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Stochastic FEM in Soil Mechanics
Part I
Discretization of Random Fields

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Abstract

The paper is devoted to discretization of continuous random fields in stochastic FEM used in soil mechanics. General formulas were derived to obtain variances and covariances of random field rectangular local averages. A numerical routine and approximation formulas to compute these parameters for analytically unintegrable random field covariance functions are described. Approximation of the variance of non-rectangular local averages is also depicted.

1. Introduction

One of the modern tools used nowadays in the analysis of random media is the stochastic FEM in which the results are in the form of parameters of random fields such as mean values, variances etc. The stochastic FEM is becoming increasingly popular in many branches of structural analysis. Although the method exhibits some similarities in all applications, each of them is specific, because the tasks for which we use it differ. Thus, the mathematical description of each issue is usually different. This paper is devoted to some specific features of application of the stochastic FEM in soil mechanics and is divided into two separate parts. In the first one, which forms this publication, attention is concentrated on some aspects of discretization of continuous random fields. In the second, which will be published next, focus is on problems of application in the analysis of a soil stratum treated as a random field.

2. Fundamentals of Monte Carlo Simulation for FEM in Soils

In general, there are two main groups of methods adapted to stochastic FEM. The first approach known as the perturbation method (Shinozuka, Yamazaki 1988) may not always be applied in soil mechanics, because it assumes only small random fluctuations of variables, which is rarely true for soil properties. What is more

important, it is difficult to apply as it needs a huge capacity of computers to calculate matrix derivatives of random fluctuations. For these two reasons the perturbation method should not be recommended for two–dimensional spaces.

The second approach bases on Monte Carlo Simulation techniques (MCS). With this method it is easy to simulate two dimensional random fields and/or loads, if necessary. Fast computers can produce quite large samples in a reasonable time so the main shortcoming of all Monte Carlo techniques is no longer as strong a limitation as it used to be. MCS therefore seems to be most convenient for two–dimensional media whose properties vary as greatly as in soils.

There is also a third group of stochastic FEM techniques which were developed in the last few years. The most promising are the orthogonal series expansion (Spanos, Ghanam 1989) and weighted integral method (Takada 1990). However, their applications are confined to simple, one dimensional problems and simple correlation functions. From the theoretical point of view they could be generalized for two or three dimensions, but it seems that mathematical difficulties are too great.

For these reasons MCS is the most handy for the purposes of soil mechanics, so it is briefly described below. The concept of MCS in FEM lies in repeated solutions of the basic FEM equation:

$$\mathbf{KV} = \mathbf{F} \tag{1}$$

where:

$\mathbf{K}$ – $m \times m$ stiffness matrix,
$\mathbf{F}$ – $m \times 1$ load vector,
$\mathbf{V}$ – $m \times 1$ displacement vector.

Equation (1) can be applied to all systems without initial deformations. It is very universal, both $\mathbf{K}$ and $\mathbf{F}$ can be random simultaneously. It needs to be solved n times to permit the obtaining of the estimates of probability distribution parameters of each displacement, strain or stress λ. In general, the routine for the MCS in FEM consists of the following steps:

I. simulation of $\mathbf{K}$ and/or $\mathbf{F}$,
II. solution of (1),
III. calculation of strains and stresses (if needed),
IV. calculation of the probability distribution parameters (after n cycles I to III).

The distribution parameters can be evaluated from formulas (2) to (5):

$$\bar{\lambda} = \frac{\sum_{i=1}^{n} \lambda_i}{n} \quad \text{(mean)}, \tag{2}$$

$$\sigma = \left(\frac{\sum_{i=1}^{n} \lambda_i^2}{n} - (\bar{\lambda})^2 \right)^{\frac{1}{2}} \quad \text{(standard deviation)}, \tag{3}$$

$$\mu_3 = \frac{\sum_{i=1}^{n} \lambda_i^3}{n} - 2\bar{\lambda}\sigma^2 + (\bar{\lambda})^3,$$

$$\gamma_1 = \frac{\mu_3}{\sigma^3} \quad \text{(third central moment and skewness)}, \tag{4}$$

$$\mu_4 = \frac{\sum_{i=1}^{n} \lambda_i^4}{n} - 4\bar{\lambda} \frac{\sum_{i=1}^{n} \lambda_i^3}{n} + (\bar{\lambda})^2 \frac{\sum_{i=1}^{n} \lambda_i^2}{n} - 3(\bar{\lambda})^4,$$

$$\gamma_2 = \frac{\mu_4}{\sigma^4} \quad \text{(fourth central moment and kurtosis)}. \tag{5}$$

It is evident from the above, that it is not necessary to store the solutions of all simulations but only the sums of $\lambda, \lambda^2, \lambda^3, \lambda^4$, and higher powers if required.

3. Random Fields in Stochastic FEM

3.1. Discretization of Random Fields

The first problem that is usually encountered in stochastic FEM is pertinent discretization of continuous random fields of material properties into finite discrete random variables. This is relatively simple providing that the fields are gaussian. Obviously, random loads do not have to be gaussian since they are represented in the form of nodal forces and are generally treated as mutually uncorrelated discrete random variables of arbitrary distribution.

Random fields of material properties should be transformed into discrete, finite random variables in such a manner that their probabilistic parameters take into account the variability of correlation of the initial continuous random field within finite elements. Neglecting this usually leads to incorrect values of $\bar{\lambda}$, σ, μ_3 and μ_4. The effects of such negligence will be shown in part II.

The theory of random field local averages must be incorporated to gain the proper mapping of a random field into random variables. This is performed below for a common model of a gaussian, first order, Markovian homogeneous and isotropic random field with an exponential function of correlation decay. The field $\underline{f}(x, y)$ has the following parameters:

– mean value $\bar{f}$,

- variance σ_p^2,
- correlation function between points "1" and "2"

and

$$\rho(d) = \exp(-\beta d), \tag{6a}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}, \tag{6b}$$

where β is the parameter of correlation decay.

According to (Vanmarcke 1983) the local average of a random field in a rectangular element $l \times t$ can be expressed as a discrete random variable by the following stochastic integral:

$$\underline{f}^m = \frac{1}{lt} \int_0^l \int_0^t \underline{f}(x, y)\, dx\, dy. \tag{7}$$

The initial field is homogeneous so the mean of $\underline{f}^m$ is unchanged:

$$E\left[\underline{f}^m\right] = \frac{1}{lt} E\left[\int_0^l \int_0^t \underline{f}(x, y)\, dx\, dy\right] = \frac{1}{lt} \int_0^l \int_0^t E\left[\underline{f}(x, y)\right] dx\, dy = \overline{f}. \tag{8}$$

Its variance can be calculated using a very well known expression:

$$\sigma_u^2 = E\left[\left(\underline{f}^m\right)^2\right] - (\overline{f})^2. \tag{9}$$

Inserting (7) into (9) one obtains:

$$\sigma_u^2 = \frac{1}{l^2 t^2} E\left[\left(\int_0^l \int_0^t \underline{f}(x, y)\, dx\, dy\right)^2\right] - (\overline{f})^2.$$

Replacing the expectancy of square of the integral by the quadruple integral of the expectancy of the product of two functions, that are the same but depend upon different variables, yields:

$$\sigma_u^2 = \frac{1}{l^2 t^2} \int_0^l \int_0^t \int_0^l \int_0^t E[\underline{f}(x, y)\underline{f}(z, u)]\, dx\, dy\, du\, dz - (\overline{f})^2. \tag{10}$$

It is known that:

$$E[\underline{f}(x, y)\underline{f}(z, u)] = cov[\underline{f}(x, y)\underline{f}(z, u)] + (\overline{f})^2. \tag{11}$$

If the points $P(x, y)$ and $Q(x+u, y+z)$ for $u, z > 0$ lie in the element and the covariance between them is $g(u, z)$, we can write (10), using (11) as:

$$\sigma_u^2 = \frac{\sigma_p^2}{l^2 t^2} \int_0^l \int_0^t \left(\int_0^x \int_0^y g\, du\, dz + \int_0^x \int_0^{t-y} g\, du\, dz + \int_0^y \int_0^{l-x} g\, du\, dz + \int_0^{l-x} \int_0^{t-y} g\, du\, dz \right) dx\, dy \tag{12}$$

hence:

$$\sigma_u^2 = \frac{\sigma_p^2}{l^2 t^2} \int_0^l \int_0^t (I_1(x, y) + I_2(x, y) + I_3(x, y) + I_4(x, y))\, dx\, dy. \tag{13}$$

$I = (I_1 + I_2 + I_3 + I_4)/lt$ expresses the correlation of two variables; the first at a point $P(x, y)$ and the second averaged over the whole finite element. The function $g(u, z)$ for relations (6a, b) equals:

$$g(u, z) = \exp\left(-\beta(u^2 + z^2)^{0.5}\right) \tag{14}$$

so analytical integration of (12) is not possible.

The derivation of integral formulas for covariances of local averages $\underline{f}^m$ and $\underline{f}^n$ is quite similar and is shown below for two identical rectangles. Providing that they do not partly overlay each other, two cases can be distinguished (Fig. 1):

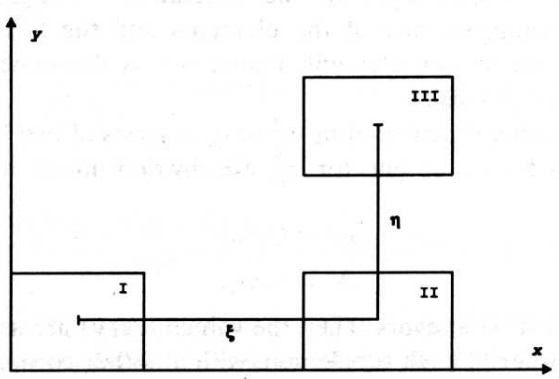

Fig. 1. Orientation of Local Averages in Stochastic FEM

I. Projections of both of them on one axis of the coordinate system fully coincide (vertical in Fig. 1). If the distance between their centroids equals ξ,

we may write the covariance as:

$$cov[I, II] = c_u = \frac{\sigma_p^2}{l^2 t^2} \int_0^l \int_0^t \left(\int_0^y \int_{\xi - x}^{\xi + l - x} g \, du \, dz + \int_0^{t-y} \int_{\xi - x}^{\xi + l - x} g \, du \, dz \right) dx \, dy \tag{15}$$

hence:

$$c_u = \frac{\sigma_p^2}{l^2 t^2} \int_0^l \int_0^t (I_1(x, y) + I_2(x, y)) \, dx \, dy. \tag{16}$$

II. Projections of local averages do not overlap at all. Thus, the projections of the distance between the centroids are ξ and η (horizontal and vertical respectively). Hence, the expression for covariance takes the form:

$$cov[I, III] = c_u = \frac{\sigma_p^2}{l^2 t^2} \int_0^l \int_0^t \int_{\xi - x}^{\xi + l - x} \int_{\eta - y}^{\eta + t - y} g \, du \, dz \, dx \, dy \tag{17}$$

so:

$$c_u = \frac{\sigma_p^2}{l^2 t^2} \int_0^l \int_0^t I_1(x, y) \, dx \, dy. \tag{18}$$

$I = (I_1 + I_2)/lt$ or $I = I_1/lt$ represent the correlation of the random field at the point $P(x, y)$ pertaining to one of the elements and the local average of the random field over the second element. Function g is the same as before so no analytical integration is feasible.

A numerical routine for calculating σ_u^2 or c_u consists of two basic steps. In the first both elements for c_u, or one for σ_u^2, are divided into rectangular subareas $\Delta l \times \Delta t$ such that:

$$\begin{aligned} \Delta l &= l/m_h, \\ \Delta t &= t/m_v. \end{aligned} \tag{19}$$

Both m_h and m_v must be integers. Then the values $I(x, y)$ are assessed as a mean covariance for a corner of each subelement with all other corners of subelements of the second (c_u) or the same (σ_u^2) element.

In the second step volume V of a solid having the base $l \times t$ and vertices $I(x, y)$ is computed. To do so a trapezium or Simpson methods can be employed. σ_u^2 or c_u are finally calculated as V/lt. Numerical experiments proved that $m_h = m_v = 20$ gives a good accuracy of results if the Simpson method is used.

3.2. Approximation of σ_u^2 and c_u

It was shown in the previous subchapter that the terms of the local average covariance matrix, for isotropic fields whose correlation function decays following (14), can not be received by analytical integration. Nevertheless, one can derive analytical approximation formulas using such a random field whose correlation function is analytically integrable. Below, the approximation formulas are derived.

One of the functions which can be integrated analytically is:

$$R(x,y) = \exp\ (-\beta_1 |x| - \beta_2 |y|)\,. \tag{20}$$

It can be a correlation function of a homogeneous but anisotropic random field. It can be presented as a product of two functions of separated variables and therefore the integration is easy.

From formulas (12), (15) and (17) we can see that c_u and σ_u^2 depend upon σ_p^2 so the approximating field, like the initial one, must have the same variance. Moreover, it must have the same mean, which is obvious, since both fields are homogeneous.

The function $g(u,z)$ for the correlation function (20) is as follows:

$$g(u,z) = \exp(-\beta_1 u - \beta_2 z)\,. \tag{21}$$

From (12), after integration we obtain:

$$\sigma_u^2 = \frac{4\sigma_p^2}{\beta_1\beta_2 lt}\left(1 - \frac{1-\exp(-\beta_1 l)}{\beta_1 l}\right)\left(1 - \frac{1-\exp(-\beta_2 t)}{\beta_2 t}\right). \tag{22}$$

In the same manner from (15) we obtained the formula for c_u for elements whose projections coincide:

$$c_u = \frac{4\sigma_p^2 \exp(-\beta_1\xi)}{\beta_1^2 l^2 \beta_2 t}(\cosh(\beta_1 l) - 1)\left(1 - \frac{1-\exp(-\beta_2 t)}{\beta_2 t}\right). \tag{23}$$

When the projections do not coincide:

$$c_u = \frac{4\sigma_p^2 \exp(-\beta_1\xi - \beta_2\eta)}{\beta_1^2\beta_2^2 l^2 t^2}(1 - \cosh(\beta_1 l))\,(1 - \cosh(\beta_2 t))\,. \tag{24}$$

The concept of approximation lies in calculation of coefficients of horizontal and vertical correlation decays β_1 and β_2. As the initial field is isotropic one can write:

$$\beta_1 = \beta_2 = \beta_R. \tag{25}$$

The value of β_R is specific for each pair of local averages. It is derived from the assumptions that relate the parameters of both random fields. For this purpose, directional scales of fluctuation and characteristic areas are computed.

The directional scale of fluctuation can be defined as (Takada 1990):

$$\delta = \int_{-\infty}^{\infty} k(r\cos\alpha, r\sin\alpha)dr, \tag{26}$$

$k(x,y)$ is the random field correlation function for which $x = r\cos\alpha$, and $y = r\sin\alpha$. The angle $\alpha \in [0,\pi]$ indicates the direction. Similarly, we define the characteristic area:

$$F = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} k(x,y)dx\,dy. \tag{27}$$

For the initial field these formulas yield:

$$\delta = 2/\beta, \tag{28}$$

$$F = 2\pi/\beta^2. \tag{29}$$

The field is isotropic so δ is direction independent. In turn, for the approximating one:

$$\delta_\alpha = 2/\beta_R(|\sin\alpha| + |\cos\alpha|), \tag{30}$$

$$F = 4/\beta_R^2. \tag{31}$$

If both scales of fluctuation are to be equal then comparing (28) and (30):

$$\beta_R = \frac{\beta}{|\sin\alpha| + |\cos\alpha|}. \tag{32}$$

It can be seen that for $\alpha = 0$ or $\alpha = \pi/2$ $\beta_R = \beta$.

If characteristic areas are assumed to be equal then:

$$\beta_R = \sqrt{\frac{2}{\pi}}\beta \approx 0.8\beta. \tag{33}$$

The variance σ_u^2 and covariances for elements having one side in common are proposed to be approximated, basing on (33), by formula (22) for σ_u^2, and (34) for c_u. It is proposed the covariances for more remote elements whose projections on the coordinate axis coincide, and elements sharing one corner, be approximated basing on (32) and (33). Both values are then averaged to obtain the approximated c_u. Where the projections coincide, covariance (23) is calculated twice, first for $\beta_R = 0.8\beta$ and next for β_r equal to the right hand side of (32), and then averaged. If in turn the corner is common, formula (24) must be similarly employed. All other covariances are received from (24) basing on (32) only, for which $|\sin\alpha| + |\cos\alpha|$ can be expressed in terms of ξ and η:

$$|\sin\alpha| + |\cos\alpha| = \frac{\xi + \eta}{\sqrt{\xi^2 + \eta^2}}. \tag{34}$$

The above assumptions result in the following approximation formulas:

– for the variance:

$$\sigma_u^2 \approx \frac{6.28\sigma_p^2}{\beta^2 l t}\left(1-\frac{1-\exp(-0.8\beta l)}{0.8\beta l}\right)\left(1-\frac{1-\exp(-0.8\beta t)}{0.8\beta t}\right) \tag{35}$$

– for the covariance when one side is common:

$$c_u = \frac{7.875\sigma_p^2\exp(-0.8\beta l)}{\beta^3 l^2 t}(\cosh(0.8\beta l)-1) \times\left(1-\frac{1-\exp(-0.8\beta t)}{0.8\beta t}\right) \tag{36a}$$

when t is common and:

$$c_u = \frac{7.875\sigma_p^2\exp(-0.8\beta t)}{\beta^3 t^2 l}(\cosh(0.8\beta t)-1) \times\left(1-\frac{1-\exp(-0.8\beta l)}{0.8\beta l}\right) \tag{36b}$$

when l is common,

– for the covariance of remote elements whose projections coincide:

$$c_u \approx \frac{3.938\sigma_p^2\exp(-\beta\xi)}{\beta^3 l^2 t}(\cosh(0.8\beta l)-1)\left(1-\frac{1-\exp(-0.8\beta t)}{0.8\beta t}\right) + \frac{2\sigma_p^2\exp(-\beta\xi)}{\beta^3 l^2 t}(\cosh(\beta l)-1)\left(1-\frac{1-\exp(-\beta t)}{\beta t}\right) \tag{37a}$$

when t coincides and:

$$c_u \approx \frac{3.938\sigma_p^2\exp(-\beta\eta)}{\beta^3 t^2 l}(\cosh(0.8\beta t)-1)\left(1-\frac{1-\exp(-0.8\beta l)}{0.8\beta l}\right) + \frac{2\sigma_p^2\exp(-\beta\eta)}{\beta^3 t^2 l}(\cosh(\beta t)-1)\left(1-\frac{1-\exp(-\beta l)}{\beta l}\right) \tag{37b}$$

when l coincides,

– for the covariance of elements whose one corner is common:

$$c_u \approx \frac{4.935\sigma_p^2\exp(-0.8\beta(l+t))}{\beta^4 l^2 t^2}(1-\cosh(0.8\beta l))(1-\cosh(-0.8\beta t)) + \frac{2\sigma_p^2\exp\left(-\beta(l^2+t^2)^{0.5}\right)}{\beta^4\frac{(l^2+t^2)^2 l^2 t^2}{(l+t)^4}}\left(1-\cosh\left(\beta l\frac{(l^2+t^2)^{0.5}}{l+t}\right)\right) \times\left(1-\cosh\left(\beta t\frac{(l^2+t^2)^{0.5}}{l+t}\right)\right) \tag{38}$$

– for all other covariances:

$$c_u \approx \frac{4\sigma_p^2 \exp\left(-\beta(\xi^2+\eta^2)^{0.5}\right)}{\beta^4 \frac{(\xi^2+\eta^2)^2 l^2 t^2}{(\xi+\eta)^4}} \left(1-\cosh\left(\beta l \frac{(\xi^2+\eta^2)^{0.5}}{\xi+\eta}\right)\right) \times \left(1-\cosh\left(\beta t \frac{(\xi^2+\eta^2)^{0.5}}{\xi+\eta}\right)\right). \tag{39}$$

To verify these approximation assumptions the calculation was carried out for a net of square elements shown in Fig. 2 and two initial random fields for which $\beta = 0.5$ and $\beta = 2$.

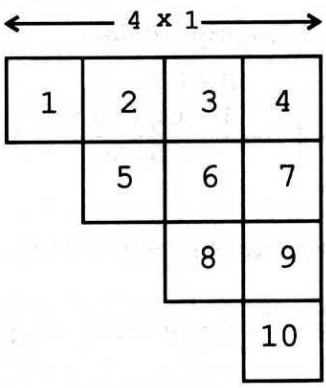

Fig. 2. Net of Square Elements for Numerical Verification of Analytical Approximations

The comparison between numerical, very accurate integration results, and approximating values of σ_u^2 and c_u computed by formulas (35) to (39) is given in Tables 1 and 2. The symbol 1-1 denotes σ_u^2, others refer to covariances of the element "1" and elements "2" to "10". These tables show that the approximation results are quite satisfactory.

Local averages over non-rectangular elements are much tougher to approximate analytically, even if the elements are triangular, for the integration limits in approximating formulas are difficult to define. Numerical routine in turn would have to be very complicated if it was to embrace local averages over elements of all different shapes.

This difficulty can relatively easily be avoided in case we want to assess only σ_u^2, for a random field averaged over a non-rectangle. The idea of this approximation consists in the replacement of such a non-rectangle with a pertinent rectangle. This rectangle should obey two conditions:

I. surface A of both elements is equal

II. the ratio k of bisectrices of the initial element:

Table 1. Results of analytical approximation
Initial random field: $\sigma_p^2 = 1,\ \beta = 0.5$

	numerical integration	analytical approximation	absolute error	relative error
1-1	0.7717	0.7726	−0.0009	0.1%
1-2	0.5904	0.5971	−0.0067	1.1%
1-3	0.3675	0.3602	0.0073	2.0%
1-4	0.2247	0.2312	−0.0065	2.9%
1-5	0.4882	0.4941	−0.0059	1.2%
1-6	0.3275	0.3346	−0.0071	2.2%
1-7	0.2073	0.2111	−0.0038	1.8%
1-8	0.2447	0.2482	−0.0036	1.4%
1-9	0.1664	0.1684	−0.0020	1.2%
1-10	0.1213	0.1224	−0.0011	0.9%

Table 2. Results of analytical approximation
Initial random field: $\sigma_p^2 = 1,\ \beta = 2.0$

	numerical integration	analytical approximation	absolute error	relative error
1-1	0.3885	0.3925	−0.0040	1.0%
1-2	0.1521	0.1559	−0.0038	2.5%
1-3	0.0253	0.0229	0.0024	9.5%*
1-4	0.0033	0.0041	−0.0080	24.0%*
1-5	0.0736	0.0747	−0.0011	1.5%
1-6	0.0149	0.0164	−0.0015	10.0%*
1-7	0.0024	0.0027	−0.0003	12.0%*
1-8	0.0047	0.0048	−0.0001	3.2%
1-9	0.0010	0.0010	$< 10^{-5}$	3.8%
1-10	0.0003	0.0003	0.0	0.0%
*high relative errors caused by c_u close to zero				

$$k = \frac{\max(a'_1, a'_2, \ldots, a'_l)}{\min(a'_1, a'_2, \ldots, a'_l)} \tag{40}$$

where l is the number of element vertices, must match the ratio of sides of the rectangle.
For these requirements the rectangle sides are as follows:

$$l_A = \sqrt{Ak}, \quad t_A = \sqrt{A/k}. \tag{41}$$

Formulas (41) were verified for local averages of triangular elements. Numerical integration for triangles and approximating rectangles were performed to compute σ_u^2. The verification was done for two different triangles and for random fields whose $\sigma_p^2 = 1$, $\beta = 0.5$, 1.0 and 2.0:

I. An equilateral triangle, whose side equals 1, is replaced by a square $l_A = t_A = 0.658$. The results were:

$\beta = 0.5$ $\sigma_u^2 = 0.8297$ (triangle),
$\sigma_u^2 = 0.8429$ (square) error 1.6%,

$\beta = 1.0$ $\sigma_u^2 = 0.6954$ (triangle),
$\sigma_u^2 = 0.7152$ (square) error 3.0%,

$\beta = 2.0$ $\sigma_u^2 = 0.4993$ (triangle),
$\sigma_u^2 = 0.5252$ (square) error 5.0%.

II. An isosceles triangle, such that $A = 1$, $\alpha = 120°$, is replaced by the rectangle $l_A = 1.564$, $t_A = 0.639$ which gave:

$\beta = 0.5$ $\sigma_u^2 = 0.7157$ (triangle),
$\sigma_u^2 = 0.7475$ (rectangle) error 4.4%,

$\beta = 1.0$ $\sigma_u^2 = 0.5335$ (triangle),
$\sigma_u^2 = 0.5714$ (rectangle) error 6.0%,

$\beta = 2.0$ $\sigma_u^2 = 0.3252$ (triangle),
$\sigma_u^2 = 0.3618$ (rectangle) error 11.0%.

The approximation, though a bit less accurate than the one from Tables 1 and 2, is fairly good. We can see that the greater the surface of the element and the weaker the random field correlation, the poorer the approximation results. In (Różyński 1992) it is shown that in order to minimize the anisotropy of rectangular local averages $l \times t$ the finite elements and the random field should obey the condition

$$\beta l \leq 0.5 \tag{42}$$

where $l \geq t$. For squares the limitation is not so strict and should be:

$$\beta l \leq 2. \tag{43}$$

Thus the maximum values of β equal:

- for the square approximating the equilateral triangle 1.32 (I),
- for the rectangle approximating the isosceles triangle 0.32 (II).

In the case of an equilateral triangle the condition (43) is satisfied for $\beta = 0.5$ and 1. However, the approximation of σ_u^2 is also good for $\beta = 2$.

In the case of an isosceles triangle the limitation (42) is not satisfied at all, but for $\beta \leq 1$ the error of approximation is not high. Thus we can see that conditions (42) and (43) which the mesh of elements should fulfil, guarantee a good approximation of σ_u^2 of non-rectangular local averages. Morever, we may intuitively predict that the accuracy should be greater for multilateral local averages like rhombs, parallelograms, hexagons etc.

4. Conclusions

I. Random fields must be discretized with local averages so as not to neglect correlations across elements.

II. Rectangular, homogeneous nets are the most convenient when the terms of covariance matrix of local averages are to be computed because of simple limits of integration.

III. Most correlation functions are not analytically integrable. However, an analytical approximation of the local average covariance matrix is feasible and gives satisfactory results. It bases on characteristic areas and directional scales of fluctuation.

IV. σ_u^2 of non-rectangular local averages can be approximated by σ_u^2 of an equivalent rectangle. Local averages of weakly correlated random fields (greater β), over elongated elements, cannot be approximated very accurately. Covariances of non-rectangles are much more difficult to approximate analytically.

References

Różyński G. (1992): *Ocena wpływu charakterystyk pola losowego parametrów podłoża gruntowego w stochastycznej metodzie elementów skończonych*, praca doktorska IBW PAN (PhD Thesis, Inst. of Hydro-Engineering of the Polish Acad. of Sci.).

Shinozuka M., Yamazaki F. (1988): *Stochastic Structural Dynamics, Stochastic Finite Element Analysis: An Introduction*, Elsevier Applied Science, London-New York.

Spanos P. D., Ghanam R. (1989): Stochastic Finite Element Expansion for Random Media, *Journal of Engineering Mechanics*, Vol. 115, No. 5, May, pp. 1035–1053.

Takada T. (1990): Weighted Integral Method in Stochastic FEM, *Probabilistic Engineering Mechanics*, Vol. 5, No. 3.

Vanmarcke E. (1983): *Random Field Analysis and Synthesis*, Cambridge Massachusetts, London England the MIT Press.