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On the Averaged Inertia Method in the MHD Viscous Flow in a Slot Between Fixed Surfaces of Revolution

Abstract

The steady laminar MHD flow of a viscous fluid through a narrow space between two fixed surfaces of revolution, having a common axis of symmetry is considered.

To solve this problem the MHD boundary layer equations are used and expressed for the axially symmetric case in a curvilinear orthogonal coordinate system x, ϑ, y connected with one of the surfaces.

The method of averaged inertia terms is used to solve the boundary layer equations.

As a result, one obtains formulae expressing such flow paremeters as the velocity components v_x, v_y and the pressure p.

Key words: MHD viscous flow, slot of small thickness, co-axial surfaces of revolution, inertia effects

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1. Introduction

The steady and unsteady laminar flows of incompressible or compressible viscous fluid in a slot between surfaces of revolution have been examined theoretically and experimentally.

In recent years, considerable attention has been paid to the potentiality of liquid metals as lubricants utilized under the high temperature at which conventional lubricants would undergo some undesirable physical changes. Although the liquid metals such as mercury, sodium, and sodium-potassium alloy, etc. have a defect as lubricants since their load-carrying capacity becomes smaller due to low viscosity, they can still be considered to be suitable as lubricants for bearings operating at high temperature because of their stability at high temperature and due to their large thermal conductivity. Moreover, since the liquid metals are good electrical conductors, it is possible to increase their load capacity by utilizing the electromagnetic field and to eliminate the foregoing defect sufficiently, thereby alleviating the drawback of low viscosity.

Snyder (1962) has made theoretical investigations and showed that this defect can be overcome, in the case of a plane slider bearing, by the application of normal (to the bearing plane) magnetic field. This has created interest in other authors to study the magnetic effects in the field of lubrication.

In the beginning, investigations have been made to analyze the effects of magnetic field in lubrication problems while neglecting inertia effects. For these occasions, it has become important to consider the inertia effects. For these occasions, it has become important to consider the inertia effects in lubrication problems, due to increase in speed and use of lubricants of small viscosity. Dowson (1961) has investigated the inertia effects (rotational lubricant inertia) in hydrostatic thrust bearings for non-magnetic fields. In the papers (Shukla and Kapur 1967, Kamiyama 1969, Agrawal 1970, Kapur and Verma 1975, Salem et al. 1982, Salem et al. 1983) the same problem has been considered to study the effects of interactions of inertia effects and magnetic fields.

The papers (Savage 1964, Che-Pen Chen 1966, Elkouh 1967, Patrat 1975) contain the theoretical analysis of the radial inertia effects in the non-magnetic flows between parallel disks. The same problem for a magnetic field is considered in the papers (Kapur and Verma 1975, Salem et al. 1982).

The experimental studies of radial non-magnetic flows are presented in the papers (Peube and Che-Pen Chen 1964, Che-Pen Chen 1966).

The more general problem of viscous incompressible fluid flow in the slot between surfaces of revolution has been considered in the papers (McAlister and Rice 1970, McAlister and Rice 1972, Walicki 1974, Walicka 1989, Walicka 1993).

The viscous throughflow between corotating and stationary surfaces of revolution, whose shapes are given by the functions satisfying any conditions for which similar solutions exist, has been solved in the papers (McAlister and Rice 1970). The same flow in a more general statement is examined in papers (Walicka 1989, Walicka 1993).

A similar problem for magnetic field is considered in papers (Walicki 1976, Walicki et al. 1978, Walicka 1990). In paper (Walicki 1976) the inertia effect due to circumferential velocity is examined and in papers (Walicki 1978, Walicka 1990) – the inertia effect due to longitudinal velocity.

This paper is an attempt to investigate the steady laminar flow of an electrically conducting and incompressible fluid in a narrow slot between fixed curvilinear surfaces of revolution, having a common axis of symmetry, in the presence of a magnetic field of constant magnitude, as shown in Fig. 1.

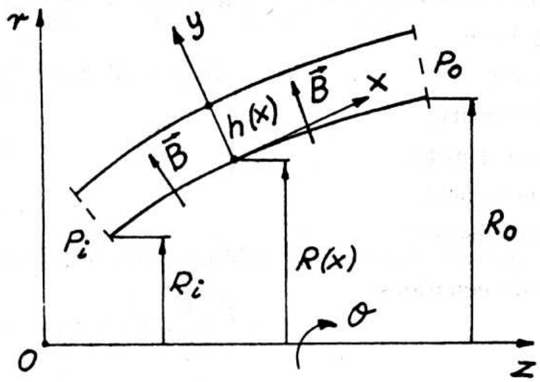

Fig. 1. Slot of small thickness between fixed surfaces of revolution. Coordinate system and geometry of surface

Basing on the method of average inertia, as in paper (Walicka 1993), we have analysed the influence of the inertia terms and magnetic field on the pressure distribution in the slot.

The problem is solved under the assumption that the Reynolds magnetic number is small which permits us to neglect the induced magnetic field (Sutton 1965).

2. Basic Equations

Let the inner surface be described by function $R = R(x)$ which denotes the radius of this surface. The thickness of the slot is described by function $h(x)$ which denotes the distance between the curvilinear surfaces measured along a normal to the inner surface. An intrinsic curvilinear orthogonal coordinate system x, ϑ, y linked with the inner surfaces is shown in Fig. 1.

The physical parameters of the flow are the velocity components v_x, v_y and pressure p. With regard to the axial symmetry of the flow, these parameters are not dependent on the angle ϑ. Let the vector of magnetic field $\overline{B}(0, 0, B_0)$ be perpendicular to the inner surface.

The equations governing the steady flow of an electrically conducting and incompressible fluid are (Sutton 1965):

the continuity equation

$$\nabla \cdot \overline{V} = 0, \tag{1}$$

the momentum equation

$$\rho(\overline{V}\nabla)\overline{V} = \rho\overline{F} - \nabla p + \mu\nabla^2\overline{V} + \overline{I} \times \overline{B}, \tag{2}$$

where:

- ρ – is the fluid density,
- $\overline{F}$ – the body force,
- p – the pressure,
- $\overline{V}$ – the fluid velocity,
- $\overline{I}$ – the current density,
- $\overline{B}$ – the magnetic field,
- μ – the fluid viscosity;

together with Maxwell's equations

$$\begin{aligned} &\nabla \times \overline{E} = 0, \\ &\nabla \times \overline{B} = \mu_e \overline{I}, \\ &\nabla \cdot \overline{E} = 0, \\ &\nabla \cdot \overline{B} = 0 \end{aligned} \tag{3}$$

and Ohm's law

$$\overline{I} = \sigma(\overline{E} + \overline{V} \times \overline{B}), \tag{4}$$

where:

- $\overline{E}$ – is the electric field,
- μ_e – the permeability of the free space,
- σ – the electrical conductivity.

Since the surfaces limiting the slot are surfaces of revolution, Lamé's coefficients H_x, H_ϑ, H_y for the above mentioned curvilinear coordinate system x, ϑ, y take the form (Walicka 1989):

$$H_x = 1, \quad H_\vartheta = R(x), \quad H_y = 1.$$

If the body force is neglected (usual assumption for the flow in a thin layer), the equation of continuity and the equations of motion in curvilinear coordinates can be expressed as:

$$\frac{1}{R}\,\frac{\partial(Rv_x)}{\partial x} + \frac{\partial v_y}{\partial_y} = 0, \tag{5}$$

$$\rho\left(v_x\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_x}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 v_x}{\partial x^2}+\frac{\partial^2 v_x}{\partial y^2}+\right.$$
$$\left.+\quad\frac{R'}{R}\frac{\partial v_x}{\partial x}-\frac{R'^2}{R^2}v_x-\frac{R''}{R}v_x\right)+I_\vartheta B_y-I_y B_\vartheta, \tag{6}$$

$$0=I_y B_x-I_x B_y, \tag{7}$$

$$\rho\left(v_x\frac{\partial v_y}{\partial x}+v_y\frac{\partial v_y}{\partial y}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v_y}{\partial x^2}+\frac{\partial^2 v_y}{\partial y^2}+\right.$$
$$\left.+\quad\frac{R'}{R}\frac{\partial v_y}{\partial x}\right)+I_x B_\vartheta-I_y B_x. \tag{8}$$

Let us assume that

$$h(x)<<R(x)$$

and the assumption on the velocity orders which can be expressed in the form

$$v_x=0(V_m)\qquad v_y=0\left(V_m\frac{h_m}{R_m}\right)$$

where: V_m is the mean value of the velocity of longitudinal flow and h_m, R_m are respectively the mean values of $h(x)$ and $R(x)$ in the slot.

Together with these usual assumption for the flows in a thin layer, the following assumptions regarding the magnetic field are made:

- the induced magnetic field B_x, B_ϑ and induced variation in B_y are assumed to be very small as compared to B_0;
- from the second equation of the set (3), the order of magnitude shows that $B_\vartheta\approx 0$ and $B_x\approx\mu_e h_m V_m B_0$, where B_0 is the applied magnetic field in direction y.

Again, it is only for the very high longitudinal velocity that an induced magnetic field would become comparable to B_0. Since the induced magnetic field is small, only terms in the equations of motion containing $B_y=B_0$ are being taken into consideration and expressed as:

$$\rho\left(v_x\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_x}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\,\frac{\partial^2 v_x}{\partial y^2}+I_\vartheta B_0, \tag{9}$$

$$0=I_x B_0, \tag{10}$$

$$0=\frac{\partial p}{\partial y} \tag{11}$$

where: y – variation in pressure is negligible.

From Maxwell's first and third equation and from Ohm's law, $\overline{E}$ vanishes everywhere and

$$I_\vartheta = -\sigma v_x B_0. \tag{12}$$

Thus equation (5) is given by

$$\rho\left(v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y}\right) = -\frac{dp}{dx} + \mu \frac{\partial^2 v_x}{\partial y^2} - \sigma B_0^2 v_x. \tag{13}$$

The problem statement is complete after specification of boundary conditions. These conditions for the velocity components v_x and v_y are the usual non-slip conditions stated as follows:

$$\begin{aligned} v_x = v_y = 0 \quad &\text{for} \quad y = 0 \\ v_x = v_y = 0 \quad &\text{for} \quad y = h \end{aligned} \tag{14}$$

Moreover, in the inlet and the outlet of the slot the boundary conditions for pressure can be written in the form

$$\begin{aligned} p = p_i \quad &\text{for} \quad x = x_i \\ p = p_0 \quad &\text{for} \quad x = x_0 \end{aligned} \tag{15}$$

thus:

x_i – denotes the inlet coordinate and

x_0 – the outlet coordinate.

3. Solution of the Reduced Equations

The equations governing the steady flow of an electrically conducting fluid in the slot between curvilinear surfaces are the equation of continuity (5) and the momentum equation (13).

These equations are the MHD boundary layer equations. Convective terms, representing the inertial contributions of the fluid, in the momentum equation make it difficult to find the solution in closed form. Following (Walicka 1992), the inertia terms are approximated by averaging them over the slot thickness. For this purpose we multiply the equation of continuity (5) by ρv_x and add the obtained expression to the momentum equation (13). As a result we have

$$\rho\left[\left(\frac{R'}{R} + \frac{\partial}{\partial x}\right) v_x^2 + \frac{\partial}{\partial y}(v_x v_y)\right] = -\frac{dp}{dx} + \mu \frac{\partial^2 v_x}{\partial y^2} - \sigma B_0^2 v_x. \tag{16}$$

Then, averaging the left-hand side of Eq. (16) across the slot thickness we can write

$$\frac{\rho}{h}\int_0^h \left[\left(\frac{R'}{R} + \frac{\partial}{\partial x}\right) v_x^2 + \frac{\partial}{\partial y}(v_x v_y)\right] dy = -\frac{dp}{dx} + \mu \frac{\partial^2 v_x}{\partial y^2} - \sigma B_0^2 v_x$$

and after integrating and taking into account boundary conditions (14), we obtain the following equation:

$$\frac{\partial^2 v_x}{\partial y^2} - k^2 v_x = f(x), \tag{17}$$

where $f(x)$ is defined as

$$f(x) = \frac{1}{\mu}\,\frac{dp}{dx} + \frac{\rho}{\mu h}\,\left(\frac{R'}{R} + \frac{\partial}{\partial x}\right)\int\limits_0^h v_x^2 dy \tag{18}$$

and

$$k = B_0\sqrt{\frac{\sigma}{\mu}} \tag{19}$$

Integrating the equation (17) we obtain:

$$v_x = \frac{f}{k^2}\left(\mathrm{ch}\,ky - 1 + \frac{1 - \mathrm{ch}\,kh}{\mathrm{sh}\,kh}\mathrm{sh}\,ky\right). \tag{20}$$

Hence from Eqs (5) and (20) the component of velocity v_y is given as:

$$v_y = -\frac{1}{R}\,\frac{\partial}{\partial x}\left\{\frac{Rf}{k^3}\left[\mathrm{sh}\,ky - ky + \frac{1 - \mathrm{ch}\,kh}{\mathrm{sh}\,kh}(\mathrm{ch}\,ky - 1)\right]\right\}. \tag{21}$$

The flow rate Q is defined as

$$Q = 2\pi R\int\limits_0^h v_x dy$$

Using the expression (20) for v_x we obtain after integration

$$Q = \frac{2\pi Rf}{k^3 M} \tag{22}$$

and hence

$$f = \frac{Qk^3}{2\pi R}M \tag{23}$$

where:

$$M = \frac{\mathrm{sh}\,kh}{M_1};\quad M_1 = 2(\mathrm{ch}\,kh - 1) - kh\,\mathrm{sh}\,kh \tag{24}$$

Applying the formula (23) in Eqs (20) and (21) we obtain the final expressions for the components of velocity:

$$v_x = \frac{Qk}{2\pi R}M(\mathrm{ch}\,ky - 1 + M_2\,\mathrm{sh}\,ky), \tag{25}$$

$$v_y = \frac{Qkh'}{2\pi R} \frac{M_3(1 - \text{ch}\, ky) - M_4(\text{sh}\, ky - ky)}{M_1^2}; \tag{26}$$

where:

$$M_2 = \frac{1 - \text{ch}\, kh}{\text{sh}\, kh}, \ M_3 = (\text{sh}\, kh - kh)(1 - \text{ch}\, kh), \ M_4 = (1 - \text{ch}\, kh)^2. \tag{27}$$

To define the pressure distribution let us go to the expression (18). By using Eqs (23) and (25) in Eq. (18) and after simple calculations, we obtain the following differential equation for pressure:

$$\frac{dp}{dx} = \frac{Qk^3\mu}{2\pi} \frac{M}{R} - \frac{\rho Q^2 k}{4\pi^2 h} \left(\frac{R'}{R} + \frac{\partial}{\partial x}\right) \frac{G}{R^2} \tag{28}$$

where:

$$G = \frac{(\text{ch}\, kh - 1)[(\text{ch}\, kh + 2)kh - 3\text{sh}\, kh]}{M_1^2}. \tag{29}$$

Integrating the equation (28) we have

$$p = C + \frac{Qk^2\mu}{2\pi} B(x) - \frac{\rho Q^2 k}{4\pi^2} D(x) \tag{30}$$

where:

$$B(x) = \int \frac{M}{R} dx, \ D(x) = \int \frac{1}{h} \left(\frac{R'}{R} + \frac{\partial}{\partial x}\right) \frac{G}{R^2} dx. \tag{31}$$

Applying the boundary conditions (10), for the slot of constant thickness we obtain

$$p = p_i + \frac{Q\mu H^3 M}{2\pi h^3} [A(x) - A_i] - \frac{\rho Q^2 HG}{8\pi^2 h^2} \left(\frac{1}{R^2} - \frac{1}{R_i^2}\right) \tag{32a}$$

or

$$p = p_0 + \frac{Q\mu H^3 M}{2\pi h^3} [A(x) - A_0] - \frac{\rho Q^2 HG}{8\pi^2 h^2} \left(\frac{1}{R^2} - \frac{1}{R_0^2}\right) \tag{32b}$$

where:

$$A(x) = \int \frac{dx}{R}; \ A_i = A(x_i), \ A_0 = A(x_0) \tag{33}$$

and $H = kh$ is the Hartmann number.

By subtracting the formulae (32a) we obtain the relation for the pressure drop in the slot:

$$\Delta p = p_i - p_0 = \frac{Q\mu H^3 M}{2\pi h^3} (A_i - A_0) - \frac{\rho Q^2 HG}{8\pi h^2} \left(\frac{1}{R_i^2} - \frac{1}{R_0^2}\right). \tag{34}$$

Note that for $H \to 0$ (i.e. also $k \to 0$) we have the following relations

$$H^3 M \to (-12), \quad HG \to \frac{6}{5}$$

and the differential equation (28) has the form

$$\frac{dp}{dx} = -\frac{6\mu Q}{\pi} \frac{1}{Rh^3} + \frac{3\rho Q^2}{10\pi^2} \frac{(Rh)'}{(Rh)^3}. \tag{35}$$

Hence the pressure distribution is given as:

$$p = C - \frac{6\mu Q}{\pi} \int \frac{dx}{Rh^3} - \frac{3\rho Q^2}{20\pi^2} \frac{1}{(Rh)^2}. \tag{36}$$

These relations are identical with those obtained in (Walicka 1993). In the above expressions the "primes" denote derivation with respect to x.

4. Example of Applications

4.1. Non-dimensional Form of the Solution

Equations (25), (26) and (32a) may be non-dimensionalized using the following parameters:

$$\begin{aligned} &\tilde{x} = \frac{x}{R_0}, \qquad \tilde{R} = \frac{R}{R_0}, \qquad \eta = \frac{y}{h}; \\ &\tilde{v}_x = \frac{v_x}{V_0}, \qquad \tilde{v}_y = \frac{v_y}{V_0}\frac{R_0}{h}, \quad \tilde{p} = \frac{(p-p_0)\rho h^4}{\mu^2 R_0^2}; \\ &R_\lambda = \left(\frac{h}{R_0}\right) Re; \quad R_e = \frac{\rho V_0 h}{\mu} \end{aligned} \tag{37}$$

where: V_0 is the average velocity in the outlet cross section of the slot defined as:

$$V_0 = \frac{Q}{2\pi} \frac{1}{R_0 h}$$

R_e and R_λ are the Reynolds and modified Reynolds numbers respectively.

The non-dimensional formulation is then (for the slot of constant thickness)

$$\tilde{v}_x = \frac{HM}{\tilde{R}}(\text{ch}\, H\eta - 1 + M_2 \text{sh}\, H\eta), \tag{38}$$

$$\tilde{v}_y = 0, \tag{39}$$

$$\tilde{p} = R_\lambda H^3 M[\tilde{A}(\tilde{x}) - \tilde{A}_0] - \frac{R_\lambda^2 HG}{2}\left(\frac{1}{\tilde{R}^2} - 1\right). \tag{40}$$

4.2. Flow Between the Conical Parallel Surfaces

For the conical parallel surfaces shown in Fig. 2 the geometric relations are given as:

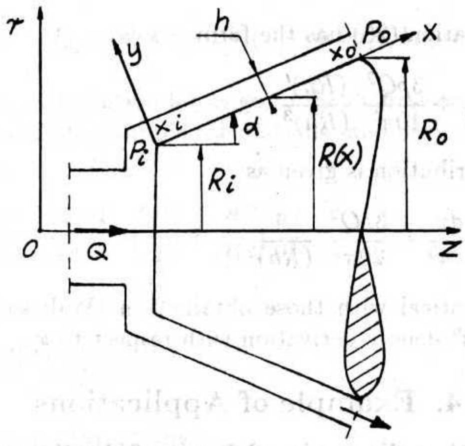

Fig. 2. Slot between conical parallel surfaces

$$R = x \sin\alpha, \ R_i = x_i \sin\alpha, \ R_0 = x_0 \sin\alpha, \ h = \text{const}.$$

The non-dimensional quantities are:

$$\tilde{R} = \tilde{x}\sin\alpha, \ \tilde{R}_0 = 1, \ \tilde{x}_0 = \frac{1}{\sin\alpha},$$

and the non-dimensional formulation is given as:

$$\tilde{v}_x = \frac{HM}{\tilde{x}\sin\alpha}(\text{ch}\, H\eta - 1 + M_2 \text{sh}\, H\eta), \tag{41}$$

$$\tilde{v}_y = 0, \tag{42}$$

$$\tilde{p} = \frac{R_\lambda H^3 M}{\sin\alpha}(\ln\tilde{x} - \ln\tilde{x}_0) - \frac{R_\lambda^2 HG}{2}\left(\frac{1}{\tilde{x}^2\sin^2\alpha} - 1\right). \tag{43}$$

Note that the solution in this case depends on the angle and is also true at the angle $\alpha = 90°$. This means that the above formulae give the solution for flow between parallel disks.

Figure 3 shows the profiles of the velocity component $\tilde{v}_x$ for different values of $\tilde{x}$ (different positions of the cross-section) and for two values of Hartmann numbers: $H = 1$ (continuous lines) and $H = 0$ (dashed line, non-magnetic flow) for the flow between parallel disks ($\alpha = 90°$). It can be seen from this figure that the differences between the velocity profiles for samll values of H are only perceptible in the cross-sections lying near the inlet to the slot; it is seen from the formula (41) that these differences increase with the increase of the magnitudes of the Hartmann number H.

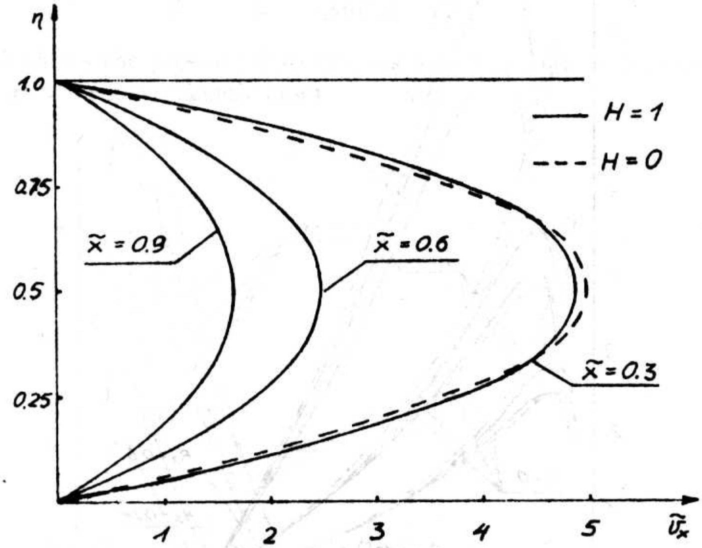

Fig. 3. Dimensionless profiles of the velocity component $\tilde{v}_x$ for the flow between parallel disks

Figure 4 shows pressure distributions for two values of Hartmann numbers ($\alpha = 90°$) : $H = 1$ and $H = 0$ (non-magnetic flow).

The dashed lines present the results of experiments given by Chen (1966) for the non-magnetic flow between parallel disks. The "dash-dot" lines present the pressure distributions without inertia effects, i.e., the pressure distributions obtained from the formula

$$\tilde{p} = R_\lambda H^3 M \ln \tilde{x}. \tag{44}$$

The pressure distributions for $H = 0$ are taken from (Walicka 1993).

4.3. Flow Between the Concentric Spherical Surfaces

For the concentric spherical surfaces shown in Fig. 5 the geometric relations are given as:

$$R = R_s \sin\varphi,\ \varphi = \frac{x}{R_s},\ R_i = R_s \sin\varphi_i,\ R_0 = R_s \sin\varphi_0,\ h = \text{const}.$$

The non-dimensional quantities are (for $\varphi = 90°$):

$$\tilde{R} = \sin\varphi,\ \tilde{R}_0 = 1,\ \tilde{x} = \varphi.$$

The non-dimensional formulation is given as:

$$\tilde{v}_x = \frac{HM}{\sin\varphi}(\text{ch}\, H\eta - 1 + M_2\, \text{sh}\, H\eta), \tag{45}$$

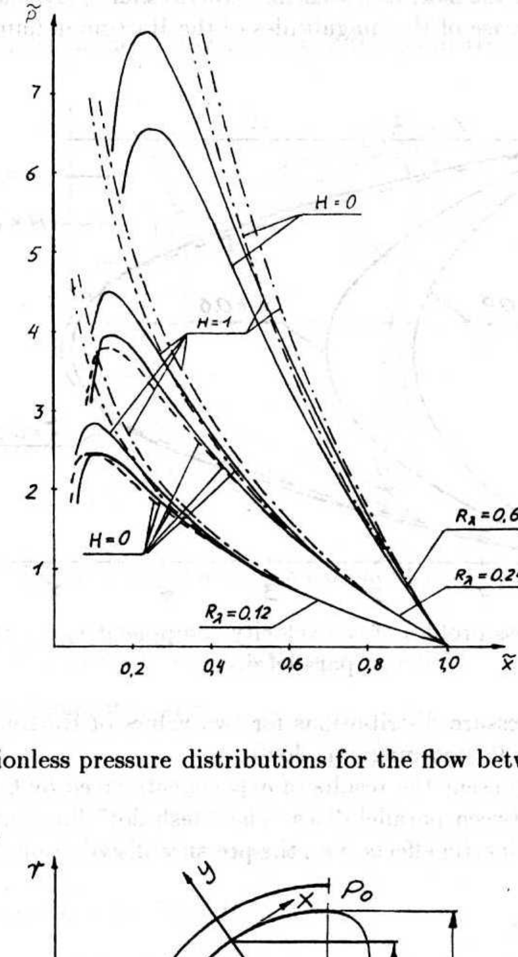

Fig. 4. Dimensionless pressure distributions for the flow between parallel disks

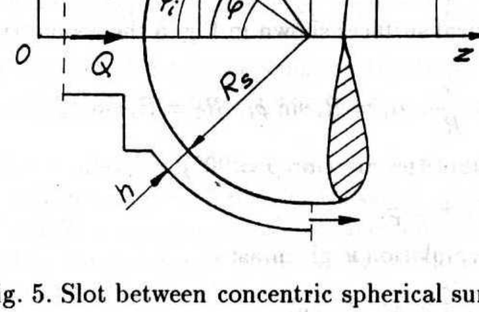

Fig. 5. Slot between concentric spherical surfaces

$$\tilde{v}_y = 0, \tag{46}$$

$$\tilde{p} = R_\lambda H^3 M \ln \tan \frac{\varphi}{2} - \frac{R_\lambda^2 HG}{2}\left(\frac{1}{\sin^2\varphi} - 1\right). \tag{47}$$

Figure 6 shows the pressure distributions for two values of Hartmann numbers: $H = 1$ and $H = 0$ (non-magnetic flow).

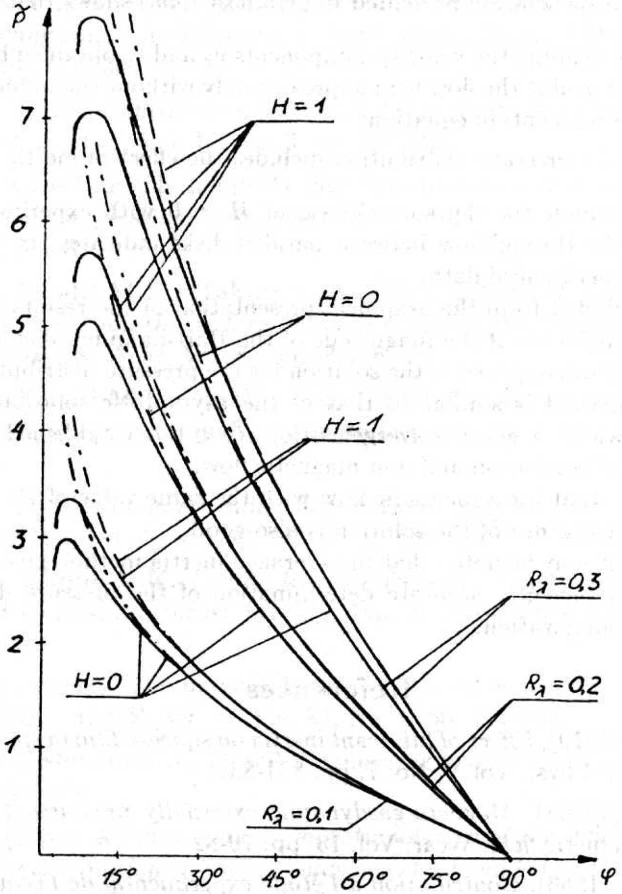

Fig. 6. Dimensionless pressure distributions for the flow between concentric spherical surfaces

The "dash-dot" lines present the pressure distributions without inertia effects, i.e., the pressure distributions obtained from the formula

$$\tilde{p} = R_\lambda H^3 M \ln \operatorname{tg} \frac{\varphi}{2}. \tag{48}$$

The pressure distributions for $H = 0$ are taken from (Walicka 1993).

5. Conclusions

Application of the method of average inertia terms in the momentum equation to study the throughflow of viscous fluid between surfaces of revolution in the presence of normal magnetic field yields the formulae defining the velocity components v_x, v_y and pressure p.

A comparison of formulae of this paper with formulae which are obtained by the method of asymptotic solution presented in (Walicka 1993) shows that:

- the formulae defining the velocity components v_x and v_y obtained by the method used here determine the flow field approximately without the influence of inertia terms of the momentum equation;
- the formula for pressure distribution includes the effect of inertia forces.

A comparison made for the special case of $H = 0$ with experimental data of Chen (1966) for the throughflow between parallel disks indicates the conformity of theoretical and experimental data.

It can be concluded from the graphical presentation of the results obtained here that the pressure increases if the magnitude of the Hartmann number H increases.

The problem of convergence of the solution for the pressure distributions obtained by the present method is similar to that of the asymptotic solution discussed in (Walicka 1989), where a good convergence for $R_\lambda < 1$ is established for the flows between surfaces of revolution and non-magnetic flow.

It may happen that for a magnetic flow with the same value of R_λ and for small $H = 0$ (1) the convergence of the solution is also good.

In conclusion, it may be noted that the averaged inertia method gives the formulae which lead to a sufficiently accurate determination of the pressure distribution in considered flow configuration.

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Summary

In the present paper the laminar magnetohydrodynamic (MHD) flow of an incompressible and electrically conducting, Newtonian fluid, in a narrow space between two fixed surfaces of revolution, having a common axis of symmetry is considered under the action of a magnetic field which vector is normal to one of the surfaces.

It is found that the equations of motion can be reduced to the MHD boundary layer equations expressed in a curvilinear orthogonal coordinate system x, ϑ, y connected with one of the surfaces.

To solve the problem the method of averaged inertia terms is used which gives an approximate solution in a simple closed form.

As a result, one obtains formulae describing the velocity components and pressure distribution in the narrow space. Two particular cases of throughflow between conical and spherical surfaces are more exactly discussed.

O metodzie uśrednienia w lepkim MHD przepływie w szczelinie między nieruchomymi powierzchniami obrotowymi

Streszczenie

W pracy rozważono ustalony laminarny MHD przepływ cieczy lepkiej w szczelinie między nieruchomymi powierzchniami obrotowymi o wspólnej osi symetrii.

Do rozwiązania zagadnienia użyto równań MHD warstwy przyściennej dla przepływu osiowo-symetrycznego, wyrażonych w krzywoliniowym ortogonalnym układzie współrzędnych x, ϑ, y związanym z jedną z tych powierzchni.

Równania warstwy przyściennej rozwiązano stosując metodę uśrednienia członów bezwładnościowych.

W wyniku otrzymano zależności określające takie parametry przepływu jak składowe prędkości v_x, v_y i ciśnienie p.