Archives of Hydro-Engineering and Environmental Mechanics
Vol. 42 (1995), No. 1–2, pp. 53–67

Bearing Capacity of Shallow Foundations on Slopes

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(Received April 04, 1995; revised August 21, 1995)

Abstract

In the paper the results of model tests and an analysis of calculation methods of bearing capacity of shallow foundations rested on the edge of slope are presented. Model tests were carried out in plane strain state with rigid rectangular strip foundations placed on homogeneous dry subsoil made of different kinds of material such as: natural gravel, dry sand and analogue material (the pin–model). Qualitative and quantitative results of slope inclination influence on bearing capacity are analysed. Some formulae and values of the slope inclination factor are recommended.

1. Introduction

Civil engineering structures are often forced to be constructed on slopes. Foundations are sometimes built on sloping side or near the top edge of a slope. Such cases are typical for both, the hydrotechnics and land site civil engineering.

To calculate the bearing capacity of such foundations there is a wide variety of methods available in the literature. These methods are based on both 1g and centrifuge model tests being carried out over the last 20 years. During the experiments the influence of ground surface inclination on bearing capacity reduction was checked and validation of theoretical solutions proposed was assessed.

Bearing capacity of foundations on inclined surfaces of a subsoil is not included in Polish Design Code PN–81/B–03020. For example, in Finland it is defined by the National Road Administration Code "Foundation Instructions in Bridge Design (1989)". The definition method, the same as in the German Code (1988) is based on Prandtl's theory of plasticity zones discussed by Hartikainen and Zadroga (1994). According to this method a bearing capacity of a slope with a 30° inclination decreases by as much as 82% when compared bearing capacity of a subsoil with a horizontal surface.

The main reason for studying the influence of slope inclination on the decrease of bearing capacity was to determine the real slope inclination factor for the foundation design.

2. Model Tests

2.1. Scope of Model Tests at Tampere University of Technology

To define the bearing capacity of a slope 30 independent experiments were carried out in the laboratory test pit. Two steel profiles: $B \times L = 0.15 \times 1.5$ m and 0.30×1.5 m were used as foundation models on a 1:1 scale to reflect real foundation conditions for falsework foundations being used as a temporary support during the construction of bridges. The experiments were made on dense gravel for three densities: $D_r = 85\%$, 90% and 95%. The foundations were situated both on the surface and at a depth of $D = 0.15$ m with three different inclinations of slope $\beta = 15°$, $22.5°$ and $30°$. Soil characteristics from triaxial apparatus for various densities being used are given in Table 1.

Table 1. Soil parameters for different densities

D_r [%]	ϕ [deg]	γ [kN/m^3]	E [MPa]
85	36	19.3	40.6
90	40	20.5	–
95	44	21.5	67.2

The main objective of the experiments was to study the bearing capacity of foundations. In addition the shape of slip surface, settlement of foundations and plastic flow of soil during loading were investigated. The ultimate load was studied by measuring the movements of the surface in the vicinity of the foundation and by interpreting stress–settlement curves.

2.1.1. Qualitative Test Results

The comparison of slip surfaces: measured during experiments and obtained from Prandtl's solution for foundation $D/B = 0.5$ is presented in Fig. 1. The theoretical solution corresponds very well to measurements from experiments in this case.

Fig. 2 shows slip surfaces observed from experiments performed on different slope inclinations and surface foundation on the top of slope. It can be seen that slope inclination influences the range of slip surfaces: the greater the inclination the deeper the slip surface.

When gravel is dense a brittle failure with a clear slip surface occurs. During excavation the slip surface measured coincides with Prandtl's solution in the form of a logarithmic spiral.

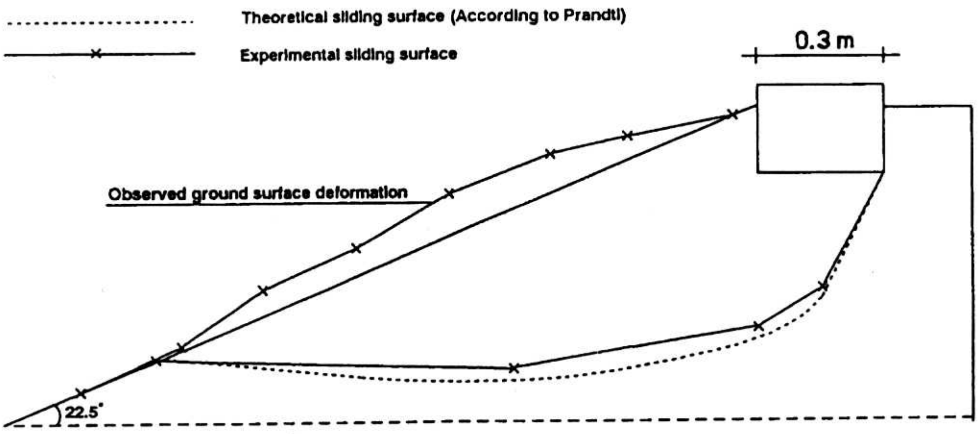

Fig. 1. Comparison of measured and calculated sliding surfaces

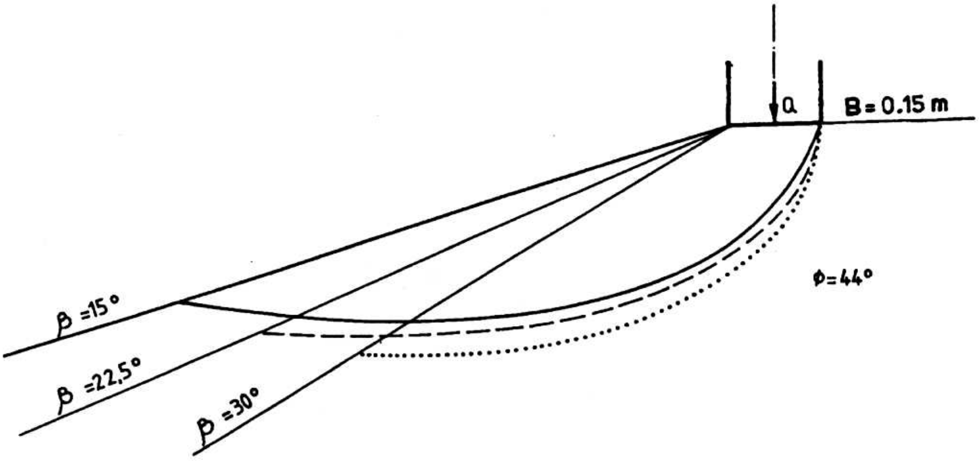

Fig. 2. Slope inclination influence on range of sliding surfaces

2.1.2. Quantitative Test Results

Typical load–settlement curves for different gravel densities, slope inclinations β and foundation characteristics B and D are presented in Fig. 3.

The ultimate load during loading was studied by measuring the movements on the surface of the surrounding ground (Fig. 1) and also by using load–settlement curves (Fig. 3).

Comparison of ultimate bearing capacity for all experiments is presented in Fig. 4. Despite some scatter of test results it can be seen that tendencies of the bearing capacity to decrease with the increase of slope inclination are similar for both surface ($D = 0$) and embedment ($D = 0.15$ m) foundations.

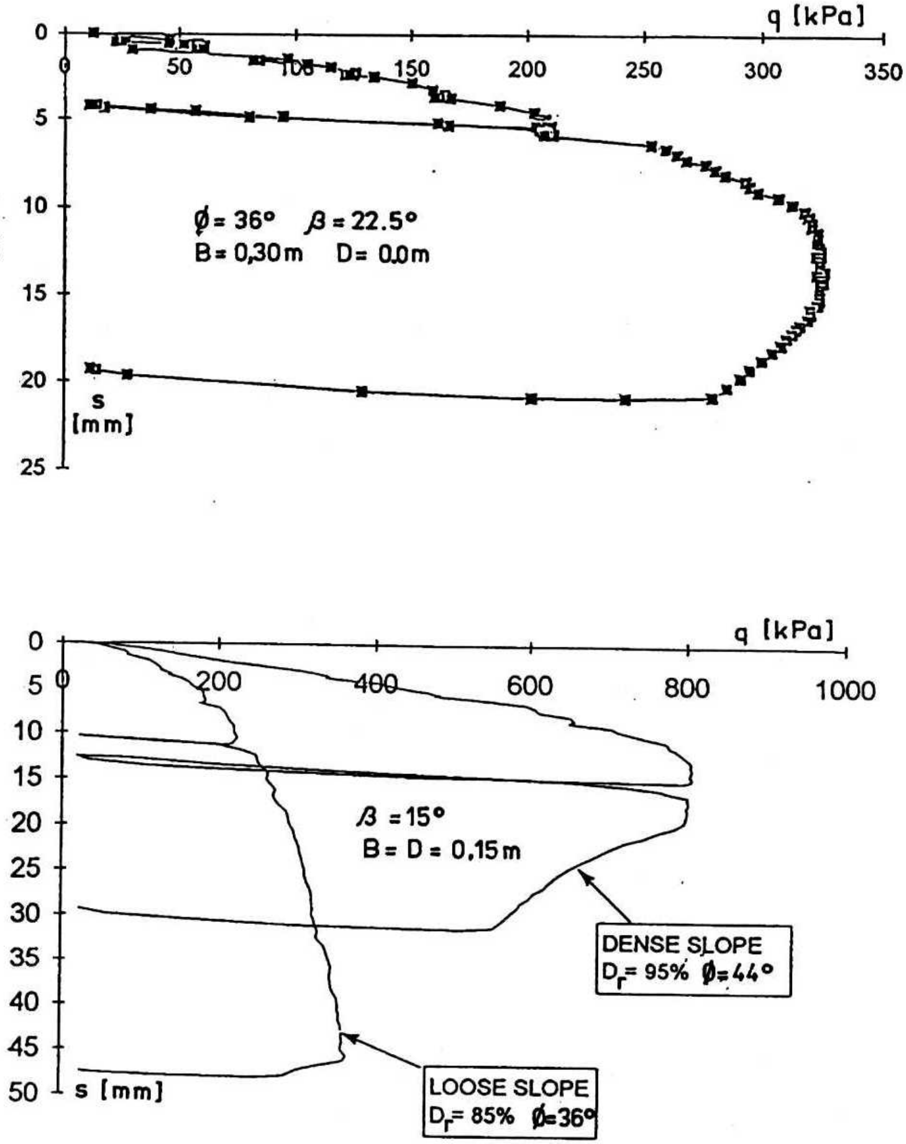

Fig. 3. Load–settlement curves obtained from experiments for loose and dense inclined subsoil

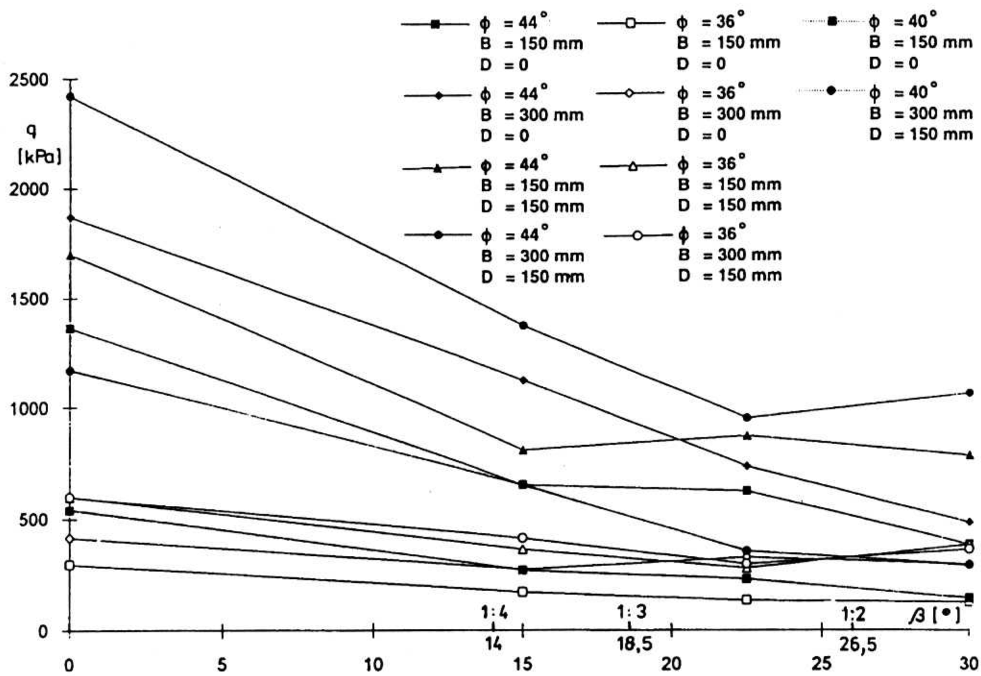

Fig. 4. Test results of ultimate bearing capacity in relation to slope inclination

2.1.3. Comparison of Model Test and Calculation Results

Test results have been compared with the results of calculations using various bearing capacity formulae. These comparisons are presented in Fig. 5.

The results of all theories applied, except Balla's theory were very much on the safe side. They took into consideration both the bearing capacity and slope inclination factors, especially those concerning steep slopes.

2.2. Scope of Model Tests Performed at Gdańsk Technical University

The goal of the large programme of model tests being carried out on both analogue (the pin–model) and natural soils was to determine:

- the ultimate bearing capacity,
- mechanism according to which an ultimate state is reached for inclined subsoil,
- shape and range of a slip surface.

The characteristics of both kinds of soils have been given in Table 2.

In the tests three steel foundations of different widths — 0.10, 0.15 and 0.20 m respectively and length of 0.5 m were used for both kinds of soils.

Experiments performed on natural soil were carried out in a box 0.5 m in width with the back wall made of steel and front wall of thick glass to observe

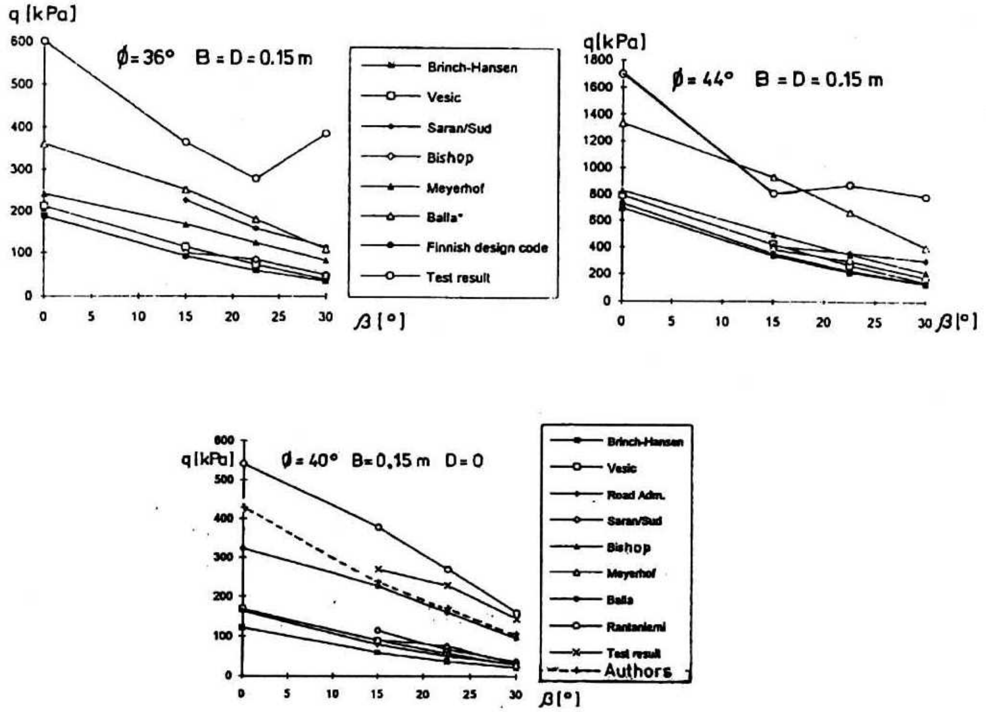

Fig. 5. Tests results in comparison to some theories and Finnish design code calculations

deformation of soil during loading. In this case plane strain conditions have been assumed. The subsoil constituted thin 10 cm layer of air–dried sand compacted by surface vibrator. Every layer was subdivided by lines of black sand located near the glass wall creating 3 cm high sublayers enabling the estimation of slip surfaces in soil. Different kinds of load were continuously applied on foundations: vertical, eccentric and inclined. The majority of experiments was carried out for foundations situated on the edge of slopes and some for foundations away from the edge for various ratios of d/B from 0 to 3.

Table 2. Soil parameters for natural and analogue soils

Kind of soil		ϕ [deg]	γ [kN/m^3]
Natural soil (dry sand)		31.5	16
Analogue soil	smooth aluminium cylinders	26	21
	sand plastered aluminium cylinders	43	15.4

2.2.1. Qualitative Test Results

A range of sliding surfaces in experiments on analogue soils for surface and embedment foundations is presented in Fig. 6.

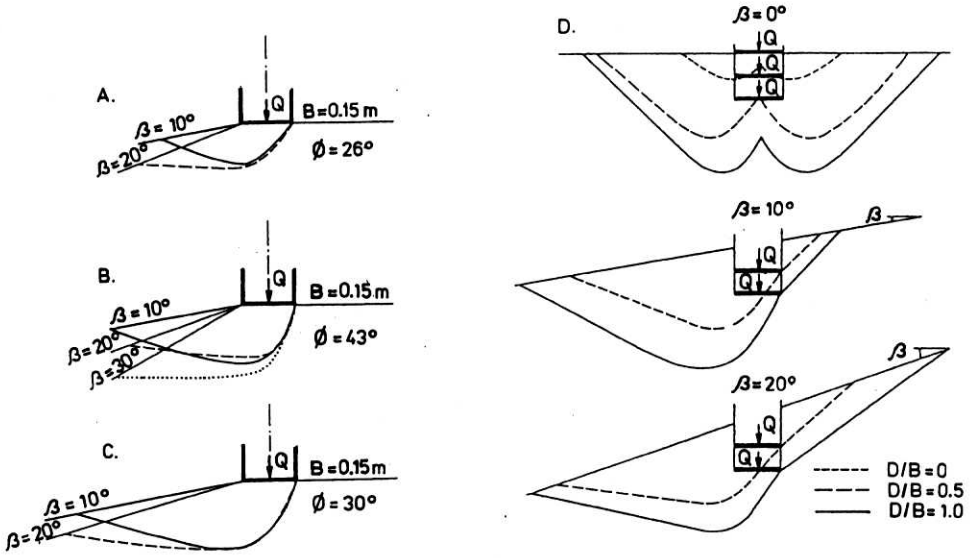

Fig. 6. Slope inclination influence on the range of sliding surfaces in analogue soils (A, B, D, authors' experiments) and in natural soil (C – Japanese experiments)

The conclusions resulting from these experiments are as follows:

- slip surfaces gradually tend to become asymmetric as with the increase of a slope inclination in comparison with horizontal surfaces; they grow larger and are more developed towards the slope side, while on the opposite side they almost disappear,
- sliding surfaces are deeper and more developed with the increase of relative foundation depth.

Comparing the independent tests carried out in Finland, Japan and Poland one can notice similar behaviour of slip surface development during loading phase.

2.2.2. Quantitative Test Results

Typical load–settlements curves for axial and eccentric loads are presented in Fig. 7.

Analysing the results of a load–settlement curve one can distinguish its three main parts:

- almost linear section with proportional strains, for which soil behaviour can be treated as elastic; parts (a) in Fig. 7,

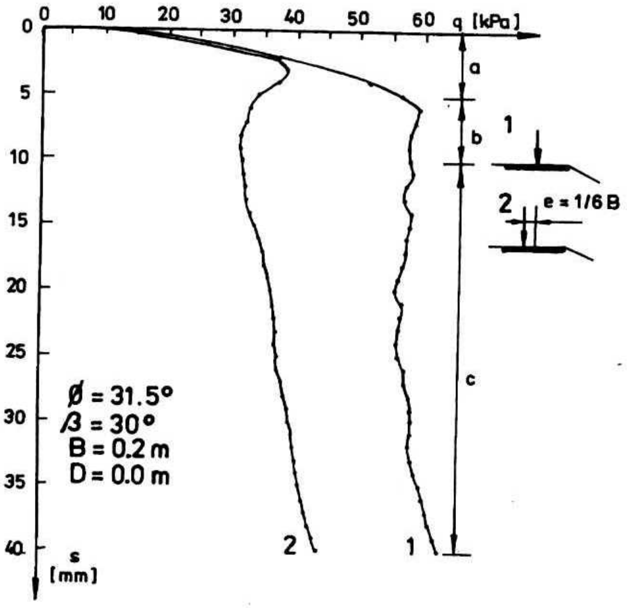

Fig. 7. Typical load–settlement curves obtained from experiments

- curved section, where plastic flow of soil begins; parts (b) in Fig. 7,
- approximately linear section corresponding to the final plastic flow; parts (c) in Fig. 7.

The curved section, where the plastic flow has begun (peak of curve) was observed for foundation settlement s/B varying from 3% to 8% for surface foundations and from 10% to 12% for embedment foundations.

It must be pointed out that only the average values of test series have been analysed due to the scatter of experimental results. Some tests were repeated two and sometimes four times.

The comparison of ultimate bearing capacity is presented in Fig. 8.

2.2.3. Comparison of Model Test and Calculation Results

The test results have also been compared with the results calculated by different bearing capacity formulae. The comparison is show graphically in Fig. 9.

The ultimate bearing capacity measured in experiments was almost two times higher than the calculated one. In the calculations the effect of friction between glass and steel walls has been included increasing the ultimate bearing capacities up to 15%.

It was also observed that the increase of slope inclination reduced the bearing capacity of the foundation as compared with the one situated on the horizontal surface. For a given slope inclination the bearing capacity value increases with the

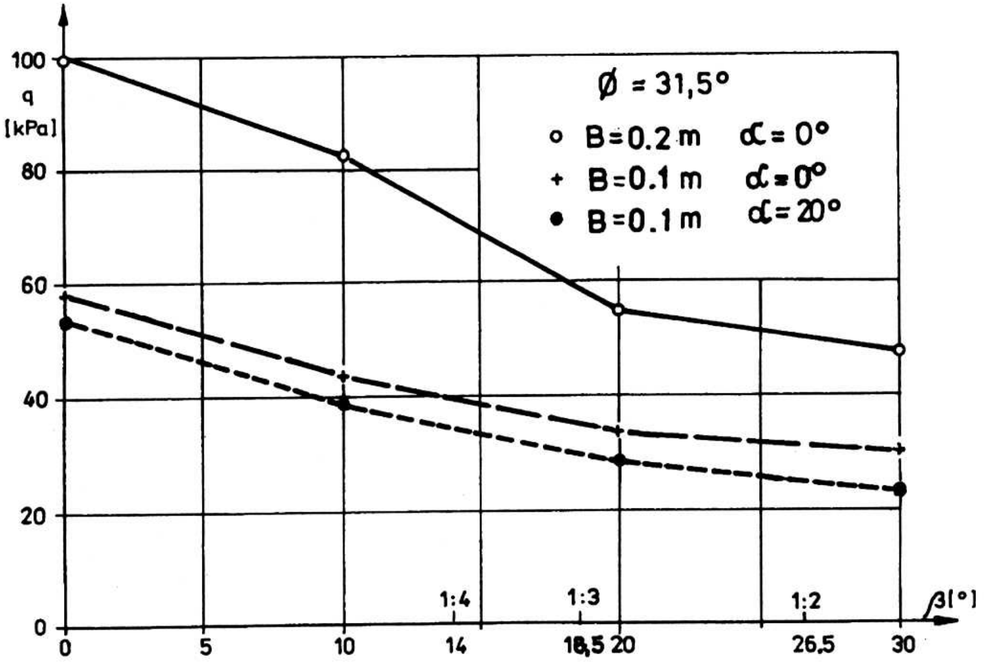

Fig. 8. Test results of ultimate bearing capacity in relation to slope inclination

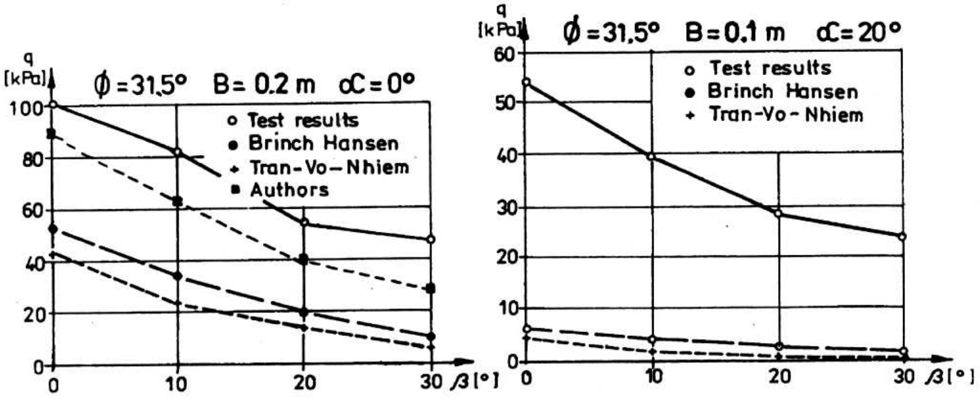

Fig. 9. Test results in comparison to some theories

increase of the relative distance of foundation from the edge of the slope. For the ratio of d/B ranging from 2.5 to 3.0 this value is identical as in the case of a horizontal surface.

3. Analysis of the Bearing Capacity Reduction Factor of Shallow Foundations on the Slope

The classical formula of bearing capacity of a foundation q resting on a slope in non-cohesive soil can be written in terms of the well-known form:

$$q = \frac{Q}{BL} = \gamma D N_q g_q + 0.5\gamma B N_B g_B, \tag{1}$$

where B and L are the width and length of foundation respectively; γ is the unit weight of soil; D is the depth of foundation, N_q and N_B are bearing capacity factors and g_q and g_B ground surface inclination factors.

The values of coefficients in Eq. (1) may differ significantly depending on the formulae recommended by various authors or particular standard regulations. The bearing capacity factor N_q can be determined for all classical calculation methods by the following formula:

$$N_q = e^{\pi \tan\phi} \tan^2\left(45 + \frac{\phi}{2}\right). \tag{2}$$

However, significant differences occur for the second bearing capacity factor N_B (see Table 3).

Table 3. Comparison of different formulae of bearing capacity factor N_B

Reference		N_B formula
DIN (1988) and EUROCODE (1993)		$2(N_q - 1)\tan\phi$
Brinch–Hansen (1970), Finnish Code (1989)		$1.5(N_q - 1)\tan\phi$
American and Norwegian Rules		$2(N_q + 1)\tan\phi$
Meyerhof (1957)		$(N_q - 1)\tan(1.4\phi)$
Chen		$2(N_q + 1)\tan\phi\tan(45 + \phi/2)$
Ingra and Baecher	$L/B = 6$	$\exp(-1.646 + 0.173\phi)$
	$L/B = 1$	$\exp(-2.046 + 0.173\phi)$
Feda		$0.01\exp(\phi/4)$
Zadroga (1994)	$L/B = \infty$	$0.657\exp(0.141\phi)$
	$L/B = 1$	$0.096\exp(0.188\phi)$

The differences between model test and calculation results of the ultimate bearing capacity of shallow foundations situated on a horizontal surface are comprehensively discussed by Zadroga (1994).

Table 4. Formulae and values of factor g_B

Method	Formula	Slope inclination β					
	g_B	0°	10°	15°	20°	22.5°	30°
Brinch-Hansen (1970)	$[1 - \tan\beta]^2$	1.0	0.67	0.53	0.40	0.34	0.18
Garnier et al. (1994)	$1 - [1.8\tan\beta - 0.9(\tan\beta)^2]$	1.0	0.71	0.58	0.46	0.41	0.26
Gemperline et al. (1984)	$1 - 0.8[1 - (1 - \tan\beta)^2]$	1.0	0.74	0.63	0.52	0.47	0.34
Weiss (1973)	$[1 - 0.79\tan\beta]^2$	1.0	0.74	0.62	0.51	0.45	0.29
Finnish Code DIN 4017 (1988)	$[1 - 0.5\tan\beta]^5$	1.0	0.63	0.49	0.37	0.31	0.18

Different formulae of ground–surface inclination factors g_B and values of these factors are given in Table 4.

Comparison of Finnish and Polish model test and calculation results with regard to ground surface inclination factors g_B are presented in Tables 5 and 6.

Table 5. Values of g_B factor from Finnish model tests

Method			Slope inclination β			
			0°	15°	22.5°	30°
Model test results on gravel	$B = 0.15$ m	$\phi = 36°$	1.0	0.59	0.46	0.43
		$\phi = 40°$	1.0	0.50	0.42	0.27
		$\phi = 44°$	1.0	0.48	0.46	0.18
		mean value	1.0	0.52	0.45	0.32
	$B = 0.30$ m	$\phi = 36°$	1.0	0.66	0.79	0.72
		$\phi = 40°$	1.0	–	–	–
		$\phi = 44°$	1.0	0.60	0.39	0.26
		mean value	1.0	0.63	0.59	0.49
Brinch-Hansen (1970)			1.0	0.53	0.34	0.18
Garnier et al. (1994)			1.0	0.58	0.41	0.26
Gemperline et al. (1984)			1.0	0.63	0.47	0.34
Weiss (1973)			1.0	0.62	0.45	0.29
Vesic (1973)			1.0	0.54	0.34	0.18
Meyerhof (1957)			1.0	0.53	0.32	0.17
Balla (1962) – estimated			1.0	0.70	0.50	0.30
Finnish Road Administration Code and DIN 4017 (1988)			1.0	0.49	0.31	0.18
Authors' recommendations			**1.0**	**0.56**	**0.40**	**0.27**

Table 6. Values of factor g_B from Polish model tests

Method			Slope inclination β			
			0°	10°	20°	30°
Model test results on sand	$\phi = 31.5°$ $\gamma = 16.0$ kN/m^3	$B = 0.20$ m	1.0	0.82	0.55	0.47
		$B = 0.15$ m	1.0	–	–	0.30
		$B = 0.10$ m	1.0	0.75	0.58	0.51
Brinch-Hansen (1970)			1.0	0.67	0.40	0.18
Garnier et al. (1994)			1.0	0.71	0.46	0.26
Gemperline et al. (1984)			1.0	0.74	0.52	0.34
Weiss (1973)			1.0	0.74	0.51	0.29
Finnish Road Administration Code and DIN 4017 (1988)			1.0	0.63	0.37	0.18
Kowalew (1964)			1.0	0.64	0.38	0.18
Meyerhof (1957)			1.0	0.64	0.40	0.18
Mizuno et al. (1960)			1.0	0.79	0.43	0.23
Tran-Vo-Nhiem (1965)			1.0	0.53	0.31	0.18
Authors' recommendations			**1.0**	**0.69**	**0.45**	**0.27**

For some calculation methods used in Tables 5 and 6 the influence of slope inclination was calculated directly in terms of bearing capacity factor N_B in Eq. (1) (Kowalew, Meyerhof, Mizuno and Tran-Vo-Nhiem).

A analysis of Finnish and Polish model tests and calculation results leads to the following conclusions:

- There is a good correlation between experimental and calculated values of ground surface inclination factors g_B for relative slope inclination $\beta/\phi < 0.6$ (the differences do not exceed 20%), while for a steep slope the values have differed over two times.
- The best correlation of experimental and calculated values of g_B was obtained for Garnier et al. (1984), Gemperline et al. (1984), Mizuno et al. (1960) and Weiss (1973) methods.

On the basis of experimental results the authors have proposed their own empirical formula for values of ground-surface inclination factors g_B:

$$\mathbf{g_B} = (1 - 0.4\tan\boldsymbol{\beta})^5. \tag{3}$$

Values of g_B factor are presented in Tables 5 and 6 and in Figure 10.

To calculate an ultimate bearing capacity of shallow foundations on a slope for non-cohesive subsoil a classical formula can be used:

$$q = \frac{Q}{BL} = 0.5\gamma B N_B g_B. \tag{4}$$

The values of N_B (Zadroga 1994) and g_B presented in Fig. 10 were obtained in terms of Eq. (3). The values recommended can be applied to the non-cohesive soils with densities and water content as in experiments presented and to surface foundations only.

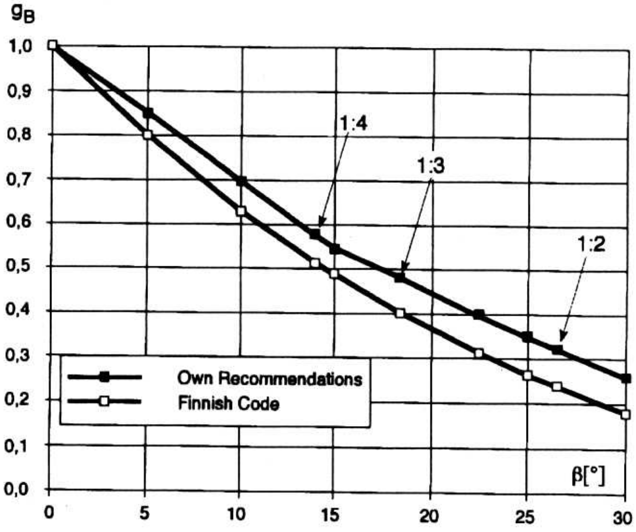

Fig. 10. Recommended values of g_B factors

4. Conclusions

The main conclusion resulting from the analysis of model test and calculations presented in this paper are as follows:

1. The values of bearing capacity of surface foundations calculated by classical methods are smaller than those measured in model tests with regard to axially and vertically loaded foundations situated near the edge of the slope in non-cohesive soil.
2. For steep slopes $\beta/\phi > 0.6$ the calculated values of ground-surface inclination factor g_B are considerably smaller (over two times) than the values obtained from experiments.
3. More realistic values of bearing capacity of shallow foundations on a slope can be obtained by applying empirical formulae of N_B and g_B factors recommended by the authors.
4. Due to the empirical character of formulae recommended it must be remembered that they concern granular soils (dry or moist sands and gravel) and shallow foundations only.

Results and conclusions presented in this paper can be a good source for potential modifications of engineering standards. It should also serve as a useful tool for engineers in choosing the appropriate bearing capacity calculation method. The extension of the investigations and analyses for granular soils with different densities and water content will enable the generalization of formulae proposed.

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