt K. (1993): Impulse generation of waves in fluid of finite depth – submitted for publication in *Zeszyty Naukowe Politechniki Gdańskiej,* Gdańsk.

Szmidt K. (1994): On the impulse response function for water waves generated in fluid of constant depth – submitted for publication in *Mechanika Teoretyczna i Stosowana,* Warsaw.

Szmidt K., Androsiuk J. (1994): Przekształcenie wąskopasmowego procesu ruchu generatora w proces falowania w kanale hydraulicznym, *XI Krajowa Konferencja Mechaniki Płynów,* WAT, Warsaw, Part I, pp. 124–129.

Wilde M. (1992): *Opis programu FILTR. ST. EXE do obróbki sygnałów stacjonarnych,* Institute of Hydro-Engineering PAS, Gdańsk.

Wilde P., Kozakiewicz A. (1993): Kalman Filter Method in the Analysis of Vibrations Due to Water Waves, *Advanced Series in Ocean Engineering.*

Wilde P., Romańczyk W. (1989): The mathematical model for the analysis of Stokes' type waves, *Archiwum Hydrotechniki,* Vol. XXXVI, No. 3–4, pp. 221–231.