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Mathematical Model of the Process of Hot Water Consumption

Abstract

The problem of stochastic modelling of hot-water intake for the needs of dimensioning of the system serving its preparation has been considered. Modelling of hot water intake was carried out using Poisson's process with variable intensity. Further, the theoretical distribution of variability of the total hot-water intake over an optional period of time has been defined For the multi-family dwellings, on the basis of experimental results, unitary hot-water consumption for workdays, free days (free Saturdays) and holidays (Sundays) as a function of the number of inhabitants and assumed probability of its being exceeded has been evaluated.

Key words: water supply of the buildings, water supply systems, hot consumptive water systems

1. Introduction

In Poland, central installations of domestic hot water are widely used in multi-family buildings. Such an installation consists of a central system for conditioning hot water, plus distribution and circulation channels.

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The fundamental problem arising when designing a hot water conditioning system is the correct determination of values characterizing hot water consumption in a building fed by the system designed. The installations serving to condition hot water are designed basing on these values. They should be chosen so as to guarantee an undisturbed supply of hot water of the required temperature and in sufficient quantity, whilst being efficiently utilized.

Characteristic of hot water consumption in residential buildings is its great variability, both over 24 hours and on particular days of the week. The values characterizing the consumption are random ones. Even in seemingly identical conditions they will never have exactly the same observed values.

In real conditions the values characterizing hot water consumption can be both greater as well as less than those assumed for calculations. Since the functioning of the system of hot water conditioning depends on these values, it is necessary to determine the character of their variability and possible effect on the performance of a system of variable values other than those assumed for the installations.

In the present paper, the author presents the stochastic model of the hot water consumption. It enables determination of the variability of values characterizing hot water consumption in multi-family houses. Stochastic processes have been used for modelling.

2. Modelling Long Term Consumption of Hot Water

The problem of probabilistic modelling the consumption of, and demand for hot water has been widely analyzed in domestic literature. Attempts have been made to use different statistical distributions to determine the variability of consumption. Normal (Pieńkowski 1982), log-normal (Jeżowiecki, Tiukało 1984, Mańkowski 1968), gamma (Siwoń 1981) and other (Mielcarzewicz, Janczewski 1984, Starinskij 1984) distributions have been applied.

In the probabilistic modelling of consumption of, and demand for water in towns, Siwoń (1981) draws the conclusions in which, among other things, he resolved that the consumption of and demand for water can be considered as random variables bounded from below. Their characteristics should be determined basing on sets of observations complying with criteria of homogeneity and independence of their elements. Empirical distributions of frequency of daily, hourly water consumption can, according to him, be approximated by gamma distribution.

The estimation of the concordance of empirical distributions with hypothetical ones has been carried out by Mielcarzewicz, Janczewski (1984) applying the tests of Kolmogorov and Pearson. They examined eleven hypothetical distributions: normal, log-normal, Weibull, gamma, Erlang, Rayleigh, Maxwell, beta, chi-square, Fisher-Tipett type I max. According to these one can apply the gamma (Pearson's type III) and Fisher-Tipett type I (distribution of extreme values type I) distributions to describe empirical water consumption distributions for various durations, irrespective of the water destination.

Jeżowiecki and Tiukało (1968) have formulated a model of demand for hot water and power in a time interval called "time of duration of the computational cycle of demand". A demand for hot water for one flat has been modelled assuming that it is subject to log-normal distribution. Identification of the parameters has been conducted basing on Suligowski's investigations (1978).

A discrete model of water consumption in interior installations has been proposed by Zaleski (1987). The author assumes that the volume of water consumed in the time determined interval $\Delta\tau$, by two different apartments of the same type M-1 is described by the same unknown probability distribution, and has then introduced a value, called by him average water consumption by an apartment of M-1 type in the time instant τ. Proceeding from the law of large numbers and theorem of Lindberg-Levy, he concludes that for a given number n of M-1 type apartments the aggregate water consumption at instant τ has an asymptotic normal distribution $N(m, \sigma)$. The standard characteristic of water consumption by one M-1 type apartment is described by a normal distribution. The standard characteristic of the nth order by an apartment (for examined n apartments) can be distributed into the sum of two components one of which is deterministic and the other random. The model of water consumption for an arbitrary cycle will be resolved to determination of the characteristics of as many time instants as the number of intervals into which the cycle has been divided.

In order to simulate the supply of water to houses, Tiukało (1987) worked out a model of consumption in which, for the purpose of simplifying the reasoning, all the real water intake points in an apartment were replaced by one determinant intake point. The number of operating determinant intake points is the function of the total stream of water taken from the installation and the streams of water running from intake points attributed to the operating determinant intake points. The variability of the stream has been heuristically modelled with four-parameter beta distribution. In order to determine the parameters of distribution minimum and the maximum water streams have been determined for particular types of apartments. The critical value of the stream has been determined making use of Sopenski's papers. When modelling many determinant points of intake, independence of intakes in particular apartments has been assumed.

Siwoń, Stanisławski (1984) propose to use Markov's process for modelling the consumption of, and demand for water. The dependence of consumption and demand on a non-random parameter – time, results in the phenomenon of water consumption being considered as a stochastic process given in the form of a random sequence $\{X_\tau\}$ adopting values from the finite and countable state space. The stochastic characteristics are obtained by the corresponding construction of a statistic test, the elements of which are the values of consumption referred to the constant time interval. In the model of daily or hourly consumption this unit is 24 hrs or one hour. The assumption accepted by the authors that the value of the random variable $X_\tau(T)$ obtained at instant T, depends only on the analogical value $T-1$ results in the water consumption being regarded as a process with memory reaching one step back, and called the normal Markov chain. It is possible to use the process with memory reaching $r > 1$ number of steps, when we obtain a Markov chain with an r-fold dependence.

In the stochastic modelling of hot water consumption one can also apply the methods of time sequence analyses (Cichalešvili 1984, Siwoń 1986) and those of signal theory analyses (Hummel et al. 1984).

Tabernacki in his work (1990) elaborated the general method of analysis of phenomena related to the flow and consumption of water in water supply installations of dwelling houses, utilizing the dynamic simulation modelling. Basing on his studies he worked out four models concerning the phenomena of flow and water consumption in installations. They concerned the use of the installation, its hydraulic operation, dynamics of the water source and occurrence of water losses. The author demonstrated the usefulness of the elaborated models in analyses of problems occurring in the installations, among other things, to determine water flow through conduits.

Various methods have been used to model water consumption over longer periods of time: probabilistic, based on Markov's process, time sequence analyses and signal theory analyses. Analysing the literature, no application of Poisson's process for modelling the consumption of utility hot water over a longer period of time has been found. In the author's opinion this process presents the physical sense of the phenomenon analyzed. The properties of Poisson's process enable the modelling of the phenomenon of hot water consumption and determination of its variability in a relatively simple way.

3. The Description of the Consumption of Hot Water as a Stochastic Process

The process of hot water distribution in a dwelling house depends on many factors. On particular kinds of days, i.e. working days, free days and holidays the consumption cycles of hot water are similar (Suligowski 1978, Tabernacki 1990).

For short periods of time however, no regularities can be noted. An exemplary graph of the break-down of hot water distribution in the function of time, obtained during measurements, is presented in Figure 1.

Analysing the consumption of hot water in a multi-family dwelling, the factors affecting its course, and the results of consumption measurements, it can be stated that this process is an undetermined – random one.

In houses provided with hot water installations, different quantities of water are used for different purposes. Consumption is effected from any intake points, by any users, at any time. For the process of hot water consumption the time τ in which the consumption is being analysed is the parameter of a random process. The parameter for every cycle is limited by its length and is determined by the beginning and the end of the cycle:

$$\tau_p < \tau < \tau_k$$

where:

τ_p – beginning of the cycle, $\tau_p \geq 0$,

τ_k – end of the cycle, $\tau_k \geq \tau_p$.

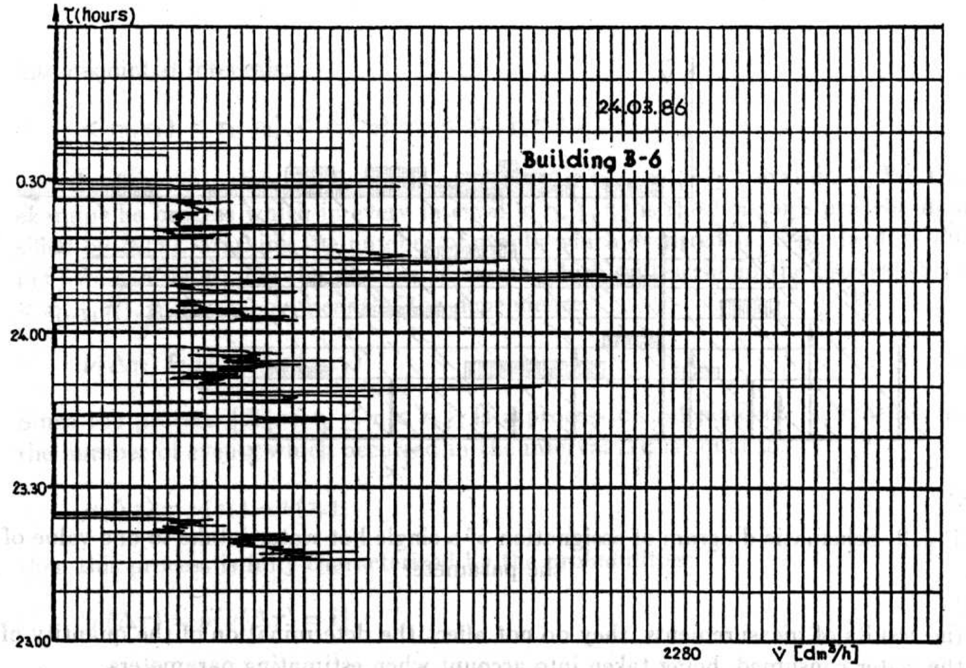

Fig. 1. Exemplary graph of hot water distribution in a multi-family dwelling in the function of time

The random variable $v(\tau)$ describes the global consumption of hot water from the initial moment of the process τ_p until moment τ. The value of parameter τ cannot be lower than zero or higher than the length of the cycle.

To simplify modelling of the phenomenon of hot water consumption, the quantity of water used at a single time has been assigned to the definite value of the parameter τ namely that, at which the consumption commenced (i.e. moment τ, at which the water began to run from the intake point). The idea of this operation has been presented in Fig. 2, where n marks the number of the consecutive intake point in the house; $\dot{v}_n(\tau)$ – intensity of outflow (volumetric flow rate) of hot water from the intake point n at moment τ; $\dot{v}(\tau)$ – intensity of hot water outflow from the central node of the utility hot water; $v(\tau)$ – total hot water outflow from intake point n during a single intake assigned as moment in time τ_i.

Assigning consumption to a particular moment in time greatly simplifies the problem, as it eliminates the necessity to consider the time of hot water outflow from intake points and the value and variability of flow – it enables consideration of only the quantities of a single consumption of hot water when utilizing the installation.

The operation described may result in some discrepancies between the accepted model and the process of consumption, e.q. by omitting consumption commenced before time τ_p or by taking into account that continued after time τ_k. These values can cancel each other out. For parameters of the model determined on the basis of

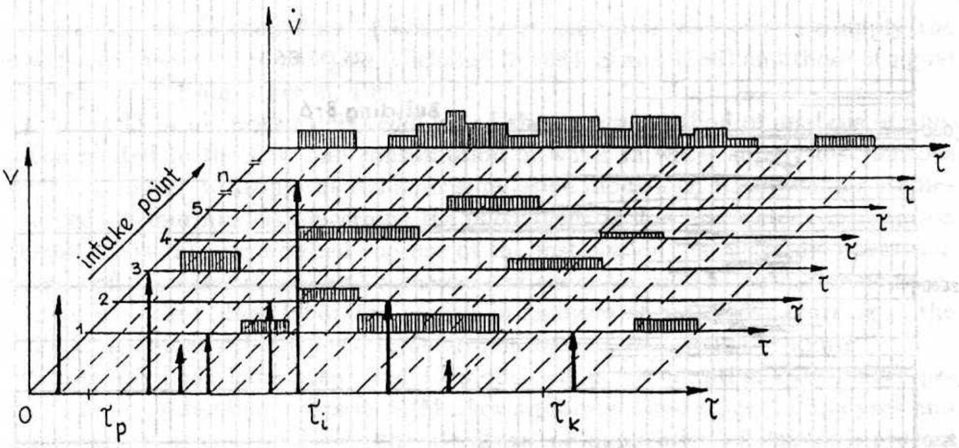

Fig. 2. Schematic diagram of assignation of a single hot water intake to one value of the parameter

the results of measurements, they do not affect the determination of the quantity of the water consumed, being taken into account when estimating parameters.

In the first phase of the analysis the sequence of random events, such as the opening of hot water bib-valves in order of occurrence is considered. Let $X(\tau)$ – be the number of times the valves are opened (number of intakes) during interval $< \tau_p, \tau)$ where $\tau_p \leq \tau < \tau_k$. Here we have to deal with a stochastic process $\{X(\tau), \tau_p \leq \tau < \tau_k\}$, where for every τ the random variable $X(\tau)$ can assume the whole, non-negative values $i = 0; 1; 2; 3 \ldots$ Moreover, for optional τ_1 and τ_2, contained in τ, and such that $\tau_1 < \tau_2$, the increment $X(\tau_2) - X(\tau_1)$ can be equal to 0; 1; 2; 3 .... It is assumed that the sequence of events considered is the process which satisfies the conditions of Poisson's process with variable intensity a. For the Poisson's process with constant intensity, intensity a is defined as a mean number of events occurring in a unit of time, while for Poisson's process with variable intensity, the intensity is defined as a function of parameter τ, satisfying condition (Klimow 1979):

$$a(\tau) = \lim_{\Delta\tau \to 0} \frac{\alpha(\tau + \Delta\tau) - \alpha(\tau)}{\Delta\tau} \tag{1}$$

where:

$a(\tau)$ – pilot function of the process, for repeatable processes it defines the mean number of events occurred from the initial moment until time τ.

The section of time $< 0, \tau_c >$ can be divided with points:

$$0 = \tau_0 < \tau_1 < \tau_2 < \tau_3 < \ldots < \tau_{n-1} < \tau_n$$

into disjointed intervals:

$$< \tau_0, \tau_1), < \tau_1, \tau_2), < \tau_2, \tau_3), \ldots < \tau_{n-1}, \tau_n)$$

and the function $\alpha(\tau), \tau \leq 0$ equals α_i, for $\tau \in < \tau_{i-1}, \tau_i), i \geq 1$ examined. We then examine he process which in every interval $< \tau_{i-1}, \tau_i)$ is the Poisson's process intensified by $\alpha_i(\tau)$. Marking the number of events which occurred from moment τ, with $\nu(\tau)$ we can determine the number of events which will occur in the interval $\Delta\tau_i = < \tau_{i-1}, \tau_i), \tau_{i-1} < \tau_i$, by means of the dependence:

$$\nu(\Delta\tau_i) = \nu(\tau_i) - \nu(\tau_{i-1}) \tag{2}$$

Since the process $\{X(\tau), \tau_p \leq \tau < \tau_k\}$, is a process of independent increments, and the number of events which occurred in the interval $\Delta\tau$ is equal to:

$$\nu(\Delta\tau) = \nu(\tau_k) - \nu(\tau_p) \tag{3}$$

then this process is fully characterized by the probabilities:

$$p_i(\Delta\tau) = p\{\nu(\Delta\tau) = i\} \tag{4}$$

where: $i = 0; 1; 2; \ldots, \Delta\tau = < \tau_p, \tau_k),\ 0 \leq \tau_p \leq \tau_k$.
It can be demonstrated (Klimow 1979) that:

$$p_i(\Delta\tau) = \frac{[\alpha(\tau_k) - \alpha(\tau_p)]^i}{i!} e^{-[\alpha(\tau_k) - \alpha(\tau_p)]} \tag{5}$$

where the pilot function of the process is defined by dependence:

$$\alpha(\tau) = E\nu(\tau) \tag{6}$$

where: E – mean value.
For the most simple process with constant intensity a, using the dependence:

$$\alpha(\tau) = \int_0^\tau a(\tau) d\tau \tag{7}$$

we obtain

$$\alpha(\tau) = \int_0^\tau a d\tau = a\tau \tag{8}$$

In a general case, if $\alpha(\tau)$ is a non-negative function and non-decreasing in the interval $< 0, \tau_c >$, it will be possible to determine the process for which the equality (6) is performed. This is called the Poisson's process with pilot function $\alpha(\tau)$ (Klimow 1979). It can thus be assumed that the process of making use of the intake points for the given cycle of consumption is the Poisson process of variable intensity with pilot function $\alpha(\tau)$, describing the mean number of openings of bib-valves from the initial moment of the cycle until the moment analysed. The distribution of probability of the number of occurrences in the Poisson process is called the Poisson distribution (Feller 1978). The probability density function of this is described by the dependence (5).

Every opening of the bib-valve entails consumption of quantity of hot water. The amount of water taken each time the valve is opened depends on the actual needs of the installation user. It thus depends on many factors, being a random variable.

The characteristic feature of hot water consumption at a single intake is that it is determined only in a set of positive numbers. It is assumed that the distribution function of single intakes is described by the distribution function $F_j(v)$, and the density by $f_j(v)$, where $v \geq 0$.

The aim of this study is determination of the global consumption of hot water by users of the given installation in an optional time interval of the cycle analysed. The global consumption from installation v is the sum of all single water intakes v_j in the time interval $\Delta\tau$ analysed:

$$v(\Delta\tau) = \sum_{j=1}^{n} v_j(\Delta\tau) \tag{9}$$

where:

- n – number of single intakes (openings of intake valves),
- v – volume of water taken from the intake point during the j-th intake.

It can be assumed that the single intakes of hot water v_j are independent values. The variable v, defining the global consumption by inhabitants in the time interval examined, is the sum of random variables describing particular intakes. Knowing the distribution of these variables it is possible to determine the distribution of the sum by computation of integrals of convolution (composition) of single distributions the necessary number of times.

$$F_{j_1+j_2}(v) = \int_0^{+\infty} F_{j_1}(v_1) f_{j_2}(v - v_1) dv_1 \tag{10}$$

where: $F_{j_1+j_2}(v)$ – distribution function of the sum of two random variables of the distribution functions $F_{j_1}(v)$ and $F_{j_2}(v)$.

The dependence (10) is also written in the form:

$$F_{j_1+j_2}(v) = (F_{j_1} * F_{j_2})(v) \tag{11}$$

If the random variables included in the composition of the sum have the same distribution:

$$F_{j_1}(v) = F_{j_2}(v) = \ldots = F_{j_n}(v) = F_j(v) \tag{12}$$

then:

$$F_{j+j}(v) = F_{2j}(v) = (F_j * F_j)(v) = F_j^{2*}(v) \tag{13}$$

and generally:

$$F_{nj}(v) = F_j^{n*}(v) \tag{14}$$

The same relation occurs for the probability density function:

$$f_{nj}(v) = f_j^{n*}(v) \tag{15}$$

The probability of using hot water k times in the time interval $< \tau_p, \tau_k)$ has been defined by the dependence (5). If it is assumed that:

$$\alpha(\tau_k) - \alpha(\tau_p) = \alpha \tag{16}$$

then:

$$p_k = \frac{\alpha^k}{k!} e^{-\alpha} \tag{17}$$

where: α – mean number of intakes (openings of valves) of hot water in the time interval $< \tau_p, \tau_k)$.
For $k = 0$ the above dependence defines the probability of non-opening of any valve in the time interval analysed.

It has been assumed that the number of intakes (openings of valves) and volume of water used during a single intake are values independent of each other. The probability that the global consumption will not, therefore, exceed the determined value m' when the valves will be opened k times is (Feller 1978):

$$p(k, v \le v') = p_k F_{kj}(v') \tag{18}$$

Introducing the dependences (14) and (17) to the above formula we receive:

$$p(k, v \le v') = \frac{\alpha^k}{k!} e^{-\alpha} F_j^{k*}(v') \tag{19}$$

Under the total probability theorem (2) it can be defined that the probability of not exceeding the given value of the consumption v by the inhabitants of the house is:

$$p(v \le v') = H(v') = \sum_{k=1}^{+\infty} p(k, v \le v') \tag{20}$$

After substituting the dependence (18) we obtain:

$$H(v') = \sum_{k=1}^{+\infty} \frac{\alpha^k}{k!} e^{-\alpha} F_j^{k*}(v') \tag{21}$$

The distribution, the function of which is described by the above formula, is called the compound Poisson distribution. With the assumptions adopted it affords a general definition (for any distribution of a single intake) the probability distribution of hot water consumption in any time interval by any group of users.

4. Proposed Model of Hot Water Consumption and Its Properties

It follows from the preceding analysis that the process of opening the valves (intakes) is described by the Poisson process with variable intensity a, of pilot function $\alpha(\tau)$, describing the mean number of events (valve openings) from the initial moment to the moment τ. The events occurring in the Poisson process create the sequence of events called the Poisson stream. Every opening of a bib-valve is assigned an intake of hot utility water. This intake is a random variable and can be described by some distribution of the function F. For further analysis it is assumed that for the given time section in all objects the variability of a single intake of hot water is defined by the same distribution. If every opening of the valve is assigned a concrete amount of the hot water v_j consumed then the distribution function F is concentrated in one point, the process of consumption is reduced to the simple Poisson process (Feller 1978), and the dependence (21) is reduced to the expression:

$$P\{X(\tau) - X(0) = kv_j\} = \frac{\alpha^k}{k!}e^{-\alpha} \tag{22}$$

On the other hand, if we assume that the number is constant and is k, then dependence (22) is reduced to the formula:

$$H(v) = F_j^{k*}(v) \tag{23}$$

As can be seen, the distribution function of variability of the amount of hot water consumed is defined by the k-fold integral of convolution of a single intake distribution function. The sum of variables described by the Poisson compound distribution in which the probability distribution of random variables forming the Poisson stream is the same, is the variable also described by the compound Poisson distribution of the parameter α equal to the sum of parameters α of the variables added.

$$\alpha = \sum_{i=1}^{n} \alpha_i \tag{24}$$

The analogue relation occurs for the difference of the Poisson streams.

The distributions of variables, the sum of which produces the same distribution, are called "regenerative" distributions. Of among the distributions varying stepwise, only the Poisson distributions (simple and compound) have such a property. This property results from that of the Poisson process, since the Poisson distribution belongs to the category of infinitely divisible distributions, which means that for any k it may be presented as a k-times composition (convolution) of a certain probability distribution. In other words, the random variable in this distribution can be presented as the sum k of certain, identical random variables of the same distribution where k can be optional. This means that if:

$$h(v) = e^{-\alpha} \sum_{k=0}^{\infty} \frac{\alpha^k}{k!}\{f\}^{k*} \tag{25}$$

and:

$$h_1(v) = h_2(v) = \ldots h_j(v) = \ldots h_n(v) \tag{26}$$

then the distribution of the variable $h_n(v)$ is described by the dependence:

$$h_n(v) = e^{-\alpha_n} \sum_{k=0}^{\infty} \frac{\alpha_n^k}{k!} \{f\}^{k*} \tag{27}$$

where:

$$\alpha_n = \frac{\alpha}{n} \tag{28}$$

The property of the compound Poisson distribution discussed can be used to model hot water consumption by any group of users. If it is assumed that the distribution of a single intake is the same for each of the users, then the parameter α of the distribution, describing the mean number of intakes by the group of users is determined by the sum of the parameters α_i (mean product of intakes) of the particular users (dependence 24). On the other hand, knowing the value of the parameter α for the group of identical users it is possible, taking advantage of this property, to determine the parameter α_j of a single user (dependence 28).

5. Determination of the Distribution of the Total Water Consumption in the Given Time Interval

It follows from the investigations that the distribution of probability of a single intake of hot water can be determined by means of the exponential distribution (Szaflik 1992). The function of the exponential distribution is defined by the formula:

$$F(v) = 1 - e^{\lambda v} \tag{29}$$

In order to define the distribution function of a compound Poisson distribution the distribution of the sum n of the identical variables of the exponential distribution must be determined. As the result of n-times computation of the convolution of the function of exponential distribution we receive the formula for the distribution function of the sum n of random variables of this distribution:

$$F(v) = F^{n*}(v) = 1 - e^{-\lambda v} \sum_{j=0}^{n-1} \frac{(\lambda v)^j}{j!} \tag{30}$$

The distribution obtained constitutes a particular case of the gamma distribution, i.e. gamma distribution of the integral exponent and is called the Erlang distribution. Using dependences (21) and (30) we receive that for the function of distribution composed of both the Poisson and exponential distributions:

$$H(v) = e^{-\alpha} \left\{ 1 + \sum_{n=1}^{\infty} \frac{\alpha^n}{n!} \left[1 - e^{-\lambda v} \sum_{j=0}^{n-1} \frac{(\lambda v)^j}{j!} \right] \right\} \tag{31}$$

From further transformations we receive:

$$h(v) = 1 - e^{-\alpha-\lambda v} \sum_{n=1}^{\infty} \frac{\alpha^n}{n!} \sum_{j=0}^{n-1} \frac{(\lambda v)^j}{j!} \tag{32}$$

The density of the distribution is defined by the dependence:

$$h(v) = \lambda e^{-\alpha-\lambda v} \sum_{n=1}^{\infty} \frac{\alpha^n}{n!} \frac{(\lambda v)^{n-1}}{(n-1)!} \tag{33}$$

For a consumption equal to zero ($v = 0$) the value of the distribution function is defined from the formula:

$$H(0) = 1 - e^{-\alpha} \sum_{n=1}^{\infty} \frac{\alpha^n}{n!} = e^{-\alpha} \tag{34}$$

This describes the probability of the bib-valve not opening, and thus the probability of the lack of outflow.

If the number of intakes in the time interval analysed is constant and equals k, then the distribution received will be transformed into the gamma distribution of the parameters k and λ, used to describe the variability of water consumption (11).

For the compound Poisson distribution presented, as also the gamma distribution, one of the parameters decides as to the form of the distribution course, the other is the scale parameter. The form of the distribution course depends on parameter α, that of λ being the scale parameter.

For the probability distribution derived one can define the dependence between the moments and the parameters of distribution. It is also possible to compute the moments from the test. Assuming that the given distribution describes properties of the variables analysed, it is possible, basing on moments from the test, to determine the parameters of the distribution and then check correctness of the assumption accepted. The mean value is defined by the formula:

$$\mu = \frac{\alpha}{\lambda} \tag{35}$$

and the square of mean deviation – variance σ^2, by dependence

$$\sigma^2 = 2\frac{\alpha}{\lambda^2} \tag{36}$$

Dependence (35) defines the volume of water consumed in the given time interval. This volume is the quotient of the mean number of openings of valves and the inverse of the mean consumption at a single opening of a valve. Assuming that the moments from the test correspond to the theoretical:

$$m = \mu \tag{37}$$

$$s^2 = \sigma^2 \tag{38}$$

we obtain the dependence between the parameters and the moments:

$$\alpha = 2\frac{m^2}{s^2} \tag{39}$$

$$\lambda = 2\frac{m}{s^2} \tag{40}$$

A computation programme to determine the theoretical distribution function according to the compound Poisson distribution has been worked out. The exemplary graph of a theoretical distribution function of probability of occurrence of diurnal parsings for the compound Poisson distribution with plottted measurement data, has been presented on the probability scale in Figure 3.

For the cases analysed conformity of the theoretical with the empirical distribution has been checked by means of Kolmogorov-Smirnov test. The conditions of the test were always satisfied, the theoretical distribution derived represents the variability of the hot water consumption in the time interval determined very well.

6. Determination of the Diurnal Distribution of Hot Water Consumption for a House with an Arbitrary Number of Inhabitants

The number of hot water intakes by one inhabitant in 24 hrs and the values of a single intake have been determined for multifamily residential houses basing on the proposed model and results of measurements. The values received have been determined basing on measurement data collected by the author (Szaflik 1985) in five house provided with bathtubs, washbasins and kitchen sinks. The smallest house had 108, and the biggest 304 inhabitants. The values interpolated for objects with the number of inhabitants outside the range examined are to be regarded as less reliable.

It follows from the preceding investigations that the distribution of hot water consumption in a residential house can be described by the compound Poisson distribution. This belongs to the category of infinitely divisible distributions, which means that it can be presented as a distribution of the sum of identical random variables. Thus the distribution of the hot water consumption in a residential house can be presented as the distributions of the sum of hot water intakes by particular residents. If the unitary distribution of hot water consumption by one inhabitant is determined, then knowing the number of inhabitants it will be possible to determine the distribution of the global hot water consumption by the whole house. If the values of the unitary parameters α_j and λ, where: α_j – the mean number of bib-valves openings by one inhabitant, λ – the mean water consumption at a single valve opening are known, then the parameters of the consumption by M inhabitants of the house are α_M and λ, where:

$$\alpha_M = M\alpha_j \tag{41}$$

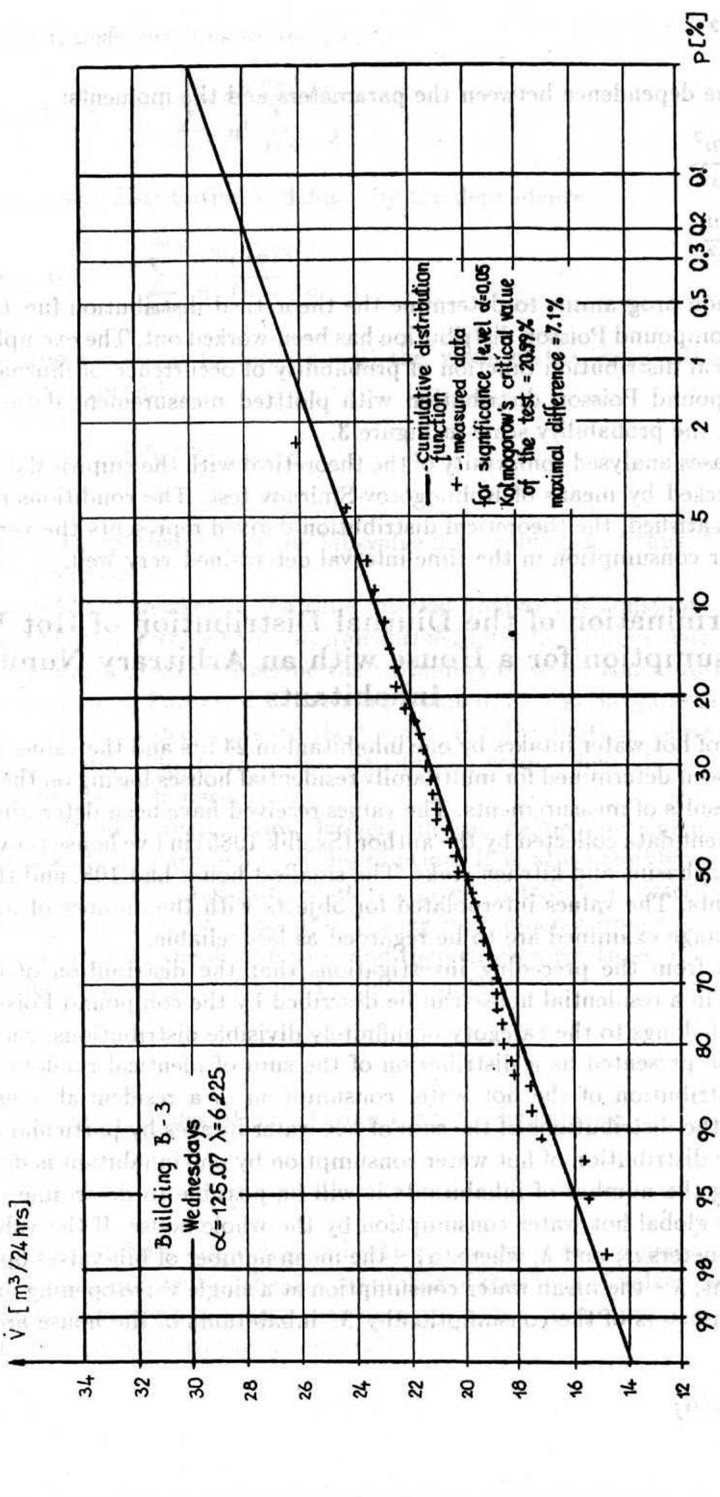

Fig. 3. Graph of the theoretical distribution function of probability of occurrence of diurnal distributions according to the compound Poisson distribution with measurement data

and the parameter λ has a constant value. It has been assumed that for a definite time section the mean single intake for all inhabitants is identical and the mean number of intakes for one inhabitant is the same. Having data from measurements it is possible to determine the distribution of demand for hot water in any time intervals, e.g. for a particular day, hours or for 24 hours.

The values for hot water consumption comprise the diurnal consumption. The averaged values of the parameters of consumption distribution for every kind of week day, determined according to the measurements, referred to one inhabitant/24 hrs, have been presented in Table 1. The averaging of the parameters have been executed assuming:

$$\sum_{i=1}^{n} m_i = \sum_{i=1}^{n} \frac{M_i \, \alpha_j}{\lambda} \tag{42}$$

$$\sum_{i=1}^{n} s^2 = 2 \sum_{i=1}^{n} \frac{M_i \, \alpha_j}{\lambda^2} \tag{43}$$

where n is the number of the objects studied. Then:

$$\alpha_j = \frac{2\left(\sum\limits_{i=1}^{n} m_i\right)^2}{\sum\limits_{i=1}^{n} M_i \sum\limits_{i=1}^{n} s_i^2} \tag{44}$$

$$\lambda = \frac{2 \sum\limits_{i=1}^{n} m_i}{\sum\limits_{i=1}^{n} s_i^2} \tag{45}$$

Table 1

Averaged unitary parameters of distribution of hot water consumption variability in the examined objects, for the week days analysed

Days	λ [m^{-3}]	$\frac{1}{\lambda}$ [m^3]	α_1 $\left[\frac{1}{m \cdot 24hrs}\right]$
Working days (Wednesdays)	5.481	0.182	0.508
Free Saturdays	3.624	0.276	0.448
Sundays	3.445	0.290	0.372

The computed values of a single intake ($1/\lambda$) presented in the table are substantial and the mean number of intakes per inhabitant relatively small ($\alpha_j < 1$). This proves that the consumption variability is, in reality, greater than results from the stochastic character of the intake process, it can be the effect of the consumption changing in an annual cycle (Mielcarzewicz, Janczewski 1984, Siwoń, Stanisławski 1984).

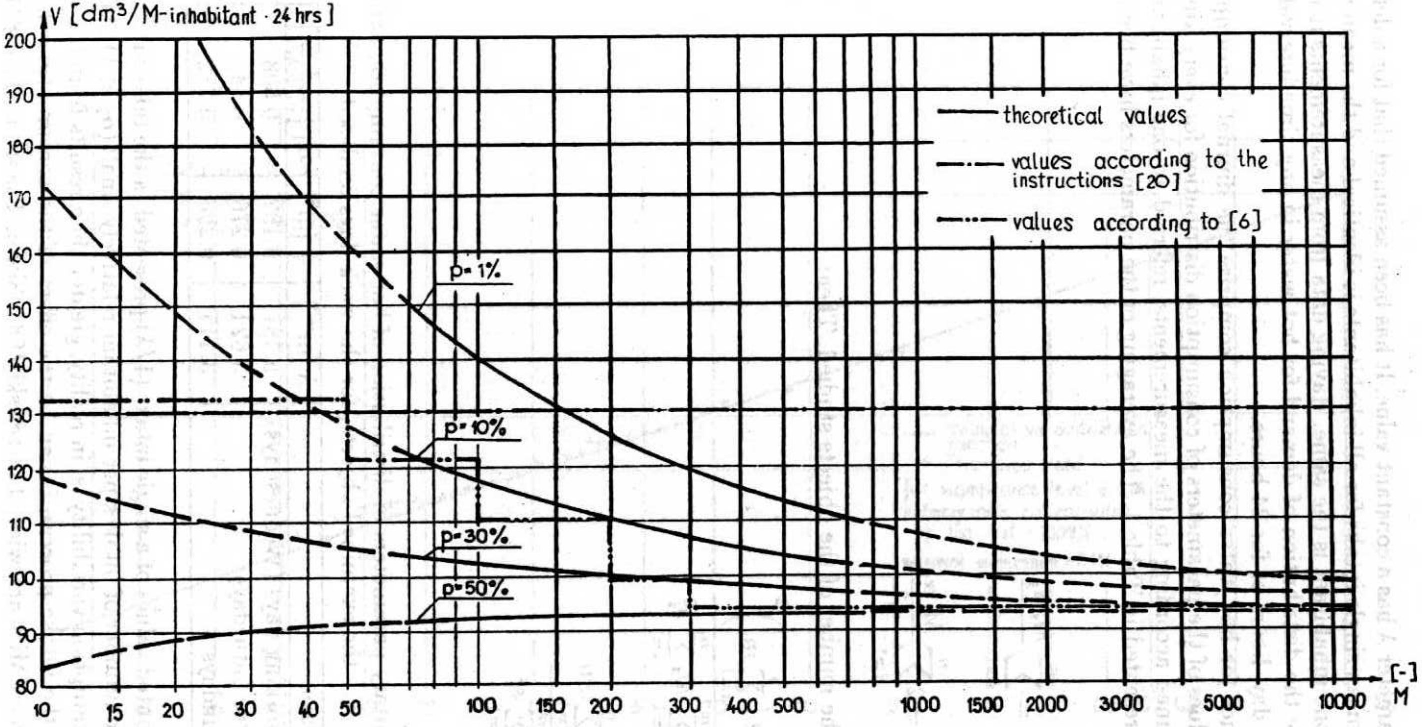

Fig. 4. Values of unitary utility hot water consumption ($dm^3/M \cdot 24$ h) of assumed values of probability of excess on working days (Wednesdays) in the function of number of inhabitants

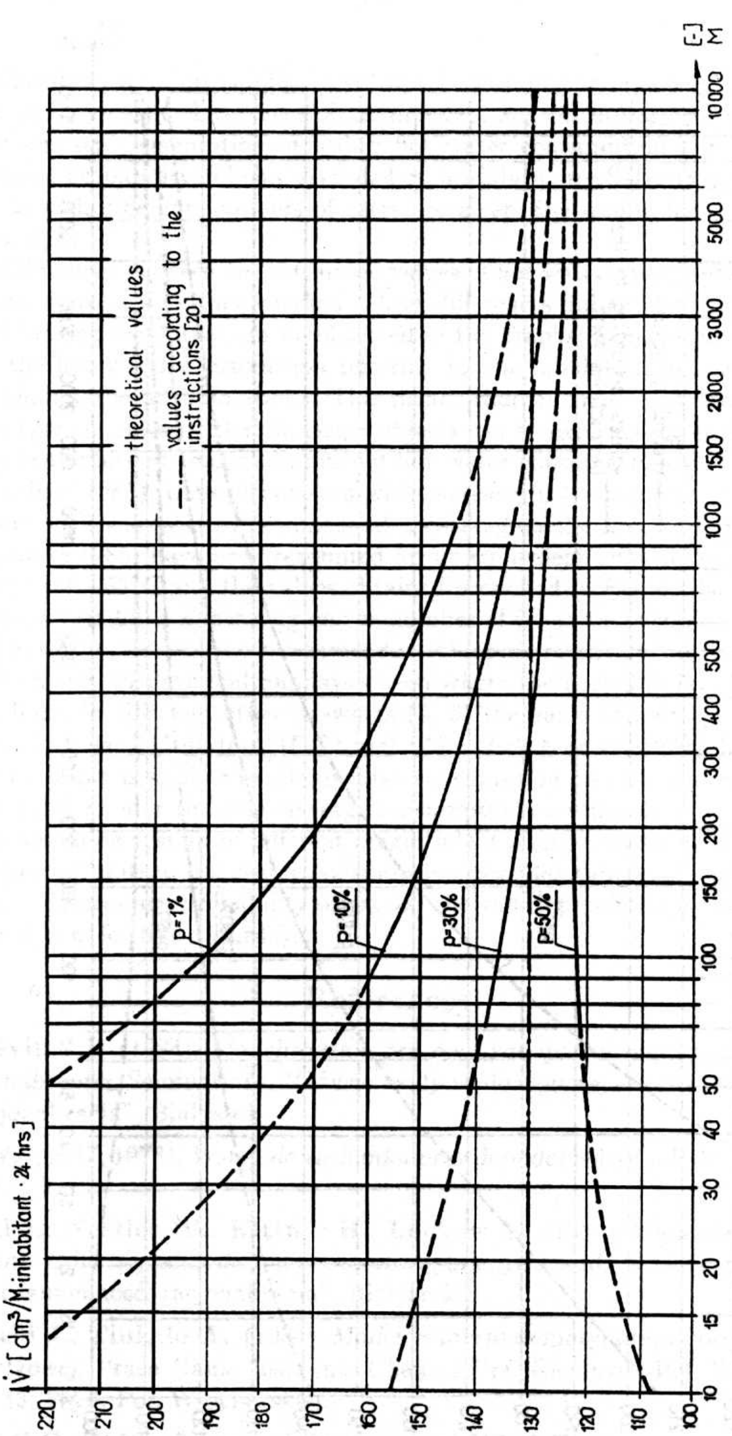

Fig. 5. Values of unitary utility hot water consumption (dm^3/$M \cdot 24$ h) of assumed values of probability of excess on free Saturdays in the function of number of inhabitants

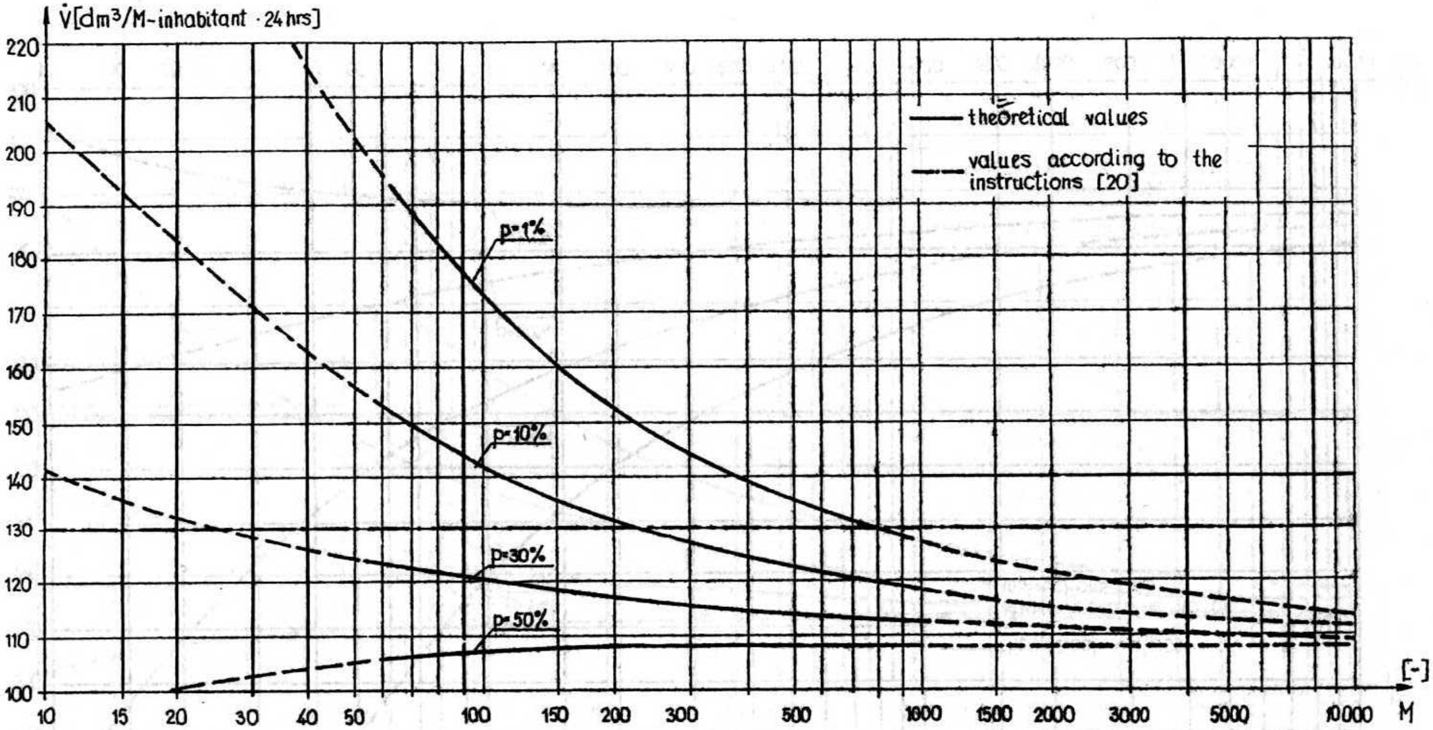

Fig. 6. Values of unitary utility hot water consumption ($dm^3/M \cdot 24$ h) of assumed values of probability of excess on Sundays in the function of number of inhabitants

The distributions of probability of hot water consumption in objects with different numbers of inhabitants have been determined basing on unitary parameters. The values of unitary consumption of utility hot water consumption ($cm^3/M \cdot 24$ hrs) with different probability of being exceeded on workdays, free Saturdays and Sundays, for objects with different numbers of users are given, the results being presented in Figures 4, 5, 6.

The differences between the estimated values of hot water consumption and those determined by tests were then checked. These differences did not exceed ten per cent.

The following regularity can be observed in the graphs: at constant value of probability, the hot water consumption referring to one inhabitant decreases with the increase the number of inhabitants. This means that in smaller objects higher consumption (per inhabitant) than in larger objects can be expected more frequently. In Germany the values of maximum diurnal hot water consumption were given (Knobloch, Lindeke 1985), these decreasing with increase in the number of inhabitants. The norms of consumption recommended according to the literature quoted (Knobloch, Lindeke 1985) have been recounted for an equivalent quantity of water with a temperature of +55°C and the values obtained presented in Fig. 4. The character of consumption variability depending on the number of inhabitants corresponds to that obtained in this paper, but the recommended values are relatively low (particularly in relation to free Saturdays and Sundays), even where the probabilities of being exceeded are high. In this and other drawings (5, 6) the value of unitary consumption recognized in Poland (130 $dm^3/M \cdot 24$ hrs) (Zarządzenie nr 47, 1974), has also been shown. This value is similar to the average consumption on non-working days (free Saturdays) and greater than the average consumption on other days.

When designing system of different magnitude it seems reasonable to assume different values of unitary hot water consumption (smaller values for larger objects) Assuming a determined probability of excess, the values presented in this paper can be accepted, e.g. for free Saturdays.

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Summary

The consumption of utility hot water in residential multi-family houses in an interval of time results from the effect of many factors, the quantitative influence of which is very difficult, if not sometimes impossible, to determine. It can be accepted that water intake is a stochastic process. The results of research presented here enable formulation of the following conclusions:

1. The process of hot water consumption can be described with the aid of the Poisson process of variable intensity, in which the events constitute single intakes of hot water, a quantity of water being assigned for each event.
2. The distribution of the amount of water used in a period of time is then described by means of the compound Poisson distribution. The parameters of this distributions are: mean number of intakes and inverse average amount of water used at a single intake. For the analysed cases of hot water consumption in a given time section, the-empirical distribution of frequency and respective theoretical distribution, for the significance level, 5% satisfied the Kolmogorov-Smirnov test conditions.
3. The properties of the process applied enable the determination on the basis of parameters of consumption distribution for a given group of users, for the distribution for a chosen group.
4. Thanks to the general character of the dependences derived the possibility exists of applying them for probabilistic modelling of cold water consumption.

Matematyczny model procesu poboru ciepłej wody

Streszczenie

W pracy zajęto się zagadnieniem stochastycznego modelowania poboru ciepłej wody użytkowej dla potrzeb wymiarowania układów służących do jej przygotowania.

W początkowej części pracy przeprowadzono analizę literatury dotyczącej modelowania i badań poboru ciepłej wody użytkowej. Wynikało z niej, że w dotychczas opracowanych stochastycznych modelach procesu poboru ciepłej wody w dłuższych odcinkach czasu nie wykorzystywano w sposób dostateczny własności procesów stochastycznych. Na tej podstawie określono cel pracy. Celem jej było opracowanie stochastycznego modelu rozbioru ciepłej wody w dłuższych odcinkach czasu, w budynkach mieszkalnych wielorodzinnych i estymacja parametrów tego modelu.

W następnej części pracy przedstawiono ogólny model procesu poboru wody w budynkach mieszkalnych wielorodzinnych. Poszczególnym rodzajom dni, takim jak dni robocze, dni wolne od pracy (wolne soboty), dni świąteczne (niedziele), przypisano odpowiadające im cykle. Założono stochastyczną powtarzalność cykli. Modelowanie procesu poboru przeprowadzono przy użyciu procesu Poissona o zmiennej intensywności. Wykazano, że przy określonych założeniach pojedyncze pobory ciepłej wody w czasie danego cyklu można traktować jako zdarzenia w pewnym procesie Poissona. Każdemu zdarzeniu przyporządkowano ilość pobranej wtedy ciepłej wody. Przyjęto, na podstawie badań, że jest ona zmienną losową o rozkładzie wykładniczym.

W dalszej części pracy określono teoretyczny rozkład zmienności sumarycznego poboru ciepłej wody w dowolnym okresie czasu. Parametrami tego rozkładu są średnia ilość pojedynczych poborów w tym okresie czasu i odwrotność średniej ilości pobranej wody przy pojedynczym poborze. Następnie przeprowadzono estymację parametrów i sprawdzono zgodność otrzymanych rozkładów z danymi pomiarowymi. Otrzymano zadowalające wyniki.

Własności procesu Poissona zastosowanego do modelowania poboru ciepłej wody, przy pewnych założeniach odnośnie poboru, pozwalają na podstawie parametrów procesu dla zmiennej liczby odbiorców określić parametry dla dowolnej ich grupy. Wymieniona własność i otrzymane wyniki z przeprowadzonych badań zużycia ciepłej wody umożliwiły dla budynków mieszkalnych wielorodzinnych określenie poboru ciepłej wody użytkowej dla dni roboczych, dni wolnych od pracy (wolnych sobót) i dni świątecznych (niedziel) w funkcji liczby mieszkańców i założonego prawdopodobieństwa przewyższenia.