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Application of the Saint-Venant Model to the Problem of the Enforced Outflow from a Drainage Channel

Abstract

The Saint-Venant open channel model is applied to the simulation of pump-enforced outflow from a drainage channel. The switching boundary conditions are formulated and used to predict the moment of flow stability loss and for stability control.

Key words: drainage channel, open-channel flow, near-critical flow, stability

1. Introduction

We consider the flow in a long, flat channel delivering water from a drainage system. The hydraulic characteristics of the channel bed, as well as the inflow intensity, are known and may vary along the channel. The water management in the drained field may require control of the outflow from the channel. This can be aided by a pump station enforcing a downstream discharge greater than the natural one. The hydraulic conditions of flow along the channel, i.e. variable roughness of the bed, as well as high values of the enforced outflow may cause disturbances of the flow. The near-critical flow regime may occur in the downstream section of the channel, consequently, the

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continuity of the stream may be interrupted. These two phenomena have a highly negative influence on the stability of the bottom and shores of the channel. The aim of the work presented is to predict the moment of the unstable flow regime occurring by means of the unsteady channel flow model.

The problem was put forward by Kowalik (1992) during the 22th Seminar on Applied Mathematics held in September 1992 at Kobyla Góra.

2. Formulation of the Problem

From the engineering point of view, a long straight, almost flat drainage channel with longitudinally distributed inflow forms a non-stationary system with one spatial dimension. The water flows into the system along the channel either from the surface of the ground, the saturated zone or from the drain pipes. As the dynamics of the drains and soil flows is much lower than those of the free-surface channel flow, the inflow intensity may be assumed to be constant in the period considered. The downstream end of the channel is a point of natural or enforced outflow Q_L. The formulation of boundary conditions should contain a restriction like $Q_L \leq Q_p$, but its exact form depends on the mathematical description of the system (in the sense of compatibility of the boundary conditions with equations governing the model).

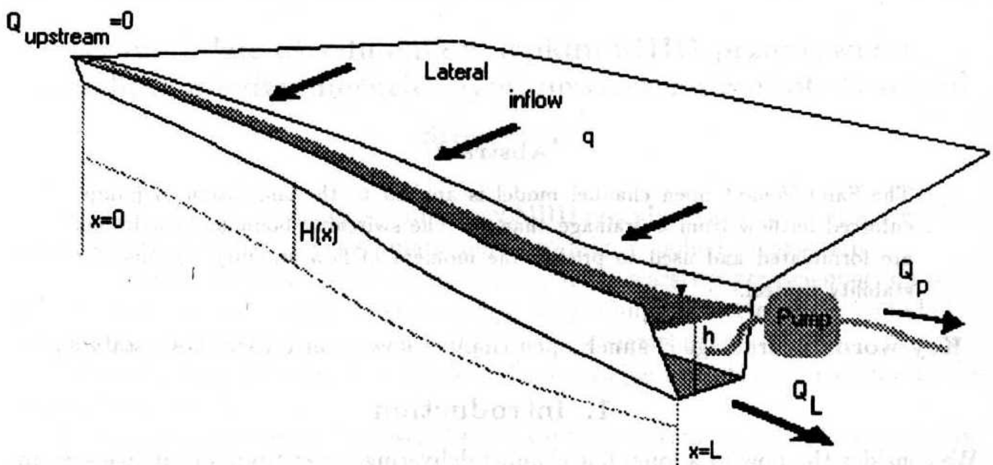

Fig. 1. Brief sketch of the prescribed system

3. Basic Equations. The Shallow Water Approach

As stated above, the equations of the mathematical model can be reduced with good tolerance to 1 spatial dimension parametrized with the variable x $(0 \leq x \leq L)$.

The equations of depth-averaged one-dimensional flow of incompressible fluid, known as Saint-Venant equations, in an open channel with trapezoidal cross-section assume the form:

$$\frac{\partial}{\partial t}\begin{pmatrix} A \\ Q \end{pmatrix} + \frac{\partial}{\partial x}\begin{pmatrix} Q \\ Q^2/A \end{pmatrix} + \begin{pmatrix} -q \\ gA\left(\frac{\partial}{\partial x}(h+H) + S_f\right) \end{pmatrix} = 0 \tag{1}$$

In fact, they are the principles of the mass and momentum conservation.

The friction term S_f is approximated by the Manning formula:

$$S_f = \frac{n^2 Q|Q|}{A^2 R^{4/3}} \tag{2}$$

Now, we should define initial and boundary conditions to make the mathematical problem well-posed. The initial condition should be given in the form $Q(x,0) = Q_0(x)\,, h(x,0) = h_0(x)$. In general, the condition of the stationary flow, according to the known source function $q(x,t)$ and a given downstream condition is sufficient approximation. It is well known (i.e. Yevjevich 1975) that the Saint-Venant system needs a number of boundary conditions equal to the number of "incoming" characteristics at each end. The upstream condition can be natural: $Q(0,t) = 0$. At the downstream end the situation is more complex. We assume the curve $Q = f(h)$ if known or simply the Manning friction law $Q = \frac{\partial H}{\partial x}\,\frac{AR^{2/3}}{n}$ for the natural outflow and the modeller-defined function $Q_p(t)$ when the forced one begins. In such a way the motion in the system is fully defined, unless the efficiency of the pump station exceeds the maximum permissible outflow from the channel. This maximum value can be established by different ways, according to the purpose of the simulation. Predicting the moment of the continuity failure needs the restriction for the flow

$$Fr \leq 1, \tag{3}$$

which is bounded by the critical flow (or *free flow*) condition, defined by the maximum possible of outflow from the system, with downstream control. In physical reality phenomena like large curvature of the free surface or discontinuity of the stream may follow the critical flow occurrence. It should be noted that the critical (or even supercritical) flow may arise spontaneously inside the domain of the channel, but this cannot be an immediate consequence of the downstream (i.e. outflow) condition.

The flow stability control procedure should rather avoid the critical or unstable flow at the downstream boundary. It would appear proper to consider the stability in terms of Froude number (direction of reverse characteristic) and Vedernikov number (spontaneous amplification of flow disturbances). According to the considered concept of stability, the boundary condition should be stated thus:

$$Fr \leq Val_{cr} \tag{4}$$

or

$$Ve \leq Val_{cr}. \tag{4'}$$

For the characteristic F of the flow set to Fr or Ve and for fixed $Val_{cr} \leq 1$ the downstream condition can be exposed briefly in the form (see Fig. 2):

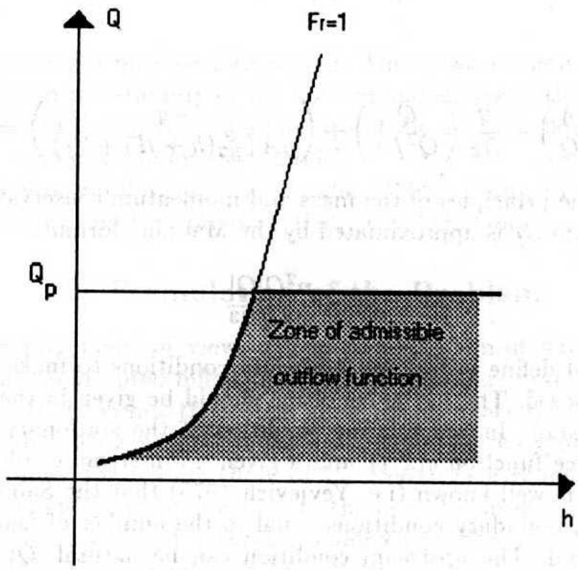

Fig. 2. Switching downstream condition for enforced outflow

$$\begin{cases} Q = -\frac{\partial H}{\partial x}\,\frac{AR^{2/3}}{n} & \text{for natural outflow,} \\ Q_L = Q_p(t) & \text{for enforced outflow, if } F(Q_p, h_L) < Val_{cr}, \\ F(Q_L, h_L) = Val_{cr} & \text{for enforced outflow, if } F(Q_p, h_L) \geq Val_{cr} \\ & \text{and } Fr(Q_p, h_L) \leq 1, \\ Fr(Q_L, h_L) = 1 & \text{otherwise.} \end{cases} \tag{5}$$

The stream interruption cases cannot be formulated correctly on the ground of the depth–averaged flow theory, and hence the Saint-Venant model cannot handle them. Fortunately, in engineering practice the prediction of the moment of stability loss and avoiding of it is more important than general modelling of the instabilities.

4. Numerical Method

The Saint-Venant equations can be solved analytically only in very simple cases, that are not applicable to real hydraulic situations. The solution should be found by numerical simulations. The author used the well known and efficient Preissman box scheme (Cunge, Liggett 1975). The advantages of this scheme are the following:

(1) the possibility of defining a non-uniform space computational step, and

(2) the implicit character of the scheme.

As was proved by Samuels & Skeels (1991), the Preissman scheme computations are stable for the range of flows defined by $|Ve| \leq 1$. The same condition is the restriction for the physical flow to be stable. The nature of the modelled situation does not allow the flow to become unstable during the simulation, so the Preissman discretization is found to be the appropriate one.

The following boundary procedure was put forward at every time level j: the moment

$$t_{cr} = \begin{cases} inf\{t > t_{j-1} : F(Q_L^j, h_L^j) = Val_{cr}\} & \text{if the condition } Q_L^{j-1} = Q_p \\ & \text{was applied,} \\ inf\{t > t_{j-1} : F(Q_L^j, h_L^j) < Val_{cr}\} & \text{if the condition } F(Q_L^{j-1}, \\ & h_L^{j-1}) = Val_{cr} \text{ was applied} \end{cases} \tag{6}$$

was estimated during the computational process. If it happened that $t_{cr} < t_{j-1} + \Delta t$, the time step was shortened to $t_{cr} - t_{j-1}$.

5. Examples of Simulation

The described numerical model was tested on a wide range of examples, differing in the shape of the bed, roughness, initial conditions and the maximum downstream outflow. The spatial variance of the roughness, slope and source term were taken into account. The first example presented was flow in a straight, 3000 m long trapezoidal channel with bed of uniform width 1.2 m and constant slope 0.001. The roughness coefficient varied with x dimension as shown in Fig. 3. A constant lateral inflow rate of 0.005 dm^2/sec was assumed. The initial depth along the channel was evaluated according to a steady flow regime. Maximum pump-enforced discharge was 50 dm^3/sec. The results of the stream-break simulation (condition $Fr \leq 1$) are presented in Fig. 4 in a form of outflow hydrograph and water stage profile at the break moment ($t = 88$ min.). Outflow from the same channel modelled with restrictions $Fr \leq 0.5$ and $Fr \leq 0.2$ are presented as well.

The trapezoidal shape of the bed results in the Ve conditions being, in general, more restrictive than the Fr conditions.

The main difficulties during the computations concerned variance of the roughnees of the bed, which caused near-critical flow between particular sections of the channel. The eventual numerical instabilities were omitted by the time step control. The shortest computational steps (length of almost 15 seconds) were used near the moment of change of condition type in the downstream section. However the results of computations were generally correct, they need to be compared with observed data.

Appendix 1. Flow Characteristics

The celerity of the Saint-Venant wave is $c = \sqrt{gA/B}$. The disturbance transforms the flow domain inside the space-time characteristic cone bounded by curves $\frac{dx}{dt} = v \pm c$ upwards and backwards. The Froude number is defined by $Fr = (\frac{v}{c})^2$. When the supercritical regime is established ($Fr > 1$), no information is transferred in a backwards direction.

The Vedernikov number $Ve = \frac{4}{3}\frac{A\,Fr}{R}\frac{dR}{dA}$ was considered by Chow (1959) and Liggett (1975) to characterize the stability of the flow. Outside the domain $|Ve| \leq 1$ perturbations of the flow are spontaneously amplified by the non-linear friction term S_f and roll waves form. In this light Ve is an indicator of large vertical disturbances of the flow.

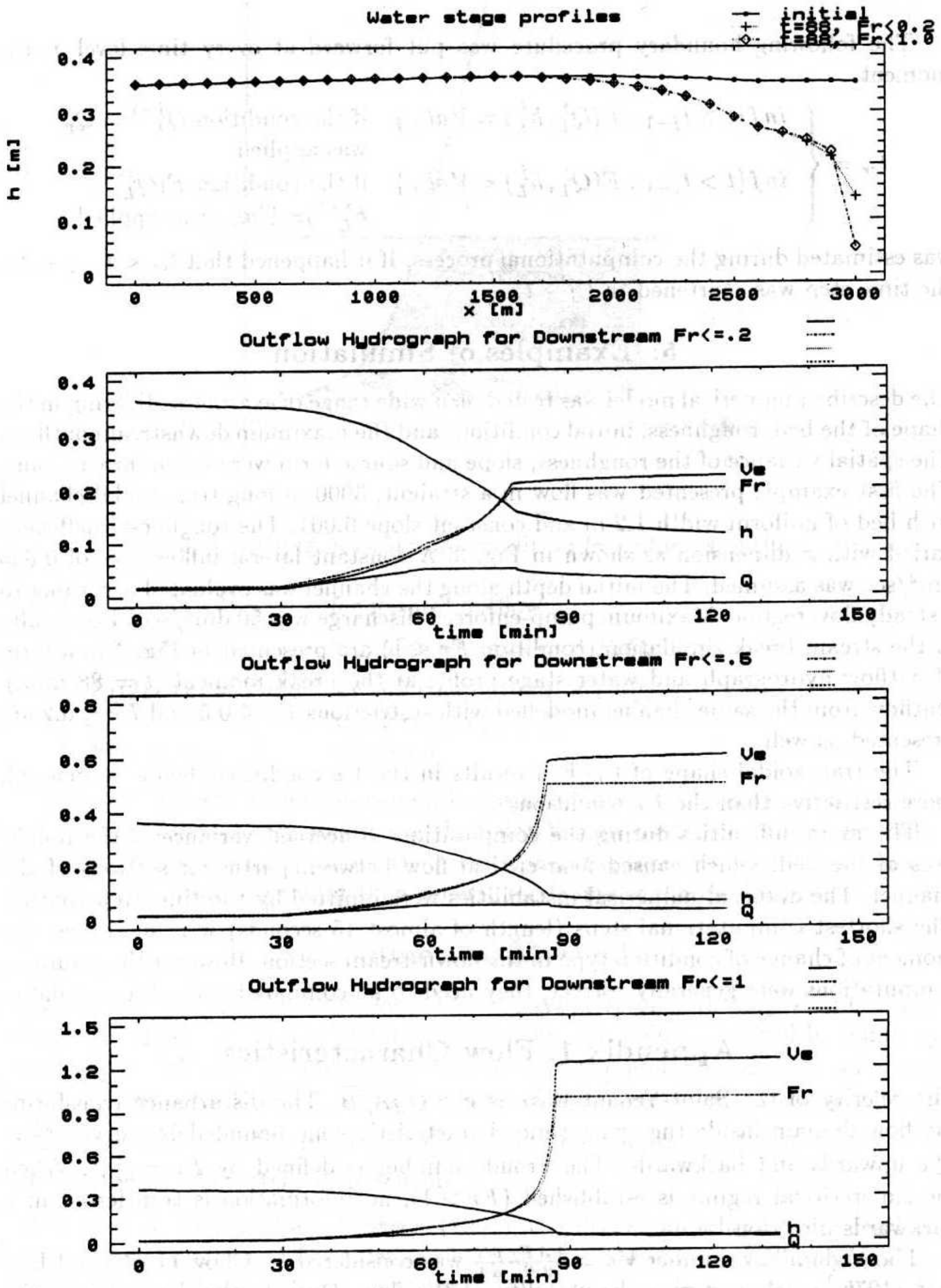

Fig. 3. Manning roughness of the bed

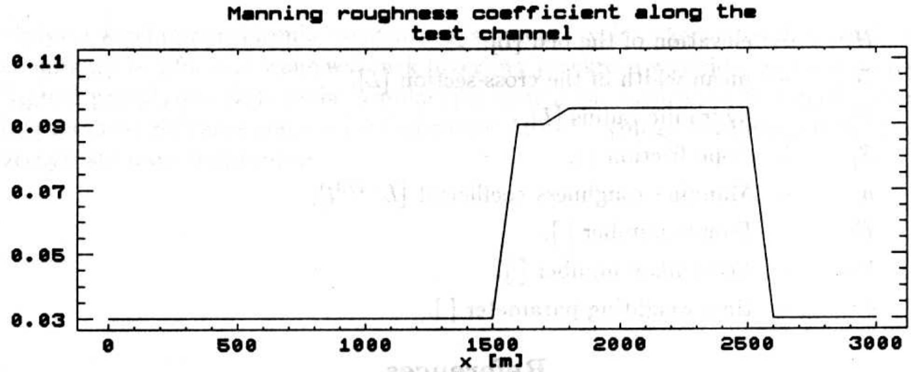

Fig. 4. Stream-break simulation for enforced outflow condition. Enforced pump discharge of 50 dm^3/sec caused stream continuity fail at $t = 88$ min. Other simulations were run with restriction $Fr \leq 0.5$ and $Fr \leq 0.2$

Appendix 2. Numerical Approximations

The Preissman scheme utilizes the following difference approximations of the differential operators:

$$\left.\frac{\partial F}{\partial x}\right|_{(i+\frac{1}{2},j+\frac{1}{2})} \approx \frac{\theta\,(F^{j}_{i+1} - F^{j}_{i}) + (1-\theta)\,(F^{j-1}_{i+1} - F^{j-1}_{i})}{\Delta x_j}$$

$$\left.\frac{\partial F}{\partial t}\right|_{(i+\frac{1}{2},j+\frac{1}{2})} \approx \frac{\frac{1}{2}\,((F^{j}_{i+1} - F^{j-1}_{i+1}) + (F^{j}_{i} - F^{j-1}_{i}))}{\Delta t}$$

The conservative nature of the basic equations suggests the difference approximation of whole terms: $\frac{\partial A}{\partial t}$, $\frac{\partial Q}{\partial x}$ in the mass balance and $\frac{\partial Q}{\partial t}$, $\frac{\partial}{\partial x}\frac{Q^2}{A}$, $\frac{\partial h}{\partial x}$ in the momentum balance equation.

The values of all functions used in the computational algorithm at the time level $t = t_j$ are evaluated at points: $(\frac{1}{2}(x_i + x_{i+1}), (1-\theta)\,t_{j-1} + \theta\,t_j)$, where (x_i, t_j) are space-time grid points. The results are better, if the non-linear style of interpolation is used. The condition of stability (Cunge & Ligget 1975) requires that $\theta > \frac{1}{2}$. Best results are obtained for $\theta \approx 0.7$.

Appendix 3. Description Symbols

x, t – spatial $[L]$ and temporal $[t]$ independent variables,

h, A – water depth $[L]$, channel active cross-section $[L^2]$,

Q, v – water discharge $[L^3\,t^{-1}]$, velocity $[L^2\,t^{-1}]$,

Q_p – pump-enforced discharge $[L^3\,t^{-1}]$,

c – wave celerity $[L^2\,t^{-1}]$,

H	–	elevation of the bed $[L]$,
B	–	mean width of the cross-section $[L]$,
R	–	hydraulic radius $[L]$,
S_f	–	slope friction [ ],
n	–	Manning roughness coefficient $[L^{-1/3}t]$,
Fr	–	Froude number [ ],
Ve	–	Vedernikov number [ ],
θ	–	time weighting parameter [ ].

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Summary

The Saint-Venant equations were used to model the drainage channel with the longitudinally distributed source term. The outflow at the downstream end could be natural or enforced by the pump station. The pump-caused outflow was modelled as the switching boundary condition, according to the physical nature of the problem. The numerical solution was evaluated by standard methods. The examples of simulation show that the moments of the break of the stream continuity and possible near-critical flow occurrence can be predicted by the model.

Model dynamiczny kanału odwadniającego z wymuszonym odpływem

Streszczenie

W celu zwiększenia efektywności działania kanału odwadniającego można wspomagać odpływ z końcowego przekroju sztucznie, np. za pomocą pompy. Zbyt wielka wydajność odpływu prowadzi do utraty stabilności przepływu i zerwania strugi. W pracy zastosowano model Saint-Venanta koryta otwartego do opisu wymuszonego

odpływu w celu przewidzenia momentu wystąpienia tych niekorzystnych zjawisk i dla ich uniknięcia. Sformułowano warunek brzegowy przekroju wyjściowego zapewniający ciągłość przepływu. Obliczenia symulacyjne oparte na szacunkowych danych przewidują moment zerwania ciągłości strugi cieczy. Model wymaga weryfikacji w oparciu o rzeczywiste dane pomiarowe.