*Archives of Hydro-Engineering and Environmental Mechanics*
Vol. 42 (1995), No. 1–2, pp. 29–51

# Modelling of Bed Shear Stress under Irregular Waves

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(Received March 01, 1995; revised June 30, 1995)

**Abstract**

A model of the turbulent boundary layer under irregular (random) waves is presented. The approach incorporates a time-invariant, two-layer eddy viscosity model including the representative parameters: friction velocity and bottom boundary layer thickness. The problem is closed by the iterative scheme for finding the wave period representing the random wave field. The scheme allows to solve the task associated with the appropriate choice of the equivalent wave period and to include the coupling effects between the harmonic components, incorporated in the eddy viscosity. Good conformity between the theoretical shear stress evaluations and experimental data is obtained.

## 1. Introduction

The possibility of obtaining a mathematically simple and fairly accurate description of the rough turbulent boundary layer under irregular (random) waves deserves considerable interest, in view of the practical importance of this physical phenomenon. In natural conditions, the bed boundary layer under random sea waves is often rough turbulent, and reliable knowledge about the varying fluid velocity near the bed and the associated bed shear stress is of primary importance when estimating, for instance, the magnitude of sediment transport.

Because bottom friction, like other characteristics of the motion resulting from surface waves, is a random process, determination of a transition function between surface waves and bottom friction in the domains of time and frequency is the basic task. The determination of the quantity $\tau_{rms}$, in response to known irregular series characterized by the root-mean-square value of the velocity at the top of the bottom boundary layer, is an important practical aspect. Analogically to theoretical solutions and semi-empirical formulae (the discussion of which can be found in Kaczmarek and O'Connor (1993b)) describing the friction coefficient for regular waves, one can expect the following relationship expressing $\tau_{rms}$:

$$\tau_{rms} = f_1\left[U_r, T_r, k_a = f_2\left(\tau_r, s, d\right)\right] \tag{1}$$

where $U_r$ is the velocity amplitude at the top of the boundary layer for sinusoidal waves – representative in the description of spectral waves, $T_r$ is the representative wave period, $k_a$ is the bottom roughness, $\tau_r$ is the representative bed shear stress, $s = \rho_s/\rho_w$ is the grain/water density ratio and $d$ is the diameter of bottom sediment grains. The proper definition of a representative wave is of great importance in the use of Equation (1).

Madsen et al. (1990), basing on a theoretical solution, defined the quantity $U_r$ as $U_{rms}$ (satisfying the energy equation):

$$U_{rms} = \sqrt{\sum_n U_n^2} \tag{2}$$

and

$$T_r = 2\pi \frac{\sqrt{\sum_n (U_n/\omega_n)^2}}{\sqrt{\sum_n U_n^2}} \tag{3}$$

where $U_n$ and $\omega_n$ denote respectively the amplitudes and frequencies of the components of Fourier series describing random signal.

They obtained a solution for the boundary layer flow for a wave motion specified by its directional spectrum using the linearized form of the boundary layer equations and a simple, time-invariant eddy viscosity formulation. The closure was obtained by requiring the solution to reduce, in the limit, to that of a simple harmonic wave, described by the parameters (2) and (3). However, while overcoming the problem of which equivalent periodic wave velocity amplitude to choose to represent the random field, their approach does not provide information on the appropriate choice of the equivalent wave period. They used a simplified version of the vertical eddy viscosity concept (which varies linearly with distance from the bottom) and therefore the equivalent wave period becomes a parameter to be fitted to the wave friction factor formula similar to that originally proposed by Jonsson (1966) by theoretical justification.

An alternative approach was taken by O'Connor et al. (1992), who showed that the period $T_r$ is better described by $T_{\max}$ than $T_z$ (zero-crossing). Their results for the mono-wave simulations using $H_s$, $T_{\max}$ and $H_s$, $T_z$ are shown in Figure 1. It is clear that the best comparison with the random wave model is obtained with $H_s$, $T_{\max}$ and not with the more traditional $H_s$, $T_z$ combination.

The main goal of the present study was to find a theoretical transition function between irregular surface wave motion and the bed shear stress in the domains of time and frequency and to find the formula (1). As there are two functional relationships $f_1$ and $f_2$ in Equation (1), a simplified case of motionless bottom is considered, with $k_a$ = const. Such a simplification results from the fact that the determination of $f_2$ in Eq. (1) requires the formulation of the model of boundary

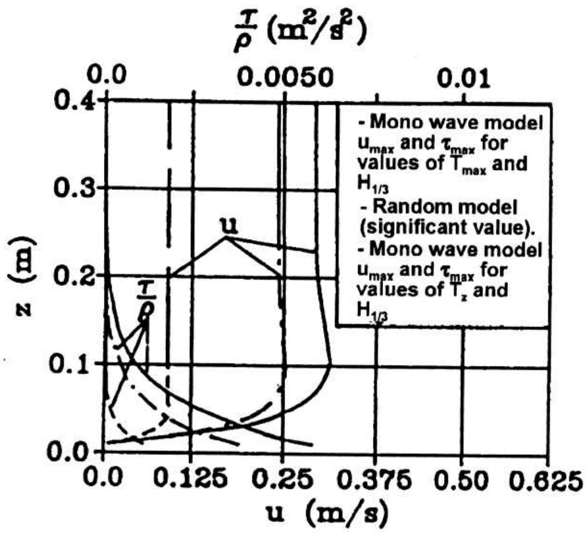



**Fig. 1.** Bed boundary layers under random wave and representative monochromatic waves, after O'Connor et al. (1992)

layer and sediment movement transport allowing for the generation of normal and tangential stresses (in the soil) by the surface waves. This is, however, very sophisticated and requires a separate analysis. It is worth referring to Kaczmarek & O'Connor (1993a, b) who took advantage of a new theoretical approach to mathematical description of moveable bed roughness under regular waves for flat and rippled bed. The iterative procedures for the determination of $f_2$ were adapted by Kaczmarek et al. (1994) for the case of irregular (random) wave and – for the description of various nonlinear effects – by Kaczmarek (1995).

To overcome the problem associated with the appriopriate choice of the equivalent wave period we start from the linear equation governing the bottom boundary layer flow. A simple time-invariant eddy viscosity formulation is still maintained to be valid, however, in contrast to Madsen's et al. (1990) study, a two-layer eddy viscosity model is proposed. Next a solution for the boundary flow is obtained for a wave motion specified by its spectrum. The problem is closed by an iterative scheme for finding the wave period representing the random wave field. The agreement between the theoretical shear stress evaluations and experimental time series supports the validity of the present spectral model.

## 2. Spectral Wave Model

### 2.1. Governing Equations

The present theory is assumed to hold in the region $z \geq z_0$, where $z = 0$ defines the theoretical bed level. If the bed is very rough, as it may be when it is covered by artificial ripples, then the theoretical bed level will lie somewhat lower than the top of the ripples, cf. Jonsson & Carlsen (1976). There seems at present to be no rule from which one can predict the magnitude of this small displacement theoretically. In practice, one determines the level $z = 0$ when fitting the observed velocity profile above the bed to a logarithmic distribution. The quantity $z_0$ will be determined from the equation:

$$z_0 = \frac{k_a}{30} \tag{4}$$

in which $k_a$ = bottom roughness parameter. The value of $k_a$ can be determined experimentally by fitting the observed velocity profile to a logarithmic profile, as shown by Jonsson & Carlsen (1976).

The basis equation of motion may be written as:

$$\frac{\partial}{\partial t}[u(z,t) - U(t)] = \frac{\partial}{\partial z}\left[\frac{\tau(z,t)}{\rho}\right] \tag{5}$$

in which $u(z,t)$ = horizontal fluid velocity, $U(t)$ = free stream velocity, i.e. at the top of boundary layer, $\tau(z,t)$ = shear stress and $\rho$ = density of water.

In Equation (5), it has been assumed that the nonlinear convective term is negligible. Moreover, it has been assumed that the boundary layer is so thin that the horizontal pressure gradient $\partial p/\partial x$ can be replaced by the term $-\rho dU/dt$ arising from potential theory.

The background and the conditions of applicability of the simplified equation of motion for the bottom boundary layer has recently been discussed by Nielsen (1992). It has been shown that the flow inside the boundary layer can often be considered to be essentially horizontal. Furthermore, in broad terms one can neglect the convective component in the equation of motion when the velocity is horizontally uniform.

The first requirement for obtaining horizontal uniformity is that the free stream velocity $U$ is uniform. This condition is fulfilled exactly over oscillating plates and in oscillatory water tunnels, while under real waves the convective term is negligibly small – as deduced by Nielsen (1992) – for small values of $a_{1m}/L$ ratio ($a_{1m}$ = amplitude of water motion at the top of boundary layer; $L$ = wave length).

The second criterion for horizontal uniformity in the boundary layer is that non-uniformities introduced by individual roughness elements should be restricted to a layer which is considerably thinner than the boundary layer itself, see Figure 2. Since the scale of the disturbances introduced by the individual roughness elements is the bed roughness $k_a$, this may be expressed by $\delta/k_a \gg 1$ which corresponds to $a_{1m}/k_a \gg 1$, as $\delta/k_a$ is an increasing function of $a_{1m}/k_a$.

Following Jonsson (1980) it is convenient to introduce the defect velocity, $u_d$, measuring the deviation from the free stream velocity:

$$u_d(z,t) = u(z,t) - U(t). \tag{6}$$

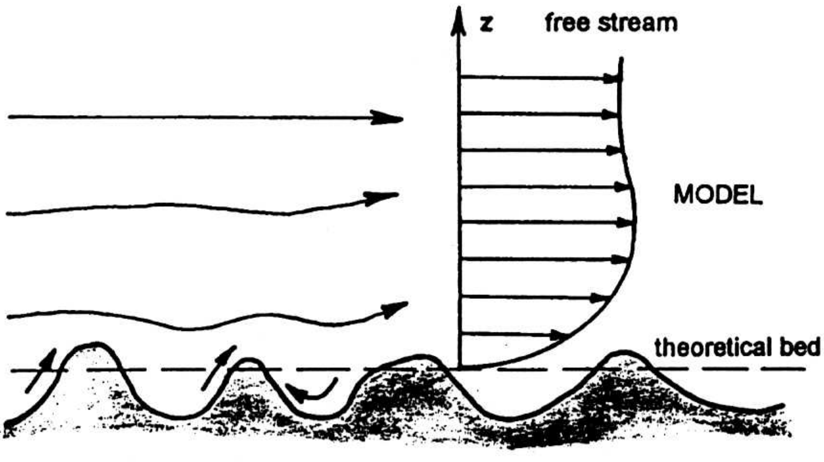



**Fig. 2.** Definition sketch

The idea of a position dependent but time independent viscosity means that one can write the shear stress as:

$$\tau(z,t) = \rho\nu_t(z)\frac{\partial u(z,t)}{\partial z} = \rho\nu_t(z)\frac{\partial u_d(z,t)}{\partial z}. \tag{7}$$

It is now convenient to introduce complex notation which determines the dependence of velocity on time and will ensure the analytical solution of the equation of motion. The free stream velocity will be written as $U(t) = U\exp(i\omega t)$, in which $U$ = real velocity amplitude. Similarly, one may write:

$$u(z,t) = u(z)e^{i\omega t} \quad \text{and} \quad u_d(z,t) = u_d(z)e^{i\omega t} \tag{8}$$

in which $u(z)$ and $u_d(z)$ are in general complex due to the phase shift relative to the free stream velocity. Equation (5) may now be written as:

$$\frac{d}{dz}\left[\nu_t(z)\frac{du_d(z)}{dz}\right] - i\omega u_d(z) = 0. \tag{9}$$

It is fortunate that this equation is explicitly solvable both when $\nu_t(z)$ varies linearly with $z$ and when it is a constant. Thus, following Brevik (1981), the two-layer eddy viscosity model is proposed:

$$\nu_t(z) = \kappa u_{fr} z \qquad \text{for} \qquad \frac{k_a}{30} < z \le \frac{\delta_r}{4} + \frac{k_a}{30} \tag{10}$$

$$\nu_t = \kappa u_{fr}\left(\frac{\delta_r}{4} + \frac{k_a}{30}\right) \qquad \text{for} \qquad z > \frac{\delta_r}{4} + \frac{k_a}{30} \tag{11}$$

where $u_{fr} = \sqrt{\tau_r/\rho}$ and $\delta_r$ are the representative friction velocity and representative bottom boundary layer thickness, respectively.

## 2.2. Solution Procedure

To solve the governing equation (9) the free stream velocity is specified as that associated with a wave spectrum, i.e.:

$$U(t) = \sum_n U_n e^{i\omega_n t} \tag{12}$$

in which the index $n$ denotes summation over frequencies. With such a representation of $U(t)$ the velocity amplitudes $U_n$ are related to the near-bottom orbital velocity spectrum and to the surface amplitude spectrum through:

$$U_n = \sqrt{2S_U(\omega_n)d\omega} = \frac{\omega_n}{\sinh(k_n h)}\sqrt{2S_\eta(\omega_n)d\omega} \tag{13}$$

in which $\omega_n$ and $k_n$ are related to each other by linear dispersion relationship.

The linearity of Equation (9) combined with the assumed time-invariant eddy viscosity concept (10) and (11) suggests a solution in the form of:

$$u(z,t) = \sum_n u_n(z)e^{i\omega t} \tag{14}$$

in which $u_n(z)$ represents the complex velocity component amplitudes and only the real part of Equation (14) constitutes the solution sought.

Introducing Equations (10) and (11) into Equation (9), one can obtain the equation for each velocity component $n$. Introducing the dimensionless variable:

$$\xi_n = \left(4\omega_n \frac{z}{\kappa u_{fr}}\right)^{\frac{1}{2}} \tag{15}$$

for the overlap layer, in which $\nu_t$ is given by Equation (10), it turns out that Equation (9) reduces to the standard differential equation for the Kelvin functions of zeroth order, with $\xi_n$ as the independent variable. It is convenient, after Brevik (1981), to write the solution as:

$$u_{dn} = U_n D_n^{-1}[A_n(\mathrm{ber}\xi_n + i\,\mathrm{bei}\xi_n) + B_n(\mathrm{ker}\xi_n + i\,\mathrm{kei}\xi_n)] \tag{16}$$

in which $D_n$ is a constant quantity defined by:

$$\begin{aligned} D_n = {} & [(\mathrm{ber}_1\xi_{\Delta n} - \mathrm{bei}\xi_{\Delta n})\mathrm{ker}\xi_{0n} - (\mathrm{bei}_1\xi_{\Delta n} + \mathrm{ber}\xi_{\Delta n})\mathrm{kei}\xi_{0n} + \\ & -(\mathrm{ker}_1\xi_{\Delta n} - \mathrm{kei}\xi_{\Delta n})\mathrm{ber}\xi_{0n} + (\mathrm{kei}_1\xi_{\Delta n} + \mathrm{ker}\xi_{\Delta n})\mathrm{bei}\xi_{0n}] + \\ & +i[(\mathrm{bei}_1\xi_{\Delta n} + \mathrm{ber}\xi_{\Delta n})\mathrm{ker}\xi_{0n} + (\mathrm{ber}_1\xi_{\Delta n} - \mathrm{bei}\xi_{\Delta n})\mathrm{kei}\xi_{0n} + \\ & -(\mathrm{kei}_1\xi_{\Delta n} + \mathrm{ker}\xi_{\Delta n})\mathrm{ber}\xi_{0n} - (\mathrm{ker}_1\xi_{\Delta n} - \mathrm{kei}\xi_{\Delta n})\mathrm{bei}\xi_{0n}]. \end{aligned} \tag{17}$$

In the above expression, $\mathrm{ber}_1$, $\mathrm{bei}_1$, etc. = first order Kelvin functions. The formulae for $\xi_{0n}$ and $\xi_{\Delta n}$ are given by Equation (15) with $z = z_0$ and $z = \Delta_r$, respectively, while $\Delta_r = \delta_r/4 + k_a/30$. The quantities $A_n$ and $B_n$ are dimensionless constants, in general complex, that have to be determined from the boundary conditions.

The solution of Equation (9) in the outer layer, in which $\nu_t$ is given by Equation (11), reads:

$$u_{dn} = -U_n T_n D_n^{-1} \exp[-(1+i)\beta_n(z-\Delta_r)] \tag{18}$$

in which:

$$\beta_n = \left(\frac{\omega_n}{2\kappa u_{fr}\Delta_r}\right)^{\frac{1}{2}}. \tag{19}$$

Further, $T_n$ is a new constant, in general complex. In writing Equation (18), use has been made of the requirement that $u_d \to 0$ when $z \to \infty$. The minus sign in front of the equation has been added for convenience, as the physical defect velocity is usually negative.

The boundary condition:

$$u_n(z_0) = 0\,; \quad u_{dn}(z_0) = -U_n$$

and the continuity of $u_{dn}$ and shear stress at $z = \Delta_r$ yields:

$$A_n = \mathrm{ker}_1\xi_{\Delta n} - \mathrm{kei}\xi_{\Delta_n} + i(\mathrm{kei}_1\xi_{\Delta n} + \mathrm{ker}\xi_{\Delta_n}),$$

$$B_n = -\mathrm{ber}_1\xi_{\Delta n} + \mathrm{bei}\xi_{\Delta_n} - i(\mathrm{bei}_1\xi_{\Delta n} + \mathrm{ber}\xi_{\Delta_n}),$$

$$T_n = (-\mathrm{ber}_1\xi_{\Delta n}\mathrm{ker}\xi_{\Delta_n} - \mathrm{bei}_1\xi_{\Delta n}\mathrm{kei}\xi_{\Delta_n} - \mathrm{ker}_1\xi_{\Delta n}\mathrm{ber}\xi_{\Delta_n} + \mathrm{kei}_1\xi_{\Delta n}\mathrm{bei}\xi_{\Delta_n}) + \\ + i(-\mathrm{ber}_1\xi_{\Delta n}\mathrm{kei}\xi_{\Delta_n} + \mathrm{bei}_1\xi_{\Delta n}\mathrm{ker}\xi_{\Delta_n} - \mathrm{ker}_1\xi_{\Delta n}\mathrm{bei}\xi_{\Delta_n} - \mathrm{kei}_1\xi_{\Delta n}\mathrm{ber}\xi_{\Delta_n}).$$

On the basis of the velocity solution, from the expression for the bottom shear stress, Equation (7), one can obtain the shear stress at $z = z_0$ (complex value) which can be expressed as:

$$\tau(t) = \tau_n \exp[i(\omega_n t + \varphi_{\tau n})] \tag{20}$$

where $\tau_n$ and $\varphi_{\tau n}$ are the bed shear stress amplitude and phase, respectively, corresponding to $n$th harmonic component $U_n$ of the input free stream velocity random series $U(t)$.

### 2.3. Closure Scheme

The solution obtained for the turbulent flow in the wave boundary layer involves the representative friction velocity $u_{fr}$ and the representative thickness of the boundary layer $\delta_r$, which are yet to be specified. Although each harmonic component of wave motion is described by the same equation, i.e. Equation (9), there is a coupling between the components incorporated in the eddy viscosity. This will appear later in the modelling of the representative values $u_{fr}$ and $\delta_r$ by iterative procedure (Scheme 1).

The velocity solution given by Equations (16) and (18) was first proposed by Brevik (1981) for regular waves. However, he has not specified the values $u_{fr}$ and $\delta_r$, which become the parameters to be fitted by comparison of model predictions and observations. The complete numerical solution of Equation (9), with the two-layer eddy viscosity described by Equations (10) and (11), has been proposed by Kaczmarek & Ostrowski (1992a). This approach has provided good results of comparison with a wide range of laboratory tests, with a similar degree of accuracy as the other models. The method is also capable of describing the bed boundary layer under nonlinear waves, as well as nonlinear waves and currents, see Kaczmarek & Ostrowski (1992b, c, d). It has been proposed that quantities $u_{fr}$ and $\delta_r$ be determined from the differential equation derived by Fredsoe (1981):

$$\frac{dz_1}{d(\omega_n t)} = \frac{30\kappa^2 U_n(\omega_n t)}{k_s \omega_n [e^{z_1}(z_1 - 1) + 1]} - \frac{z_1(e^{z_1} - z_1 - 1)}{e^{z_1}(z_1 - 1) + 1} \frac{1}{U_n} \frac{dU_n}{d(\omega_n t)} \tag{21}$$

from which the function $z_1(t)$ was obtained and the time distributions of the friction velocity $u_f$ and the boundary layer thickness $\delta(t)$ calculated thereafter on the basis of the following equations:

$$z_1 = \frac{U_n \kappa}{u_f}, \tag{22}$$

$$\delta = \frac{k_a}{30}\left(e^{z_1} - 1\right). \tag{23}$$

Further, the representative friction velocity $u_{fr}$ and thickness of the boundary layer $\delta_r$ have been specified as:

$$u_{fr} = u_{f\,\max}, \tag{24}$$

$$\delta_r = \delta_m = \max(\delta_1, \delta_2) \tag{25}$$

where $u_{f\,\max}$ is the maximum value of bed shear velocity during the wave period, that is $\max[u_f(\omega t)]$ and $\delta_1$ and $\delta_2$ the boundary layer thicknesses at the moments corresponding to maximum and minimum velocity at the top of the turbulent boundary layer.

Here, the combination of the solution given by equations (16) and (18) is proposed with the determination of the values $u_{fr}$ and $\delta_r$ given by Equations (21)–(25).

It seems to be worth checking the combination for the case of regular waves only. The results of computations are plotted in Figure 3, together with the results of the other theoretical and experimental approaches within friction factor considerations. Close agreement is found among the various theoretical models.

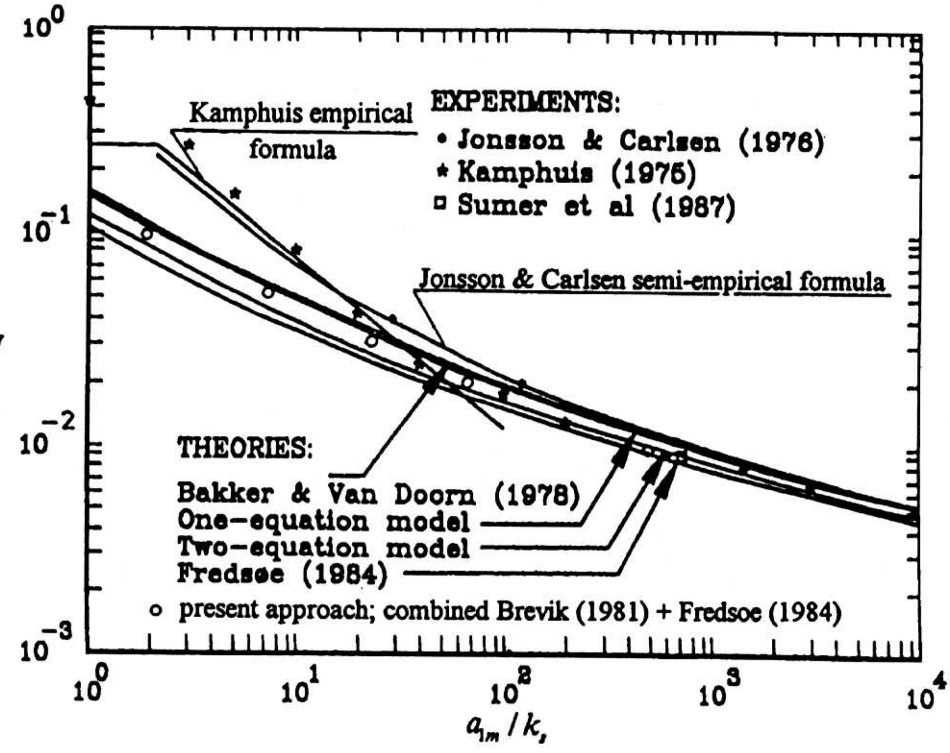



**Fig. 3.** Friction factor diagram

For the wave motion given by free stream irregular series the iterative procedure is proposed, shown in Scheme 1, to determine the representative period $T_r$, friction velocity $u_{fr}$ and the boundary layer thickness $\delta_r$, on the basis of Equations (21)–(25).

## 3. Comparison of the Theoretical Results With the Laboratory Data

### 3.1. Experimental Set-up

The measurements were carried out in the IBW PAN wave flume. The wave flume, 0.5 m wide and about 20 m long, is equipped with a programmable wave maker and can be filled with water up to 0.7 m. The reinforced concrete slabs were placed on a steel frame to create a 12 m slope. The slope of mean inclination 1:20 represents a cross-shore profile with underwater bar. The experimental set-up is sketched in Figure 4, and has been described in detail by Ostrowski (1993).

The device has been constructed to measure the bed shear stress. The moveable plate is susceptible to very small shear forces while it does not respond to any vertical load. The plate is buoyant and returns to its equilibrium when the extorting force vanishes. The device is placed in a steel casing with a square hole

Scheme 1. Computation of bed shear stress under irregular waves

[1] Fourier decomposition of the input $U(t)$

$$U(t) = \sum_n U_n \sin(n\omega t + \varphi_n) + \frac{1}{2}U_0$$

[2] Calculation of the input root mean square value:

$$U_{rms} = \sqrt{\sum_n U_n^2}$$

[3] Assumption of representative period $T_r$
[4] Determination of parameters of representative eddy viscosity distribution: $u_{fr}$ & $\delta_r$ (running Fredsoe's (1981) model with $U_{rms}$ & $T_r$ as an input)
[5] Computation of representative shear stress amplitude $\rho u_{fB}^2$ (using Brevik's (1981) approach with $U_{rms}$, $T_r$ & eddy viscosity distribution from step 4. as an input)
[6] Computation of bed shear stress components $\tau_n$ & $\varphi_{\tau n}$ using Brevik's approach with $U_n$, $n\omega$ (from step 1.) and representative eddy viscosity (determined in step 4.) as an input
[7] Calculation of bed shear stress root mean square value:

$$\tau_{rms} = \sqrt{\sum_n \tau_n^2}$$

[8] Checking whether $\rho u_{fB}^2$ (step 5.) = $\tau_{rms}$ (step 7.)

if NO ⟶ correction of $T_r$ and going to step 4.
if YES ⟶ going to step 9.

[9] Calculation of output time series (bed shear stress):

$$\tau(t) = \sum_n \tau_n \sin(n\omega t + \varphi_n + \varphi_{\tau n})$$

[10] Computation of input and output spectra (optionally)

on top, uncovering the plate active surface, i.e. the surface on which the shear stresses act. These stresses cause a plate displacement proportional to the inducing shear force. The relationship between the displacement of the plate and the shear stresses was determined using static and dynamic calibration, taking into account such effects as inertia, damping and resonance. Discussion on the above can be found in Ostrowski (1993).

The measuring device was located in the bottom at a depth of 0.385 m, between the slope toe and the offshore bar, i.e. in the zone where waves were subject to transformation without breaking. The active surface of the moveable plate coincided with the bed surface. The video taping of the plate displacement with a frequency of 50 frames per second was facilitated by provision of a glass wall section in the flume. The video data is processed thereafter, providing the bed shear stress series of 0.1 s interval between samples. The free surface elevation is

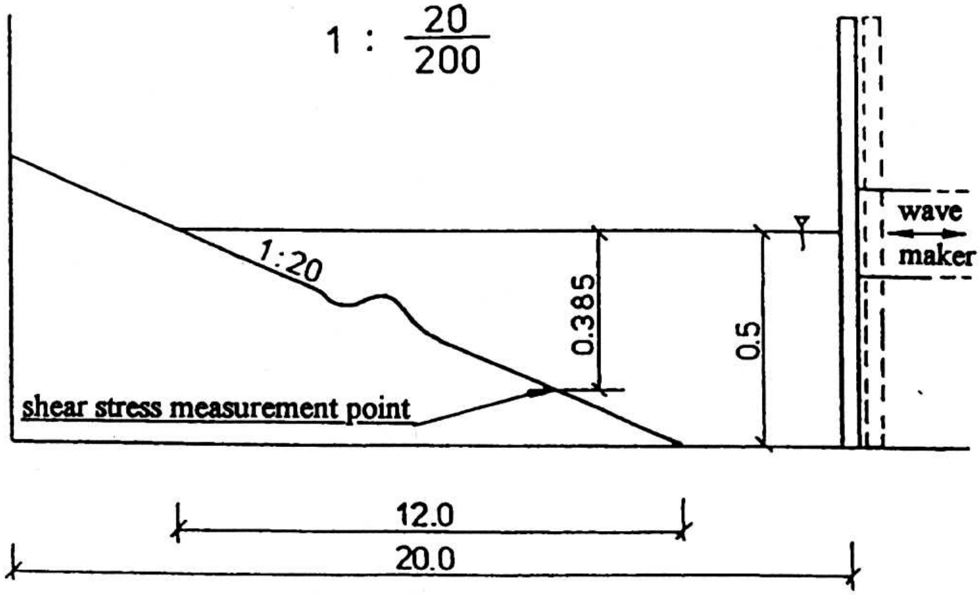



**Fig. 4.** Experimental setup

also registered, using a wave gauge, as well as the horizontal velocity component of orbital wave motion, using a micro-propeller. The latter is measured at a number of points at the measurement section, with the lower-most point located 0.013 m above the bed. The velocity series at this point has been assumed to represent input free stream velocity at the top of the boundary layer.

### 3.2. Time Series Results

The bed shear stress and free stream velocity series were measured for three irregular wave tests of the parameters given in Table 1.

**Table 1.** Irregular wave parameters at the measurement section (depth $h = 0.385$ m, registration period 300 s)

| Test | $H_s$ [m] | $T_p$ [s] |
|---|---|---|
| 1 | 0.125 | 1.5 |
| 2 | 0.189 | 2.0 |
| 3 | 0.168 | 1.5 |

Taking the free stream velocity as an input, the bed shear stress series were computed using the procedure presented in Scheme 1. The fragmentary results of computations, together with the free stream velocity series, for Tests Nos. 1, 2 and 3 are shown in Figures 5, 6 and 7, respectively. The time lag of free stream velocity with respect to shear stress is visible in all cases. This phase shift amounts to about 30° and corresponds fairly well to the quantities measured/computed for the case of turbulent boundary layer under monochromatic waves. The high frequency fluctuations of computed bed shear stress reflect small scale fluctuations of free stream velocity (less evident).

The fragmentary results of computations in comparison with measured bed shear stress for Test Nos. 1, 2 and 3 are depicted in Figures 8, 9 and 10, respectively. The conformity is good, although no secondary fluctuations of the shear stress were measured, as the measuring device damps the displacements induced by high frequency components of extorting force.

It should be noted that the representative wave period resulting from computations amounted exactly to spectral peak period ($T_r = T_p$) for all analysed tests. Furthermore, it was confirmed by computations that the representative period does not depend on assumed bed roughness height $k_a$.

### 3.3. Spectral Characteristics

Both the free stream velocity (on the basis of which the shear stress was computed) and the measured shear stress were registered in the period of 300 s. The sampling intervals for measured free stream velocity (input) and measured shear stress (output) were $\Delta t = 0.05$ s. and $\Delta t = 0.1$ s, respectively. Thus the number of samples was $N = 6000$ and $N = 3000$, respectively. However, the parameters of spectral computations were chosen so as to provide the same frequency band width for theoretical results and laboratory data.

The spectral density functions were determined for computed and measured bed shear stress series. The results are depicted in Figure 11. The conformity is good for Tests 1 & 3, satisfactory – for Test 2.

The bed shear stress $\tau$, as a response to the free stream velocity $U$, represents a physical system in which the velocity and the stress are output and input series, respectively. Basic information on the spectral characteristics of the input-output system is provided by the transmittance function:

$$H_{U,\tau}(\omega) = |H_{U,\tau}(\omega)| \exp[-i\varphi_H(\omega)] \tag{26}$$

in which $|H_{U,\tau}(\omega)|$ = amplification factor (transmittance function modulus), $\varphi_H(\omega)$ = phase factor. It is not possible to compute a cross-spectrum for measured velocity and shear stress (necessary for the determination of complex transmittance $H_{U,\tau}(\omega)$), as two different techniques were employed to register the velocity (input) and shear stress (output), while the amplification factor $|H_{U,\tau}(\omega)|$ can be determined on the basis of input and output spectra, $G_U(\omega)$ and $G_\tau(\omega)$, using the following formula:

$$|H_{U,\tau}(\omega)| = \left(\frac{G_\tau(\omega)}{G_U(\omega)}\right)^{\frac{1}{2}} \tag{27}$$

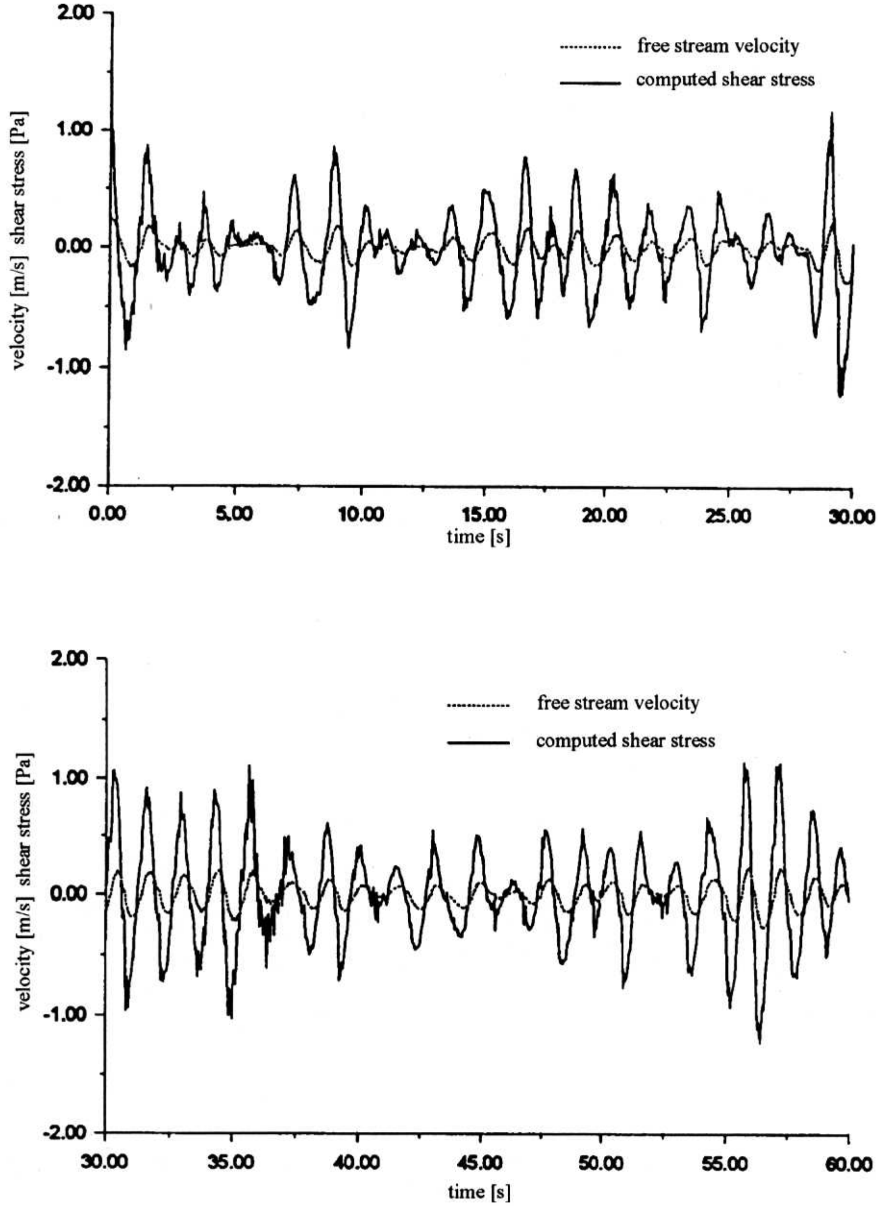
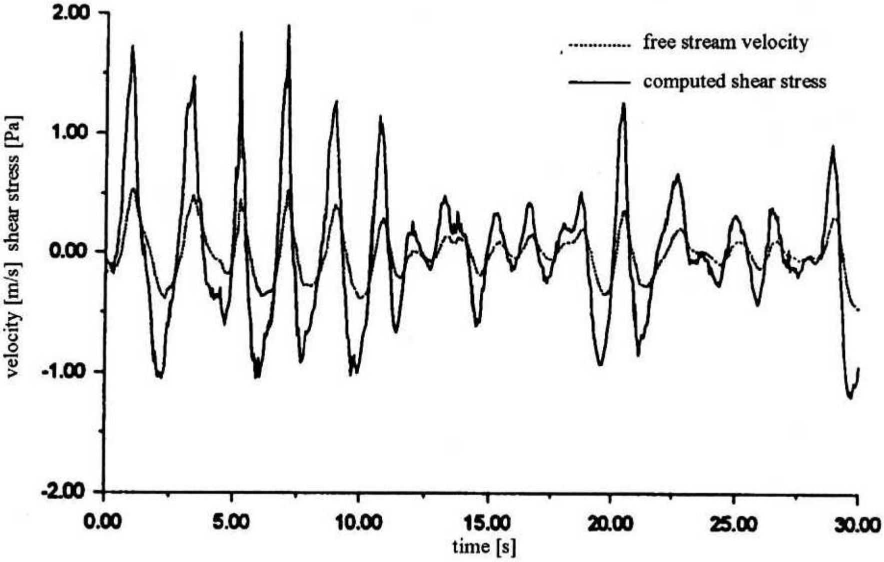



**Fig. 5.** Measured free stream velocity and computed bottom shear stress under irregular waves (parameters of Test No. 1, Ostrowski 1993)



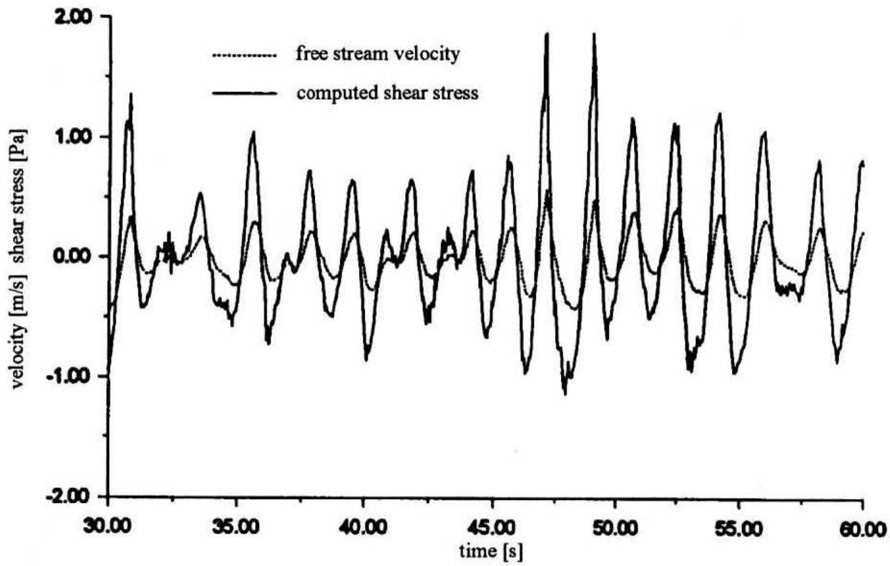




**Fig. 6.** Measured free stream velocity and computed bottom shear stress under irregular waves (parameters of Test No. 2, Ostrowski 1993)

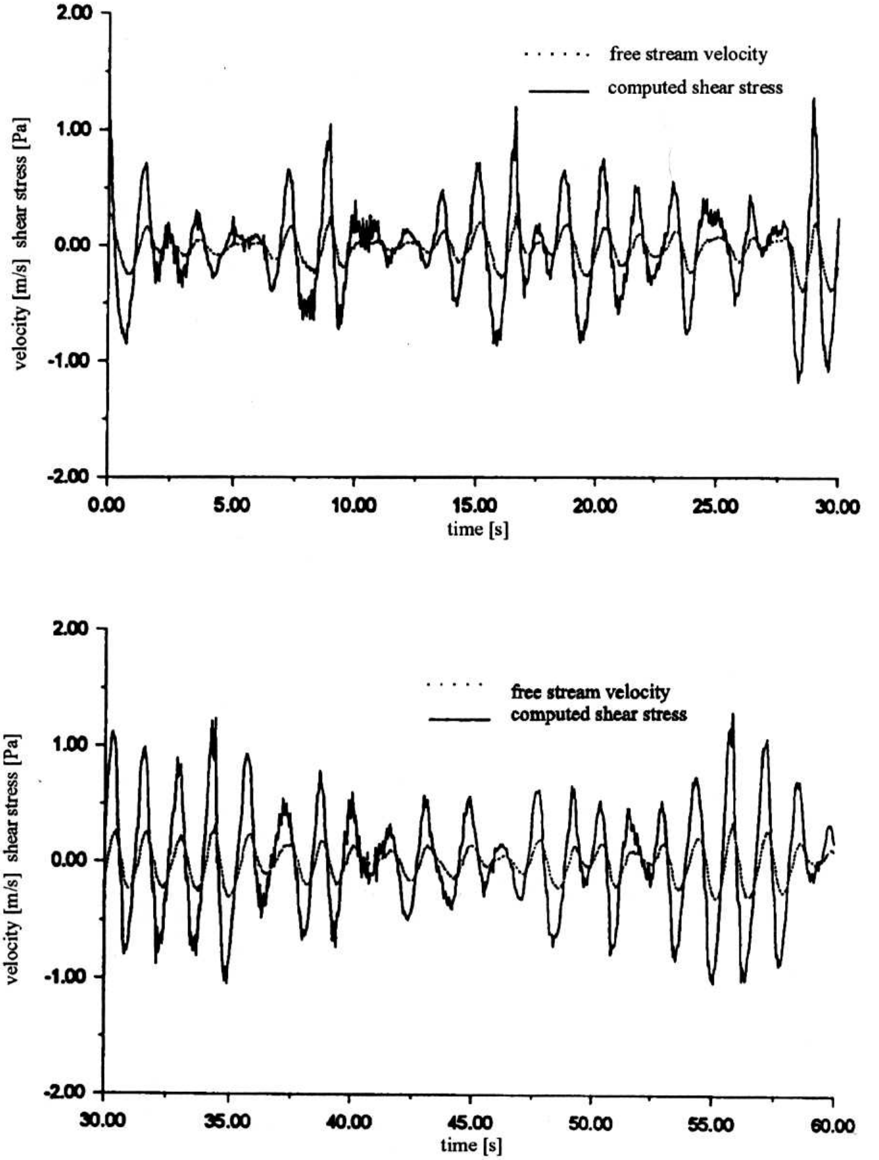



**Fig. 7.** Measured free stream velocity and computed bottom shear stress under irregular waves (parameters of Test No. 3, Ostrowski 1993)

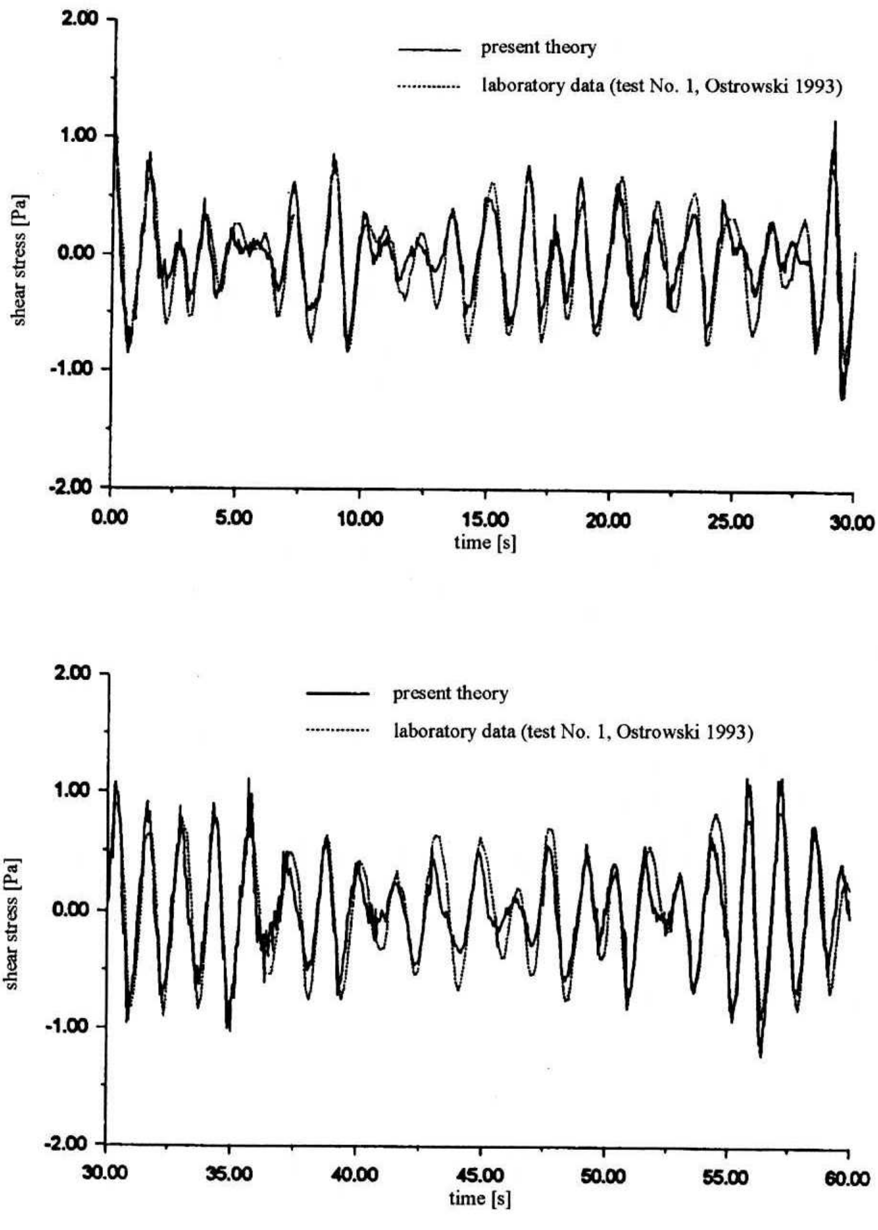



**Fig. 8.** Computed and measured bottom shear stress under irregular waves (parameters of Test No. 1, Ostrowski 1993)

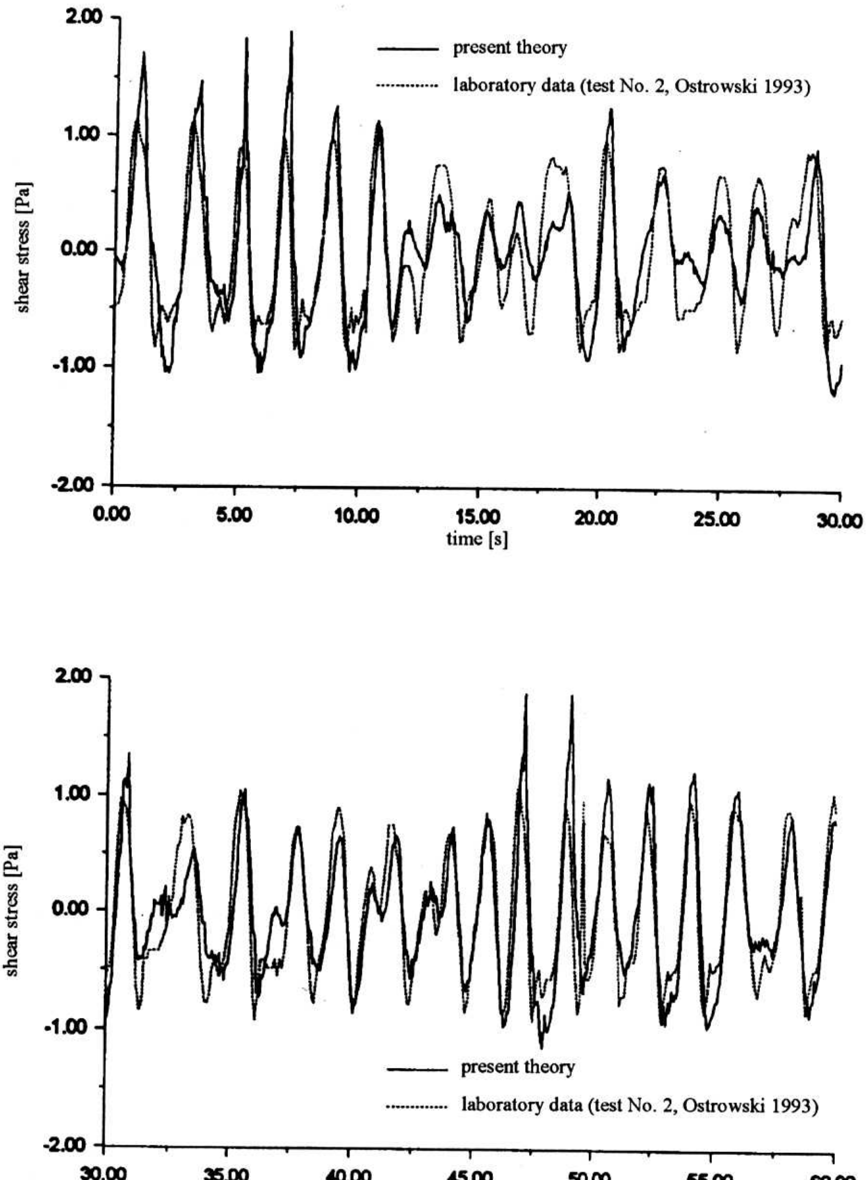



**Fig. 9.** Computed and measured bottom shear stress under irregular waves (parameters of Test No. 2, Ostrowski 1993)

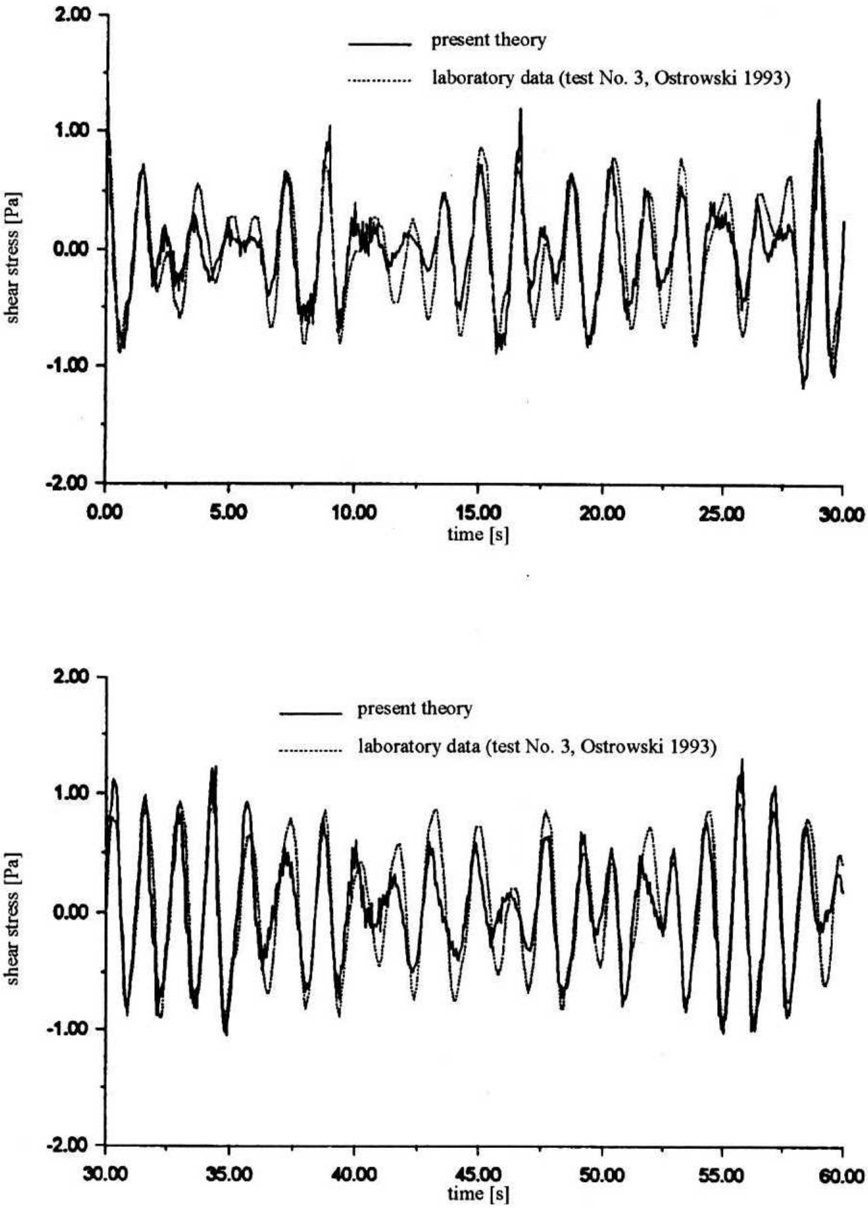



**Fig. 10.** Computed and measured bottom shear stress under irregular waves (parameters of Test No. 3, Ostrowski 1993)

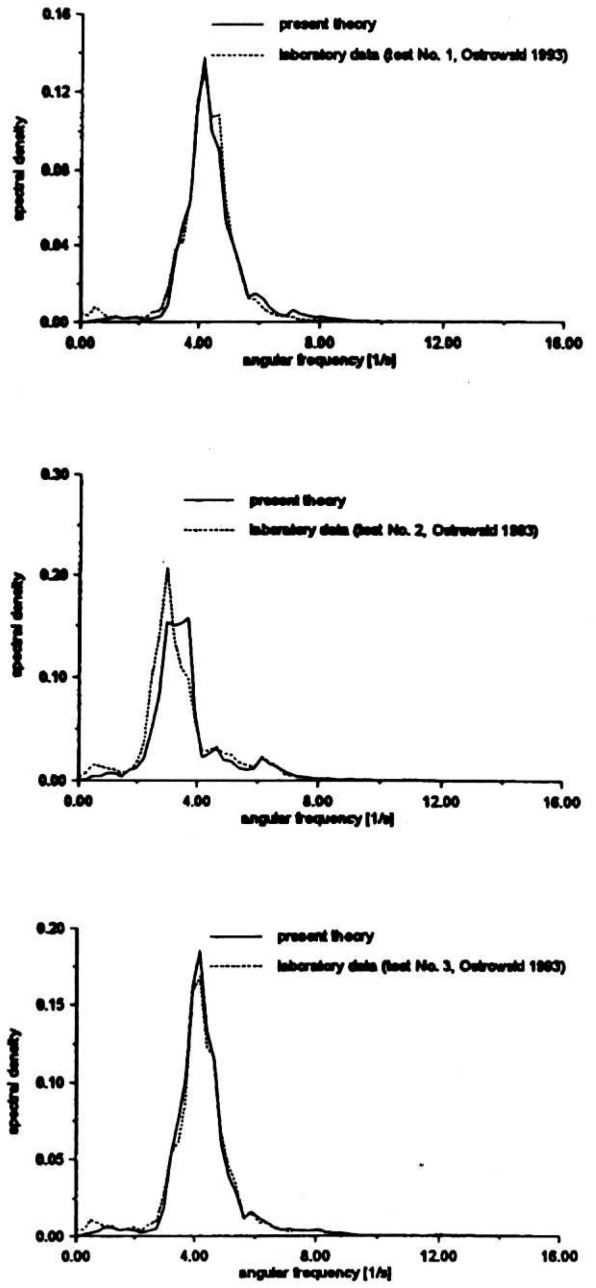



**Fig. 11.** Bottom shear stress spectra for computed and measured shear stress (parameters of Tests Nos. 1, 2 and 3, Ostrowski 1993)

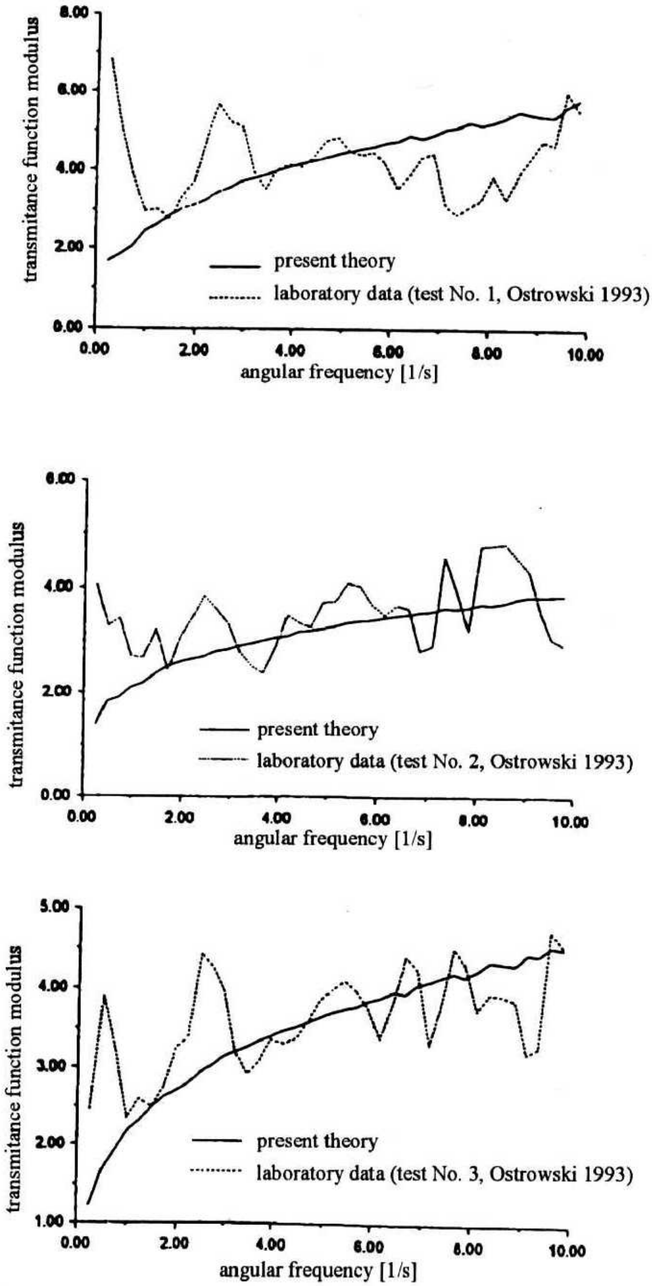



**Fig. 12.** Transmittance function modulus for bed shear stress and measured free stream velocity (parameters of Tests Nos. 1, 2 and 3, Ostrowski 1993)

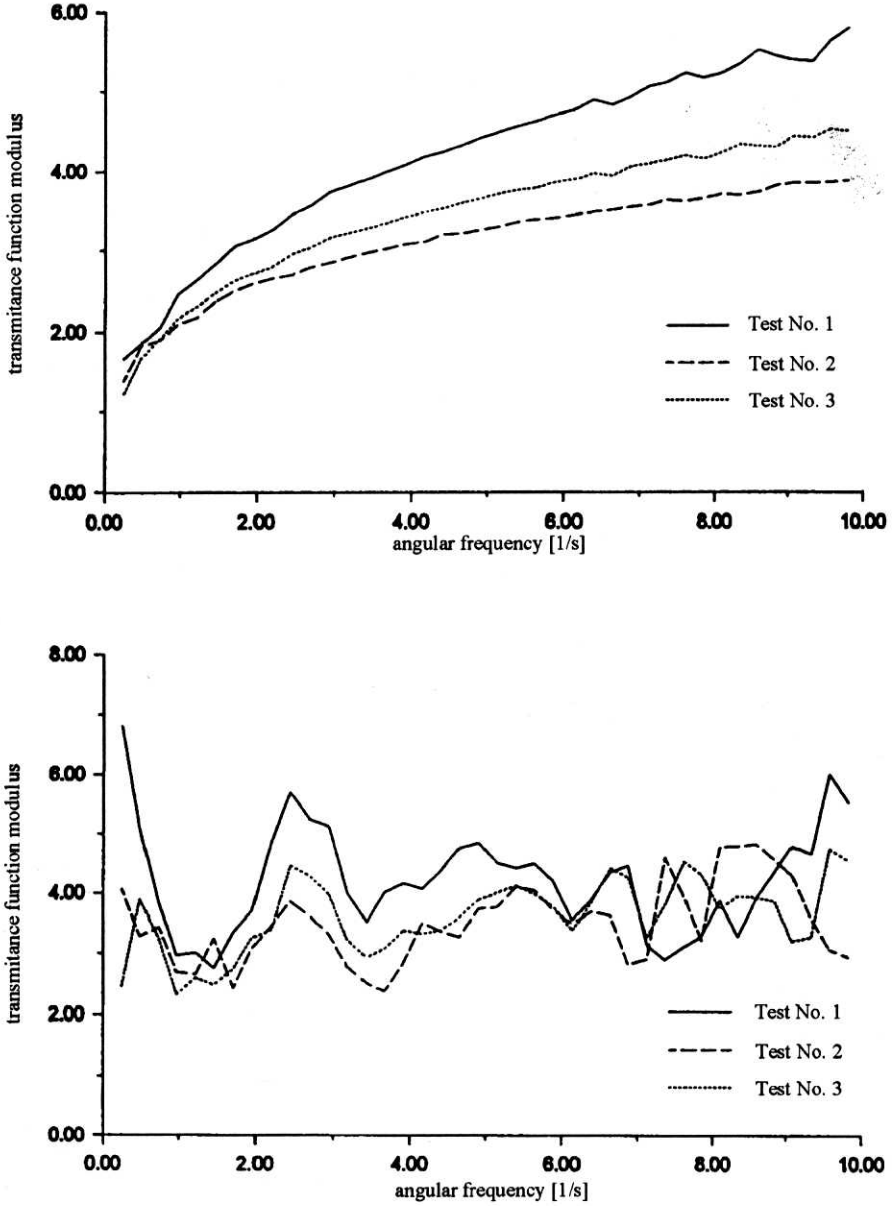



**Fig. 13.** Transmittance function moduli with computed (top) and measured (bottom) bed shear stress as the output series
(parameters of Tests Nos. 1, 2 and 3, Ostrowski 1993)

taking either computed or measured shear stress series to calculate the output spectrum.

Computed transmittance function moduli $H_{U,\tau}(\omega)$, with the output measured and determined using the present theory, are shown in Figure 12. The full set of transmittance curves is depicted in Figure 13. The experimental curves are not as smooth as their theoretical counterparts but they oscillate about certain constant values, while theoretical functions increase distinctly. Both the theoretical and experimental results in Fig. 13 show that the transmittance function modulus depends on the parameters of an experiment (wave conditions). This implies the nonlinearity of the system: velocity $\longrightarrow$ shear stress. The transmittance of the linear system would be only a function of frequency $\omega$ and would not depend on the parameters of input series.

## 4. Summary and Conclusions

The main goal of the present study was to give a mathematically simple and fairly accurate description of the rough turbulent boundary layer under irregular (random) waves, ensuring the formulation of a theoretical transition function between irregular surface wave motion and the bed shear stress in the domains of time and frequency.

To obtain the above we started from the linear equation governing the bottom boundary layer flow with a simple time-invariant eddy viscosity model. However, in contrast to Madsen's et al. (1990) study, a two-layer eddy viscosity formulation was proposed. Next, a solution for the boundary flow was obtained for a wave motion specified by its irregular series. The Fourier decomposition was used.

The problem has been closed by the iterative scheme for finding the wave period representing the random wave field. The scheme affords a solution to the task associated with the appriopriate choice of the equivalent wave period and includes the coupling effects between the harmonic components, incorporated in the eddy viscosity.

Conformity of theoretical shear stress evaluations and experimental data which supports the validity of the present model was obtained.

For the range of the available experimental data the representative period was found to be equal to $T_p$ (spectral peak period), irrespective of the bed roughness $k_a$.

## Acknowledgements

The study was sponsored by KBN and PAN, Poland, under programme 2 IBW PAN, which is hereby gratefully acknowledged. The Authors wish to thank Prof. R. Zeidler for helpful suggestions throughout the study.

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