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Analysis of Variability as Sum of Simple Stochastic Processes

1. Introduction

Performing the measurement of any arbitrary soil parameter such as moisture content, unit weight, shearing resistance, cone tip resistance etc. along a defined straight line, the results of the measurement may by presented in a form of a stochastic process (Sulikowska 1987). Fig. 1 illustrates such examplary processes. If the measured property is such as, for instance, moisture content, the problem consists in determination the process parameters, taking into account measurement errors if they can be assessed. However, if the property is a sum of two, sometimes mutually dependent values then the problem becomes more complicated. Let us consider, as an example, a classical case of shearing resistance given by a simple Coulomb criterion:

$$\tau = \sigma * \tan(\phi) + c \tag{1}$$

Measured value of τ is a sum of two components: friction – $\tan(\phi)$ and cohesion – c. Thus the resultant process τ is a sum of these component processes. Generally, those processes are correlated, and the correlation should be negative similarly as for common random variables. Additionally, the friction component is multiplied by a normal stress σ, which can be a process itself, but this complicates the issue considerably.

Processes represented in figs 1a, b, c are not stationary. Their expected value increases with depth and scattering of results, described by the variance of the process, can also vary. Let us assume that we succeed in transforming the examined process into a stationary one by either subtracting the trend or if such an approach fails, performing

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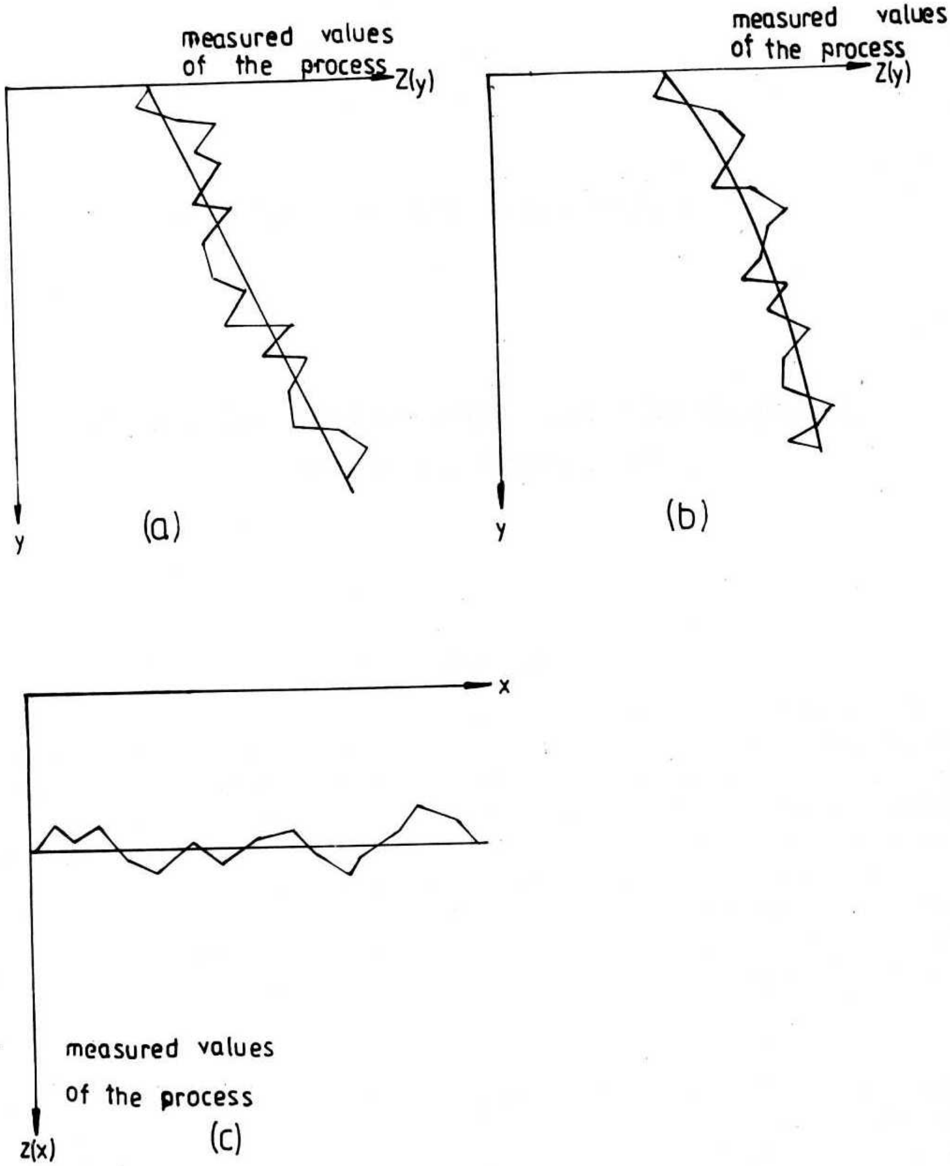

Fig. 1. Soil properties as stochastic processes

the differential operations until the expected value is constant. Let us also assume that the expectancy $E[z]$ of the process Z, which is a resultant of two other processes, was approximated by a straight line $E[Z|y] = by + c$ (fig. 1a) or by a parabola $E[Z|y] = ay^2 + by + c$ (fig. 1b). Further procedure should include two steps:

1. resolving the conditional expectation of the resultant process into conditional expectations of the components, for instance – in case of shearing resistance into the friction and cohesion;
2. resolving the fluctuations about the expectancy of the resultant process into the pertinent component fluctuations.

The first step can be solved traditionally by determination of the friction and cohesion components at different depths and then evaluation of their relative contribution in global shearing resistance. The knowledge of such contribution allows to resolve the resultant shearing resistance expectancy into the friction and cohesion expectances.

In the further part of this paper the second item will be examined. A special attention will be drawn to the following issues:

- possibilities of the resolution of the resultant fluctuation process into the component ones,
- examination of the conditions required of the resultant process to get resolved into more simple ones.

The whole problem can be simplified when two component processes are examined and one of them is treated as measurement errors. The resolution of the resultant process will then result in the removal of those errors.

To perform the calculation, one must first determine formulae that relate the parameters of the component processes to the resulting one. Main processes utilized in engineering practice are the gaussian ARIMA (p, d, q) ones. They are autoregressive integrated moving average processes where p represents the order of autoregression, d is the order of differential operation and q indicates the order of moving average. Summation and resolution of such processes will be described generally and the first and second order processes disturbed by measurement errors will be discussed more thoroughly.

2. Basic relations for summation of processes

In the great majority of practical applications one will have to deal with the summation of not more than three processes, one of which usually describing the measurement errors. That is why the sum of only three processes will be discussed here. Obviously, the formulae for any arbitrary number of component processes can be derived, but these formulae become increasingly complex.

An ARIMA (p, d, q) process is given by the expression (1):

$$\phi(B)\nabla^d X_t = \theta(B)a_t \tag{2}$$

where:

- X_t is a process value
- B is a backshift operator such that

$$BX_t = X_{i-t} \tag{3}$$

- ∇ is a differential operator such that

$$\nabla X_t = X_t - X_{t-1} = (1 - B)X_t \tag{4}$$

$$\nabla^2 X_t = \nabla(\nabla X_t) = X_t - 2X_{t-1} + X_{t-2} \tag{5}$$

- $\phi(B)$ and $\theta(B)$ are operational polynomials of p-th and q-th order

$$\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \ldots - \phi_p B^p \tag{6}$$

$$\theta(b) = 1 - \theta_1 B - \theta_2 B^2 - \ldots - \theta_q B^q \tag{7}$$

- a_t is a gaussian white noise process with zero expectancy and variance equal to σ_a^2,
- t represents a position of X_t in the whole realization.

ARIMA $(p, 0, q)$ is a stationary autoregressive and moving average process ARMA (p, q). ARMA $(p, 0)$ becomes an autoregressive process AR (p) of p-th order, ARMA $(0, q)$ reduces to a moving average MA (q) process of q-th order.

Let us now assume that three ARIMA processes $X_t^{(1)}$, $X_t^{(2)}$, $X_t^{(3)}$ are given and that their orders are (p_1, d_1, q_1), (p_2, d_2, q_2), (p_3, d_3, q_3) correspondingly. One of them, in the most simple case, can be a white noise ARIMA (0, 0, 0) and can be treated as an uncorrelated process of measurement errors. In a general case however, all three of them can be mutually correlated.

The task is to determine the character and order of the resultant process Z_t which is the sum of the components. Also its parameters related to those of the component processes should be found.

$$Z_t = X_t^{(1)} + X_t^{(2)} + X_t^{(3)} \tag{8}$$

If both sides of the expression (2) are multiplied by the operator $\phi^{-1}(B)\nabla^{-d}$, the following will be received:

$$X_t = \phi^{-1}(B)\nabla^{-d}\theta(B)a_t \tag{9}$$

Basing on formula (9), the component processes $X_t^{(i)}$ for $i = 1$ to 3 can be written as:

$$X_t^{(i)} = \phi_i^{-1}(B)\nabla^{-d(i)}\theta_i(B)a_t^{(i)} \tag{10}$$

ϕ_i^{-1} are inverses of the operational polynomials of p_i-th order given by the expression (6), $\theta_i(B)$ are operational polynomials of q_i-th order, $a_t^{(i)}$ are white noises with zero expectances and variances equal to σ_{ai}^2. These white noises can be mutually correlated it is their covariances may not be equal to zero so also the component processes will be correlated. Substituting (10) into (8) and in order to get rid of the negative powers multiplying both sides by $\phi_1(B)\ \phi_2(B)\ \phi_3(B)\ \nabla^{d(1)}\ \nabla^{d(2)}\ \nabla^{d(3)}$, the following formula is received:

$$\begin{aligned}&\phi_1(B)\phi_2(B)\phi_3(B)\nabla^{(d(1)+d(2)+d(3))}Z_t =\\&= \phi_2(B)\phi_3(B)\theta_1(B)\nabla^{(d(2)+d(3))}a_t^{(1)}+\\&+\phi_1(B)\phi_3(B)\theta_2(B)\nabla^{(d(1)+d(3))}a_t^{(2)}+\\&+\phi_1(B)\phi_2(B)\theta_3(B)\nabla^{(d(1)+d(2))}a_t^{(3)}\end{aligned} \tag{11}$$

The polynomial $\phi_1(B)\phi_2(B)\phi_3(B)\nabla^{(d(1)+d(2)+d(3))}$ is of $(p_1+p_2+p_3+d_1+d_2+d_3)$-th order and polynomials situated beside are of $(p_2+p_3+q_1+d_2+d_3)$, $(p_1+p_3+q_2+d_1+d_3)$ and $(p_1+p_2+q_3+d_1+d_2)$ orders correspondingly. Thus, the expression (11) defines an ARIMA (p,d,q) process:

$$\phi(B)\nabla^d Z_t = \theta(B)u_t \tag{12}$$

in which:

- the order of autoregression equals $p_1+p_2+p_3$,
- the order of differential operation equals $d_1+d_2+d_3$,
- the order of moving average equals the greatest order of polynomials on the right hand side of the equation (11).

As can be seen from the above consideration, summing the ARIMA processes yields a new ARIMA process of higher orders. Not only does the order of autoregression increase but so do the orders of differential operation and moving average. Even a sum of simple processes produce a higher order result and make the calculation procedure tedious.

To preserve the generality of considerations a general ARIMA process will be discussed first. By assuming that some or all of pertinent numbers p, d, q are equal to zero, it can be then reduced to ARMA, AR, MA or even to a white noise. Because of laborious calculations when dealing with high order processes, a detailed analysis will be performed only for relatively simple autoregressive processes disturbed by a white noise.

3. Single ARIMA process disturbed by measurement errors

Let us assume that a process Z_t is measured and it is a sum of a process $X_t^{(1)}$ – ARIMA (p_1, d_1, q_1) and a white noise $X_t^{(2)}$ it is ARIMA (0, 0, 0). Expression (11) will then take a shape:

$$\phi_1(B)\nabla^{d(1)}Z_t = \theta_1(B)a_t^{(1)} + \phi_2(B)\nabla^{d(1)}a_t^{(2)} \tag{13}$$

(11) represents the process in which $p = p_1$, $d = d_1$, $q = \max(q_1, p_1 + d_1)$. Using formula (12) it can is seen that u_t is a white noise having a zero expectancy and a variance which depends upon the variances and parameters of the component processes.

Two particular cases will be discussed in detail; an AR (1) process disturbed by a white noise and a sum of two AR(1) also with a white noise interference.

3.1. Superposition of an AR(1) process and a white noise

AR(1) is the most simple autoregressive process whose order $p = 1$. If a white noise is added to it then:

$$\begin{array}{lllll} p_1 = 1; & \phi_1(B) = 1 - \phi_1^{(1)}(B); & \theta_1(B) = 1; & d_1 = 0; & q_1 = 0; \\ p_2 = 0; & \phi_2(B) = 0; & \theta_2(B) = 0; & d_2 = 0; & q_2 = 0; \end{array}$$

and the resultant process will be characterized by:

$$p = p_1 = 1, d = 0, q = \max\ (q_1, p_1 + d_1).$$

thus if a white noise is added to an AR(1) process, the resultant is an ARIMA $(1,0,1)$ = ARMA (1, 1) process.

When an assumption is made that a soil property, which is analysed, is described by the AR(1) process, and an uncorrelated random errors are made during the measuring, the resultant process will be ARMA (1, 1). According to (13) it can be written as:

$$\left(1 - \phi_1^{(1)}B\right) Z_t = a_t^{(1)} + \left(1 - \phi_1^{(1)}B\right) a_t^{(2)} \tag{14}$$

so:

$$Z_t = \phi_1^{(1)} Z_{t-1} + a_t^{(1)} + a_t^{(2)} - \phi_1^{(1)} a_{t-1}^{(2)} \tag{15}$$

By analogy, utilizing (16) for $\phi(B) = 1 - \phi_1(B)$ and $\theta(B) = 1 - \theta_1(B)$ (ARMA (1, 1)) there will be:

$$Z_t = \phi_1 Z_{t-1} + u_t - \theta_1 u_{t-1} \tag{16}$$

It is very easy to notice that as (15) and (16) describe the same process so $\phi_1 = \phi_1^{(1)}$. To determine θ_1 and a variance σ_u^2 of u_t process either autocovariance functions of processes (15) and (16) or their spectra must be compared. Autocovariance functions were compared here.

An autocovariance function $\gamma(k)$ for the process (18) is defined as an expected value:

$$\begin{aligned} \gamma(k) &= E[Z_t Z_{t-k}] = E\left[\phi_2^{(1)} Z_{t-1} Z_{t-k} + a_t^{(1)} Z_{t-k} + a_t^{(2)} Z_{t-k} - \phi_1^{(1)} a_{t-1}^{(2)} Z_{t-k}\right] \\ \gamma(k) &= \phi_1^{(1)} \gamma(k-1) + \gamma_{az}^{(1)}(k) + \gamma_{az}^{(2)}(k) - \phi_1^{(1)} \gamma_{az}^{(2)}(k-1) \end{aligned} \tag{17}$$

where:

$$\begin{array}{lll} \gamma_{az}^{(1)}(k) = \sigma_{a1}^2 & = v_1 & \text{for} \quad k = 0 \\ & = 0 & \text{for} \quad k > 0 \\ \gamma_{az}^{(2)}(k) = \sigma_{a2}^2 & = v_2 & \text{for} \quad k = 0 \\ & = 0 & \text{for} \quad k > 0 \end{array} \tag{18}$$

and $\phi_1 = \phi_1^{(1)}$.
After substituting (18) into (17) one receives for $k = 0$:

$$\gamma(0) = \phi_1\gamma(-1) + v_1 + v_2 - \phi_1\gamma_{az}^{(2)}(-1)$$

Knowing that the autocovariance function is symmetrical it is $\gamma(-1) = \gamma(1)$:

$$\gamma_{az}^{(2)}(-1) = E\left[a_{t-1}^{(2)} Z_t\right] = E\left[\phi_1^{(1)} a_{t-1}^{(2)} Z_{t-1} + a_{t-1}^{(2)} a_t^{(1)} + a_{t-1}^{(2)} a_t^{(2)} - \phi_1 a_{t-1}^{(2)} a_{t-1}^{(2)}\right]$$

$$\gamma_{az}^{(2)}(-1) = \phi_1 v_2 - \phi_1 v_2 = 0$$

Hence:

$$\gamma(0) = \phi_1\gamma(1) + v_1 + v_2 \tag{19}$$

For $k = 1$ after the substitution into (20):

$$\gamma(1) = \phi_1\gamma(0) + \gamma_{az}^{(1)}(1) + \gamma_{az}^{(2)} - \phi_1\gamma_{az}^{(2)}(0)$$

$$\gamma(1) = \phi_1[\gamma(0) - v_2] \tag{20}$$

and for $k > 1$:

$$\gamma(k) = \phi_1^{(1)}\gamma(k-1) = \phi_1\gamma(k-1) \tag{21}$$

Similarly, the autocovariance function of the same compound process but given by (16) takes a form:

$$E\left[Z_t Z_{t-k}\right] = E\left[\phi_1 Z_{t-1} Z_{t-k} + u_t Z_{t-k} - \theta_1 u_{t-1} Z_{t-k}\right] \tag{22}$$

For consecutive $k = 0, 1$ and $k > 1$ one receives:

- for $k = 0$

$$\gamma(0) = \phi_1\gamma(1) + v - \theta_1\gamma_{uz}(-1) \tag{23}$$

where $v = \sigma_u^2$

$$\gamma_{uz}(-1) = E[u_{t-1}Z_t] = E\left[\phi_1 Z_{t-1}u_{t-1} + u_t u_{t-1} - \theta_1 u_t u_{t-1}\right]$$

$$\gamma_{uz}(-1) = v(\phi_1 - \theta_1) \tag{24}$$

Now substituting (24) into (23):

$$\gamma(0) = \phi_1\gamma(1) + v\left(1 - \theta_1\phi_1 + \theta_1^2\right) \tag{25}$$

- for $k = 1$

$$\gamma(1) = \phi_1\gamma(0) + \gamma_{uz}(1) - \theta_1\gamma_{uz}(0)$$

and because $\gamma_{uz}(1) = E[u_{t+1}Z_t] = 0$ as the current value of the process does not depend upon the future random impulse, so:

$$\gamma(1) = \phi_1\gamma(0) - \theta_1 v \tag{26}$$

- for $k > 1$

$$\gamma(k) = \phi_1\gamma(k-1) \tag{27}$$

Now one should compare the expressions for autocovariances of the processes given by (15) and (16). This must be done for consecutive k. If $k = 0$ then from (19) and (26):

$$\phi_1\gamma(1) + v_1 + v_2 = \phi_1\gamma(1) + v\left(1 - \theta_1\phi_1 + \theta_1^2\right)$$

$$v_1 + v_2 = v\left(1 - \theta_1\phi_1 + \theta_1^2\right) = A \tag{28}$$

One can conclude from the above formula that the variance v of the white noise generating the resultant process, is not just a simple sum of the variances v_1 generating AR(1) and v_2 of measurement errors, but also depends on the resultant process parameters θ_1 and ϕ_1.

For $k = 1$ formulae (20) and (26) are compared and:

$$\phi_1\left(\gamma(0) - v_2\right) = \phi_1\gamma(0) - \theta_1 v$$

$$v_2\phi_1 = v\theta_1 \tag{29}$$

For $k > 1$, using (21) and (27), an identity is received.

It is easy now, to find the values of the parameters of the resultant process if the parameters of the component processes are known. From (18) $\phi_1 = \phi_1^{(1)}$ and from (29):

$$v = \frac{\phi_1 v_2}{\theta_1} \tag{30}$$

Substituting (30) into (28) produces a quadratic equation of θ_1:

$$\theta_1^2 - \theta_1\left[\phi_1 + \frac{v_1 + v_2}{\phi_1 v_2}\right] + 1 = 0 \tag{31}$$

The solution consists of two roots whose product equals one. Only one root fulfils the requirement of the process inversibility as only one root lies in the interval $-1 < \theta_1 < 1$. The resultant process has got a variance which can be computed by taking an expectancy of a square of the expression (16):

$$E[Z_t Z_t] = v\frac{1 - 2\phi_1\theta_1 + \theta_1^2}{1 - \phi_1} \tag{32}$$

It is worth noting that the inverse problem (resolution of the ARMA (1,1) into AR(1) and white noise) is not always possible. It is seen from (30) that the variance v_2 of the white noise added to the AR(1) can be calculated from the formula:

$$v_2 = \frac{\theta_1 v}{\phi_1} \tag{33}$$

As v and v_2 must be always positive, it implies that θ_1 and ϕ_1 of the resultant process must be of the same sign. The above is indispensable if one wants to get an AR(1) process after subtraction of a white noise from ARMA (1, 1). If this condition is not satisfied, a given ARMA (1, 1) process can be resolved but only into an AR(1) and a moving average process of the first order MA(1) (Knabe 1987). The relations presented above make it easy to separate measurement errors as a white noise if it is known that the process to which those errors are added is AR(1), so the process which is recorded is ARMA(1, 1).

3.2. Sum of two AR(1) processes and a white noise

It was proved in the point 3.1 that the resultant of an AR(1) and a white noise is an ARMA (1, 1) process. Hence the sum of two AR(1) and a white noise can be treated as a sum of of an AR(1) and ARMA (1, 1). Let us assume that the measured process Z_t is a sum of a process X_t - ARMA(1, 1) and a process Y_t - AR(1). These processes are described by the following formula:

$$\left.\begin{aligned} (1-\phi_x B)X_t &= (1-\theta_x B)u_t \\ (1-\phi_y B)Y_t &= w_t \end{aligned}\right\} \tag{34}$$

According to (11) the resultant process $Z_t = X_t + Y_t$ satisfies the relation:

$$(1-\phi_x B)\ (1-\phi_y B)Z_t = (1-\theta_x B)\ (1- = \phi_y)u_t + (1-\phi_x B)w_t \tag{35}$$

Hence:

$$\begin{aligned} Z_i &= (\phi_x+\phi_y)Z_{t-1} - \phi_x\phi_y Z_{t-1} + u_t + w_t - (\theta_x+\phi_y)u_{t-1} - \phi_x w_{t-1} + \\ &+ \theta_x\phi_y u_{t-2} \end{aligned} \tag{36}$$

Expression (36) implies that the resultant process is ARMA (2, 2) so:

$$Z_t = \phi_1 Z_{t-1} + \phi_2 Z_{t-2} + e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} \tag{37}$$

Such a process depends upon five parameters $\phi_1, \phi_2, \theta_1, \theta_2, \sigma_e^2$. They can be determined by means of the parameters $\phi_x, \phi_y, \theta_x, \sigma_u^2, \sigma_w^2$ of the component processes. As before, an autocovariance functions of (36) and (37) are used to derive the relations for calculating the parameters of the resultant process. The autocovariance function $\gamma(k)$ of Z_t given by (37) is an expectancy:

$$\gamma(k) = E[Z_t Z_{t-k}] = [\phi_1 Z_{t-1}Z_{t-k} + \phi_2 Z_{t-2}Z_{t-k} + e_t Z_{t-k} - \theta_1 e_{t-1}Z_{t-k} - \theta_{t-2}Z_{t-k}]$$

For $k = 0, 1, 2$ one receives:

$$\gamma(0) = \phi_1\gamma(1) + = \phi_2\gamma(2) + \sigma_e^2\left[1 + \theta_1^2 + \theta_2^2 - \phi_1\theta_1 - \phi_2\theta_2 - \theta_2\phi_1^{(2)} + \phi_1\theta_1\theta_2\right] \tag{38}$$

$$\gamma(1) = \left[\phi_1\gamma(0) + \sigma_e^2(\theta_1\theta_2 - \theta_1 - \theta_2\phi_1)\right] / (1-\phi_2) \tag{39}$$

$$\gamma(2) = \phi_1\gamma(1) + \phi_2\gamma(0) - \theta_2\sigma_e^2 \tag{40}$$

For $\phi_2 = 0$ and $\theta_2 = 0$ formulae (38), (39) and (40) reduce to (25), (26) and (27). If in turn the expression (36) is used to define the autocovariance function, one gets:

$$\begin{aligned}\gamma(0) &= (\phi_x + \phi_y)\gamma(1) - \phi_x\phi_y\gamma(2) + \sigma_u^2\{1 + (\phi_x - \phi_y)(-\theta_x - \phi_y + \phi_x\phi_y\theta_x)\} + \\ &+ \sigma_w^2(1 - \phi_x\phi_y)\end{aligned} \tag{41}$$

$$\gamma(1)(1 + \phi_x\phi_y) = \sigma_u^2[-\theta_x + \phi_y + \theta_x\phi_y(\phi_x - \theta_x)] - \phi_x\sigma_w^2 + (\phi_x + \phi_y)\gamma(0) \tag{42}$$

$$\gamma(2) = (\phi_x + \phi_y)\gamma(1) - \phi_x\phi_y\gamma(0) + \theta_x\phi_y\sigma_u^2 \tag{43}$$

Again for $\phi_x = \phi_1, \theta_x = 0, \phi_y = 0$ and $\sigma_w^2 = v_2$, the above formulae reduce to (19), (20) and (21).

Treating (41), (42) and (43) as a set of simultaneous equations, $\gamma(0), \gamma(1)$ and $\gamma(2)$ can be calculated as functions of ϕ_x, ϕ_y, θ_x, σ_w^2 and σ_u^2. This is done as follows:

$$\gamma(0) = \frac{W_0}{W},\ \gamma(1) = \frac{W_1}{W},\ \gamma(2) = \frac{W_2}{W} \tag{44}$$

where:

$$\left.\begin{aligned}W_0 &= a_1[1 + \phi_x\phi_y] + a_2[\phi_x + \phi_y]\ [1 - = \phi_x\phi_y] - a_3[1 + \phi_x\phi_y]\phi_x\phi_y \\ W_1 &= a_1[\phi_x + \phi_y] + a_2\left[1 - \phi_x^2\phi_y^2\right] - a_3[\phi_x + \phi_y]\phi_x\phi_y \\ W_2 &= a_1[(\phi_x + \phi_y)^2 - \phi_x\phi_y(1 + \phi_x\phi_y)] + a_2[\phi_x + \phi_y]\ [1 - \phi_x\phi_y] + \\ &+ a_3[(1 + \phi_x\phi_y) - (\phi_x + \phi_y)^2] \\ W &= [(1 + \phi_x\phi_y)^2 - (\phi_x + \phi_y)^2]\ [1 - \phi_x\phi_y]\end{aligned}\right\} \tag{45}$$

and:

$$\left.\begin{aligned}a_1 &= \sigma_u^2[1 - (\phi_x - \theta_x)(\theta_x + \phi_y - \phi_x\phi_y\theta_x)] + \sigma_w^2[1 - \phi_x\phi_y] \\ a_2 &= \sigma_u^2[-\phi_y - \theta_x + \theta_x\phi_y(\phi_x - \theta_x)] - \sigma_w^2\phi_x \\ a_3 &= \sigma_u^2\theta_x\phi_y\end{aligned}\right\} \tag{46}$$

Comparing (36) to (37) yields:

$$\phi_1 = \phi_x + \phi_y,\quad \phi_2 = -\phi_x\phi_y \tag{47}$$

Let us assign basing on (41), (42) and (43):

$$\left.\begin{aligned}b_1 &= \gamma(0) - \phi_1\gamma(1) - \phi_2\gamma(2) = \\ &= \sigma_e^2[1 + \theta_1^2 + \theta_2^2 - \phi_1\theta_1 - \phi_2\theta_2 - \theta_2\phi_1^2 + \phi_1\theta_1\theta_2] \\ b_2 &= -\phi_1\gamma(0) + [1 - \phi_2]\gamma(1) = \sigma_e^2[-\theta_1 - \theta_2\phi_1 + \theta_1\theta_2] \\ b_3 &= \gamma(2) - \phi_1\gamma(1) - \phi_2\gamma(0) = \sigma_e^2\theta_2\end{aligned}\right\} \tag{48}$$

Hence:

$$\theta_2 = -\frac{b_3}{\sigma_e^2} \tag{49}$$

$$\theta_1 = \frac{b_3\phi_1 - b_2}{b_3 + \sigma_e^2} \tag{50}$$

It is straightforward to verify that $a_1 = b_1$, $a_2 = b_2$, $a_3 = b_3$. Thus it is not necessary to calculate the autocovariances $\gamma(0)$, $\gamma(1)$ and $\gamma(2)$ to obtain b_1, b_2 and b_3.

Having inserted (49) and (50) into the first equation of (48) a non-linear equation for the variance σ_e^2, a white noise generating the resultant process, is obtained. If we assign $\sigma_e^2 = v$, the equation will take a shape:

$$\begin{aligned} 1 \ &+ \ \frac{(a_3\phi_1 - a_2)^2}{(a_3 + v)^2} - \frac{\phi_1(a_3\phi_1 - a_2)}{a_3 + v} \left[\frac{a_3}{v} + 1\right] + \\ &+ \ \frac{\frac{a_3^2}{v}(\phi_2 + \phi_1^2) - a_1}{v} = 0 \end{aligned} \tag{51}$$

It has two constraints that limit the root searched:

$$v > 0 \text{ and } v \neq -a_3 \tag{52}$$

To get θ parameters formulae (49) do (50) are applied. In this way a set of five parameters of the resultant ARMA (2, 2) process being the sum of ARMA (1, 1) and AR(1) is computed.

An inverse issue consists in a resolution of the ARMA (2, 2) process into two component processes mentioned above. The procedure is as follows:
utilizing (47) a quadratic equation in ϕ is received, roots of which are ϕ_x and ϕ_y:

$$\phi^2 - \phi_1\phi - \phi_2 = 0 \tag{53}$$

To preserve the stationarity of the process these roots must lie in the interval between -1 and 1.
In the next stage b_1, b_2 and b_3 are calculated from (47) and they are correspondingly equal to a_1, a_2 and a_3. a_1, a_2, a_3 are defined by a set of simultaneous equations (46). From the last of these equations:

$$\theta_x = \frac{a_3}{\sigma_u^2\phi_y} \tag{54}$$

A transformation of the second one yields:

$$\sigma_w^2 = \frac{a_3}{\phi_2}\left[1 + \phi_2 + \frac{a_3}{\sigma_u^2}\right] + \frac{\phi_y\sigma_u^2 - a_2}{\phi_x} \tag{55}$$

Putting (54) and (55) into the first one forms a non-linear equation from which σ_u^2 can be calculated:

$$\begin{aligned} a_1 \ &= \ \sigma_u^2\left[1 - \left(\phi_x - \frac{a_3}{\sigma_u^2\phi_y}\right)\left(\frac{a_3}{\sigma_u^2\phi_y} + \phi_y + \frac{\phi_2 a_3}{\phi_y\sigma_u^2}\right)\right] + \\ &+ \ \left[\frac{a_3}{\phi_2}\left(1 + \phi_2 + \frac{a_3}{\sigma_u^2}\right) + \frac{\phi_y\sigma_u^2 - a_2}{\phi_x}\right](1 + \phi_2) \end{aligned} \tag{56}$$

Numerical solution of this equation permits to evaluate $\sigma_u^2 > 0$, then in the next step $\sigma_w^2 > 0$ from (55) and $-1 < \theta_x < 1$ from (54).

4. Numerical experiments

Theoretical formulae were presented in previous chapters for calculation of the parameters of a resultant process if its components were given or vice versa; ways of resolving the resultant process into its components were also discussed. However, accurate values of the parameters are practically never known and only a set of numbers being a realization of a certain process is at the disposal. Basing on it stochastic models are tried to match the observed series. They are tried by the estimation of their parameters and finally the one matching best is accepted. Nevertheless, its parameters are burdened with errors. The following numerical experiment was performed in order to find out how the length of the realization influences the accuracy of estimated parameters of both the resultant and the component processes:

- two realizations of an AR(1) process were generated. Their variances were identical: $\sigma_z^2 = 5$ but autoregressive parameters were different and equal to 0.8 and 0.5. ϕ_1 are taken positive because the autocorrelation function $\rho(k)$ of such AR(1) process is a decaying, positive exponential curve. For negative ϕ_1 the sign of autocorrelations alters in consecutive steps being positive when k is even and negative for odd k. Such models are not very useful in geotechnical applications. Moreover, another assumption was made that the intervals between consecutive steps are identical for both realizations. If they are not the same they must be stated and in a particular case when their ratio is equal to $\ln(0.05)/\ln(0.8)$, the same autoregressive process will be received. Lengths of both realizations are the same $n = 5000$;
- a process of white noise was generated with a zero expectation and a variance $v = 1$ which is equal to 20% of the variances of autoregressive processes. As above $n = 5000$.

The described basic processes can be written as:

$$Z_t = 0.8\ Z_{t-1} + a_t^{(1)} \tag{57}$$

$$Z_t = 0.5\ Z_{t-1} + a_t^{(2)} \tag{58}$$

$$Z_t = a_t^{(3)} \tag{59}$$

Variances of random impulses of these processes are: $\sigma_{a1}^2 = 1.8$, $a_{a2}^2 = 3.75$, $\sigma_{a3}^2 = 1$. Two resultant processes were analysed and an attempt to resolve them back on basic processes was made afterwards:

- an ARMA (1, 1) process, being a combination of AR(1) process (60) and a white noise (62),
- an ARMA (1, 1) process, being a combination of AR(1) process (61) and a white noise (62).

4.1. AR(1) process disturbed by a white noise

If two basic processes (57) and (58) are combined, the following resultant process is received:

$$Z_t = 0.8\ Z_{t-1} + u_t^{(1)} - 0.2467\ u_{t-1}^{(1)} \tag{60}$$

Its variance $\gamma(0) = 6.0$ and its white noise variance $\sigma_{u1}^2 = v_1 + 3.2426$. When basic processes (58) and (59) are merged then:

$$Z_t = 0.5\ Z_{t-1} + u_t^{(2)} - 0.101\ u_t^{(2)} \tag{61}$$

and $\gamma(0) = 6.0, \sigma_{u2}^2 = v = 4.9495$.
The following samples were analysed:

(a) first 100 elements (d) 100 elements from 3001 to 3100
(b) first 500 elements (e) 500 elements from 3001 to 3500
(c) first 2000 elements (f) last 2000 elements.

Parameters of processes represented by samples (a) – (f) were estimated by means of the autocovariance function, assuming that they represent a theoretical model of ARMA (1, 1). At first the effect of the length of the samples (100, 500, 2000) on the estimates of the parameters was examined. Secondly the behaviour of the estimates (accuracy, standard errors) for the same sample length was scrutinized. Results are shown in table 1 and 1a.

Column 5 in table 1 contains the estimates $\hat{\phi}_1$ and estimates of the standard deviations; the exact value $\phi_1 = 0.8$ does not differ much from the estimates and in each case is situated in the confidence interval $\hat{\phi}_1 \pm \hat{\sigma}\phi_1$ (column 2 in table 1a). Estimates of θ_1 are much less accurate. In four of six cases the exact value is outside $\hat{\theta}_1 \pm \hat{\sigma}\theta_1$ (column 3 in table 1a), but always within $\hat{\theta}_1 \pm 2\hat{\sigma}\theta_1$. In one case (sample (a) of 100 elements) the standard deviation estimate is greater than the estimate $\hat{\theta}_1$ itself ($\hat{\sigma}\theta_1 = 0.156, \hat{\theta}_1 = 0.063$). $\theta_1 = 0$ hypothesis could not be rejected in this case even for a high level of significance.

The knowledge of the estimates of the resultant process parameters (table 1) makes it possible to calculate the estimates of the component processes parameters. Using (29) one can determine the estimate of the white noise variance that disturbs the initial AR(1) process. However one must realize that direct inserting of $\hat{\phi}_1, \hat{\theta}_1$ and $\hat{\sigma}_{us}^2$ into (29) and calculating v_2 might cause serious errors. To get more accurate results and to verify the calculated values of v_2 a procedure applying a Monte Carlo method was used. For each sample sets of $\phi_1, \hat{\theta}_1$ and $\hat{\sigma}_{u1}^2$ values were generated 20000 times and each set put into (29). A mean and a variance for 20000 results were then calculated. The results are presented in table 2. As can be noticed from table 2, the calculated (column 3) estimates of the variance of the white noise added to the initial AR(1) are scattered considerably about the theoretical value of v_2 equal to one. This refers especially to short series ($n = 100$). A Monte Carlo method could have been applied here because the exact value of $v_2 = 1.0$ was known in advance. However the use of Monte Carlo method did not improve the accuracy of v_2 considerably. Of course, in practical applications, the exact value of σ_{u1}^2 is not known and, in order to calculate

Table 1

Estimates of the ARMA (1, 1) process parameters for $\phi_1 = 0.8$, $\theta_1 = 0.2467$, $\sigma_u^2 = 3.2426$, $\gamma(0) = 6.0$, $\bar{z} = 0.0$.

Sample mark	Number of elements	Sample mean	Sample variance	Parameters with their standard errors		Correlation coefficient between $\hat{\phi}$ and $\hat{\theta}$	Random impulse variance
	n	$\hat{z}$	$\hat{\gamma}(0)$	$\frac{\hat{\phi}_1}{\hat{\sigma}\phi_1}$	$\frac{\hat{\theta}_1}{\hat{\sigma}\theta_1}$	$\hat{r}$	$\hat{\sigma}_{u1}^2$
1	2	3	4	5	6	7	8
a	100	-0.447	5.173	0.74 0.117	0.063 0.156	0.7762	2.252
d	100	0.125	5.305	0.761 0.145	0.26 0.193	0.7878	3.320
b	500	-0.011	5.680	0.76 0.049	0.173 0.066	0.7629	3.132
e	500	-0.030	5.407	0.815 0.057	0.347 0.08	0.7556	3.354
c	2000	-0.118	5.629	0.78 0.021	0.222 0.029	0.7588	3.138
f	2000	0.009	5.851	0.817 0.026	0.31 0.036	0.7442	3.301

Table 1a

Confidence intervals for $\hat{\phi}_1$ and $\hat{\theta}_1$ from table 1

Sample mark	$\hat{\phi}_1 \pm \hat{\sigma}\phi_1$ $\hat{\phi}_1 \pm 2\hat{\sigma}\phi_1$	$\hat{\theta}_1 \pm \hat{\sigma}\theta_1$ $\hat{\theta}_1 \pm 2\hat{\sigma}\theta_1$
1	2	3
a	0.623 – 0.857 0.505 – 0.974	-0.093 – 0.218 -0.248 – 0.347
d	0.672 – 0.907 0.471 – 1.052	0.0674 – 0.453 -0.126 – 0.646
b	0.712 – 0.808 0.663 – 0.857	0.107 – 0.239 0.042 – 0.305
e	0.758 – 0.812 0.701 – 0.929	0.267 – 0.426 0.188 – 0.506
c	0.759 – 0.800 0.738 – 0.821	0.193 – 0.250 0.164 – 0.279
f	0.791 – 0.843 0.786 – 0.868	0.273 – 0.346 0.237 – 0.383

Table 2

Estimates of a disturbing white noise variance (accurate value $\sigma_{a3}^2 = v_2 = 1.0$)

Sample mark	Sample length n	White noise variance estimate $\hat{v}_2$	Estimate of standard deviation of $\hat{v}_2$ $\hat{\sigma}v_2$	Estimate of $\hat{v}_2$ from direct calculation not using Monte-Carlo method $\tilde{v}_2$
1	2	3	4	5
a	100	0.1908	0.7070	0.218
d	100	1.0037	0.7620	1.134
b	500	0.7213	0.2690	0.714
e	500	1.3548	0.3170	1.426
c	2000	0.9103	0.1638	0.892
f	2000	1.2144	0.2070	1.252

the estimate of v_2, one must use the values from column 8 of table 1. Table 3 shows how the ARMA (1, 1) process was resolved on two component processes AR(1) and a white noise for the results presented in table 1.

Table 3

Estimates of component processes calculated on the basis of estimates of resultant process samples

Sample mark	Sample length n	AR(1) autoregressive parameter $\hat{\phi}$	AR(1) white noise variance $\hat{\sigma}_{a1}^2$	AR(1) variance $\hat{\gamma}\ (0)$	White noise variance from (29) $\hat{\sigma}_{a3}^2$
1	2	3	4	5	6
a	100	0.740	2.230	4.930	0.218
d	100	0.761	1.752	4.171	1.134
b	500	0.760	2.100	4.970	0.714
e	500	0.815	1.380	3.980	1.426
c	2000	0.780	1.890	4.740	0.892
f	2000	0.817	1.530	4.600	1.252

Estimates of the autoregressive parameter ϕ_1 are quite accurate. This allows to state that if ϕ_1 is great enough (close to 1.0) what means that the neighbouring values of the process are considerably correlated, then the proposed method is reasonably accurate. It should be expected that for ϕ_1 closer to zero, the separation of measurement errors will be less effective as AR(1) with smaller ϕ_1 becomes similar to a white noise. This can also be deduced from the fact that AR(1) autocorrelation function $\rho(k) = \phi^k$ approaches zero rapidly for $k > 1$ when ϕ_1 tends to zero. To show this a similar analysis was done for $\phi_1 = 0.5$ in which the correlation between the neighbouring points is about 2.5 times smaller than for $\phi_1 = 0.8$. The results of the estimates of the parameters calculated on a basis of the realizations of the resultant process given by formula (61) are quoted in table 4 and 4a.

Table 4

Estimates of the ARMA (1, 1) process parameters for $\phi_1 = 0.5$, $\theta_1 = 0.101$, $\sigma_u^2 = 4.95$, $\gamma(0) = 6.0$, $\overline{z} = 0.0$.

Samples mark	Number of elements	Sample mean	Sample variance	Parameters with their standard errors		Correlation coefficient between $\hat{\phi}_1$ and $\hat{\theta}_1$	Random impulse variance
	n	$\hat{z}$	$\hat{\gamma}(0)$	$\frac{\hat{\phi}_1}{\hat{\sigma}\phi_1}$	$\frac{\hat{\theta}_1}{\hat{\sigma}\theta_1}$	$\hat{r}$	$\hat{\sigma}_{u1}^2$
1	2	3	4	5	6	7	8
a	100	-0.840	5.047	0.324 0.376	0.013 0.393	0.968	5.169
d	100	0.152	5.163	0.991 0.017	0.924 0.030	0.674	5.142
b	500	0.0006	5.289	0.374 0.117	0.025 0.123	0.954	4.651
e	500	0.2625	5.623	0.513 0.224	0.171 0.242	0.9327	4.950
c	2000	0.0253	5.605	0.460 0.053	0.106 0.058	0.9327	4.841
f	2000	0.0567	6.058	0.469 0.047	0.069 0.051	0.929	5.037

It is obvious at the first glance, that both ϕ_1 and θ_1 estimates are disturbed by considerable errors and it would be purposeless to separate a white noise and an AR(1) process. So it seems reasonable at the stage of identification to assume that one deals with an AR(1) process and to estimate its parameters knowing that these estimates will have greater standard deviations then those of the initial AR(1). Such a procedure ought to be performed because in the majority of cases the hypothesis of $\theta_1 = 0$ can not be rejected at the significance level of 5% (confidence interval $\hat{\theta}_1 \pm 2\hat{\sigma}\theta_1$).

4.2. ARMA (2, 2) process as a combination of two AR(1) and a white noise

When processes (57), (58) and (59) are added together the resultant process is ARMA (2, 2). It has the following theoretical parameters: $\phi_1 = 1.3$, $\phi_2 = -0.4$, $\theta_1 =$

Table 4a

Confidence intervals for $\hat{\phi}_1$ and $\hat{\theta}_1$ from table 4

Sample mark	$\hat{\phi}_1 \pm \hat{\sigma}\phi_1$ / $\hat{\phi}_1 \pm 2\hat{\sigma}\phi_1$	$\hat{\theta}_1 \pm \hat{\sigma}\theta_1$ / $\hat{\theta}_1 \pm 2\hat{\sigma}\theta_1$
1	2	3
a	-0.052 – 0.700	-0.380 – 0.406
	-0.428 – 1.076	-0.773 – 0.800
d	0.974 – 1.008	0.894 – 0.954
	0.957 – 1.025	0.864 – 0.984
b	0.257 – 0.491	0.098 – 0.148
	0.140 – 0.608	-0.022 – 0.271
e	0.289 – 0.737	-0.071 – 0.413
	0.065 – 0.961	-0.313 – 0.655
c	0.407 – 0.513	0.048 – 0.164
	0.354 – 0.566	-0.010 – 0.222
f	0.422 – 0.516	0.018 – 0.120
	0.375 – 0.563	-0.033 – 0.171

0.764, θ_2 = -0.0564, $\gamma(0) = 11$, $\sigma_u^2 = 7.09$ and $\bar{z} = 0$. $N = 5000$ elements of such process were generated and the following samples were taken:

- first 100, 500, 1000, 2000, 3000 and 4000 elements,
- the whole realization.

These samples were estimated on the assumption that they are the realizations of the ARMA (2, 2) process. The results are presented in table 5.

The following conclusions can be drawn from the above table:

- there is no use applying a compound ARMA (2, 2) model (5 parameters) to shorter series – below 2000 elements, as either the estimates are inaccurate or the estimation is not feasible,
- more simple models can be used to fit the series as e.g. an ARMA (1, 1) one and a separation of a white noise (see 4.1) can be done,
- long samples fit quite well even though the standard errors of the estimates decrease very slowly with the increase of the lengths of the realizations.

To illustrate how the process ARMA (2, 2) is resolved, calculations were carried out for the estimation obtained for $n = 5000$. Accurate, theoretical values are in brackets: Generated process satisfies the equation:

$$Z_t = 1.3Z_{t-1} - 0.4Z_{t-2} + u_t - 0.764u_{t-1} + 0.0564u_{t-2}$$

After having resolved it into AR(1) + AR(1) + white noise:

Table 5

Estimates of ARMA (2, 2) parameters for $\phi_1 = 1.3$, $\phi_2 = -0.4$, $\theta_1 = 0.764$, $\theta_2 = -0.0564$, $\gamma(0) = 11$, $\bar{z} = 0$, $\sigma_u^2 = 7.09$

Sample length n	Mean $\hat{z}$	Variance $\gamma(0)$	White noise variance $\hat{\sigma}_u^2$	Autoregressive parameters $\frac{\hat{\phi}_1}{\hat{\sigma}\phi_1}$	Autoregressive parameters $\frac{\hat{\phi}_2}{\hat{\sigma}\phi_2}$	Moving average parameters $\frac{\hat{\theta}_1}{\hat{\sigma}\theta_1}$	Moving average parameters $\frac{\hat{\theta}_2}{\hat{\sigma}\theta_2}$
1	2	3	4	5	6	7	8
100	0.65	10.58	6.73	0.35 / 1.60	0.22 / 1.26	-0.175 / 1.51	-0.09 / 0.46
500	0.23	10.85	ARMA(2, 2) parameters can not be estimated – initial autoregressive estimates out of the range of stationarity				
1000	0.15	9.85					
2000	0.11	10.34					
3000	0.065	10.68	6.74	1.41 / 0.27	-0.46 / 0.2	0.96 / 0.22	-0.086 / 0.044
4000	-0.04	10.99	7.00	1.56 / 0.041	-0.58 / 0.035	1.029 / 0.042	-0.083 / 0.032
5000	-0.04	11.02	7.04	1.31 / 0.18	-0.395 / 0.12	0.775 / 0.155	-0.0455 / 0.05

- first component – AR(1) $\phi_1 = 0.472$ (0.5), $\sigma_{a1}^2 = 4.71$ (4.95)
- second component – AR(1) $\phi_1 = 0.857$ (0.8), $\sigma_{a2}^2 = 0.61$ (1.8)
- third component – white noise $\sigma_{a3}^2 = 0.81$ (1)

As can be seen, except the variance of the white noise in the second AR(1) process, other parameters were approximated quite accurately.

4.3. Application of Kalman filter in resolving ARMA (1, 1) into AR(1) and white noise

Estimates of the ARMA (1, 1) process parameters given in the table 1 allow to determine estimates of the components: $\hat{\phi}_1$, $\hat{v}_1$ in AR(1) and v_2 of the white noise, given by (29) – table 3, column 6.These estimates could be calculated because $\hat{\theta}_1$, $\hat{\phi}_1$ and v of the ARMA (1, 1) were known. The question appears whether it would be reasonable to use the Kalman filter to remove a white noise from the initial series ARMA (1, 1) having in mind that the formula (29) is applied to get the estimate of v_2. After filtering the remaining series can be treated as AR(1) and its parameters estimated. Such procedure consists of the following stages:

- estimation of parameters as ARMA (1, 1),
- resolution into AR(1) and a white noise (see 4.1),
- filtering off the white noise from the initial series using Kalman's filter,

– estimation of parameters as AR(1) for the remaining series.

The method was tested using 100 element samples taken from the process being a sum of (57) and (58) formerly generated. The results are shown in table 6.

Table 6

Estimates of AR(1) parameters for $\phi_1 = 0.8$, $v_1 = 1.8$, $\gamma(0) = 5$ after having filtered off a white noise

Order numbers in 5000 element realiza-tion	$\hat{\phi}_1$ and standard deviation after filtering	AR(1) white noise variance after filtering	Added white noise variance	$\hat{\phi}_1$ and standard deviation before filtering	AR(1) white noise variance before filtering
	$\hat{\phi}_1/\hat{\phi}_1$	$\hat{v}_1$	$\hat{\sigma}^2_{a3}$	$\hat{\phi}_1/\hat{\sigma}\phi_1$	$\hat{v}_1$
1	2	3	4	5	6
501 – 600	0.826 0.06	2.11	0.99	0.85 0.22	1.83
601 – 700	0.65 0.08	2.37	0.90	0.64 0.20	2.45
1401 – 1500	0.584 0.08	3.38	0.2	0.56 0.43	3.47
1901 – 2000	0.796 0.06	1.59	1.43	0.83 0.19	1.09
2501 – 2600	0.742 0.07	1.40	1.41	0.84 0.26	1.00
3001 – 3100	0.70 0.07	1.59	1.19	0.70 -0.16	1.43
3501 – 3600	0.85 0.05	1.63	1.15	0.89 0.25	1.15

Estimates $\hat{\phi}_1$ and standard deviations are placed in the 5-th column. Estimates of $\hat{\phi}_1$ with standard deviations after filtering are positioned in the 2-nd column. Estimates of $\hat{\phi}_1$ after filtering are a bit more accurate when compared to those derived from not filtered series. The average difference between the estimates and the theoretical value of $\phi_1 = 0.8$ is 15% smaller for filtered series than for unfiltered. It is much more important that the filtration reduced standard deviations $\hat{\sigma}\phi_1$ od $\hat{\phi}_1$ nearly four times and therefore the estimates $\hat{\phi}_1$ of filtered series acquire more confidence. It is also worth noting that the estimates of an AR(1) white noise variance v_1 improve considerably. Mean standard deviation of the estimates in column 3, calculated with regard to the accurate value of $v_1 = 1.8$, is 22% lower when compared to that computed for column 6, and decreased from 0.91 to 0.71.

Summarizing, it can be stated that Kalman filtration of a white noise from ARMA (1, 1) following the initial resolution of the ARMA (1, 1) process into AR(1) and a white noise and then the estimation of parameters of the remaining series as AR(1) improves the final evaluation of AR(1) parameters.

5. Final conclusions

The method of stochastic processes separation discussed in the paper can be applied when:

- quite a great number of observations is at the disposal; the more simple process the shorter realization is required; for example 100 elements can be sufficient for AR(1) process and even 1000 elements may not be sufficient for ARMA (2, 2),
- simple stochastic model can be used to describe the observations,
- one is mainly interested in separating measurement errors from processes described by relatively simple models such that the resultant process is also relatively simple,
- neighbouring points of the series are highly correlated.

It is not recommended to apply the method if:

- number of observations is small,
- neighbouring elements in the series are not strongly correlated,
- estimates of the resultant series, before resolution, are characterized by high standard deviations,
- some estimates of the parameters of the observed process are close to zero; it is much better to simplify the model of the resultant process instead of attempting to extract the measurement errors.

In general one should say that the presentation of soil properties as stochastic processes requires additionally to the presentation of the estimates of the parameters also the presentation of their standard deviations. Great standard deviations (errors) occur even if the resultant process is a sum of relatively simple components. There is therefore rather little chance to determine, in the nearest future, stochastic parameters of such processes as for instance shearing resistance which are composed of two or more processes and should be resolved into basic components. Although theoretical calculations are feasible and not very complicated, practical applications require long series of measurements which are not yet available.

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Summary

Summing and separation of the ARIMA stochastic processes, used in description of the subsoil properties, have been discussed in the paper. Autocovariance functions of the sum of the component processes and the resultant process have been compared in order to find formulae relating the parameters of the components and the resultant process. A special attention has been drawn to extract a white noise of measurement errors from the resultant process assuming that the resultant process is a sum of simple AR(1) processes and that white noise.

Numerical experiments have been performed and described so as to verify the accuracy of theoretical formulae when applied in practice – estimates of the parameters used instead of exact, theoretical values.

Streszczenie

W pracy przedstawiono sumowanie i rozdzielanie procesów stochastycznych ARIMA, których używa się do opisu własności podłoża gruntowego. W tym celu porównano funkcje autokowariancji procesów składowych i wynikowego w celu znalezienia wzorów wiążących parametry procesów składowych z parametrami procesu wynikowego. Szczególny nacisk położono na wyodrębnienie białego szumu spowodowanego przez błędy pomiarów przy założeniu, że proces wynikowy jest sumą procesu AR(1) i tego białego szumu.

Wykonano eksperymenty numeryczne, które opisano w celu weryfikacji wzorów teoretycznych stosowanych w praktyce, estymaty tych parametrów zastąpiły tu wartości teoretyczne.