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Application of finite-difference method to modelling of the spreading of chemical pollutants in the aeration zone

1. Introduction

The process of contamination migration in a one-dimensional porous medium can be expressed by the hydrodynamic dispersion equation:

$$\frac{\partial(c\theta + \rho S)}{\partial t} = \frac{\partial}{\partial z}\left(\theta D_s \frac{\partial c}{\partial z} - qc\right) - \mu_w \theta c - \mu_s \rho S + \gamma_w \theta + \gamma_s \rho \tag{1}$$

where:

- c – contamination concentration [ML^{-3}],
- S – adsorption isotherm [$M\ M^{-1}$], accepted as $S = k\ c$, $k = \text{const}$,
- D_s – dispersion coefficient [$L^2\ T^{-1}$],
- q – flow of liquid $q = v * \theta$ [LT^{-1}],
- ρ – soil bulk volume [ML^{-3}],
- μ_w, μ_s – constants characterising substance decay in the medium's liquid and solid phases [T^{-1}],
- γ_w, γ_s – constants characterising substance formation in the medium's liquid and soild phases [T^{-1}],
- z, t – variables: spatial [L], temporal [T].

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In this equation, the dispersion coefficient is made to depend on bulk humidity – θ and the speed of liquid flow in the soil – v (Maciejewski 1984):

$$D_s = (-a\theta + b)v + d, \quad a, b, d - \text{constants.}$$

There is an initial condition imposed on the equation's solution:
$c(z,0) = c_p(z), \quad (0 \leq z \leq L),$
and the following boundary conditions:

$$\text{Dirichlet} - \quad c(z_o, t) = c_i(t), (t > 0),$$

$$\text{Neuman} - \quad q_c(t) = \left(-D_s\theta \frac{\partial c}{\partial z} + v\ \theta\ c\right)\Big|_{z=z_o}$$

$$\text{or} \quad \frac{\partial c}{\partial z}(L,t) = 0,$$

where: L – thickness of considered soil profile,

$$z_o = 0 \quad \text{or} \quad z_o = L.$$

An approximate solution of the hydrodynamic dispersion equation is possible provided we know the flow speed field v and values of humidity θ. Most often, they are determined with the help of the Fokker-Planck (diffusion) equation (Vauclin et al. 1979). For this purpose, the created model assumes a convergent and stable differential scheme with constant or variable space interval (Samarskij 1984).

In order to attain an approximate solution of the hydrodynamic dispersion equation, the formula was converted to reach the form:

$$r \frac{\partial c}{\partial t} = \frac{\partial}{\partial z}\left(\theta D_s(\theta, v) \frac{\partial c}{\partial z}\right) - q \frac{\partial c}{\partial z} - \mu c + \gamma \tag{2}$$

where:

$$r = \theta + \rho k, \quad \mu = \mu_w + \mu_s \rho k, \quad \gamma = \gamma_w \theta + \gamma_s \rho$$

utilising the continuity equation for liquid flow.

The paper submits the methods for the approximate solution of so-formulated issue, know from literature, as well as some methods worked out by the author. For this issue, it is essential to choose an appropriate computation algorithm due to the influence of a so-called advection term occurring in the equation. If the issue's approximation method is inapt, there often occurs a phenomenon of approximation solution oscillation or impulse broadening in the numerical dispersion process.

Another crucial item is the selection of a method depending on the values displayed by the parameters in this equation and also on digitisation of temporal and spatial variables assumed when coming upon the approximate solution. The paper submits the following methods for solving the issue: a globally stable differential scheme elaborated by the author, the van Genuchten (1974) method (most often made use of in practice), the Galerkin-Pietrov method. Presented are also two new methods: a modified

Galerkin-Pietrov and an algorithm solving after introducing co-called migratory coordinates. These methods will be compared referring to their accuracy, the occurrence of oscillation and numerical dispersion for different space and time intervals. For this purpose, the analytical solution of issue (Bear 1972) was utilised:

$$\frac{\partial c}{\partial t} = D_s \frac{\partial^2 c}{\partial z^2} - v\frac{\partial c}{\partial z},$$

imposing the following conditions in the solution:

$$c(z_o, 0) = M_o\sigma(z_o), \quad \sigma(z_o) - \text{delta function}$$

$$\lim_{z\to+\infty} c(z) = 0, \quad \int_{-\infty}^{\infty} c(z,t)dz = M_o;$$

The solution is made up by the function:

$$c(z,t) = \frac{M_o}{\sqrt{4\pi D_s t}} \exp -\frac{(z-vt)^2}{4D_s t}$$

Accepted were the following values of the parameters, selected in order that the above-cited effects manifest themselves the more markedly:

$$M_o = 0.1998, \quad D_s = 77.144, \quad q = 288, \quad L = 70, \quad \theta = 0.5,$$

proces time 0.042.

Figure 1 displays the initial condition and the result after the process, adopted for the comparisons.

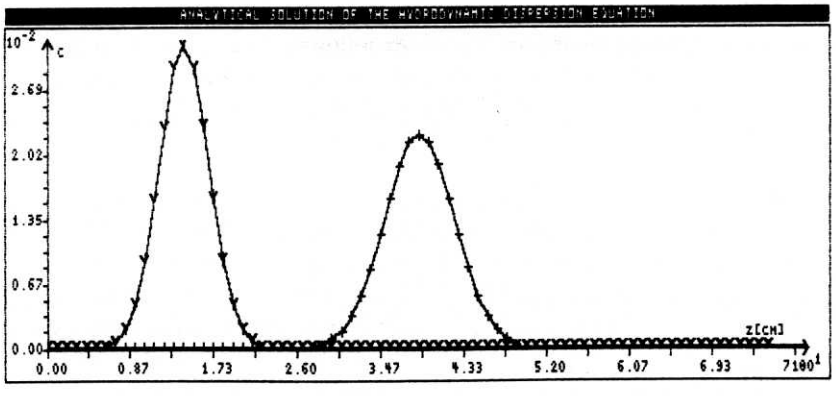

Fig. 1. Analytical solution of the hydrodynamic dispersion equation
Y initial condition
+ concentration after the process

2. A characteristic of the approximate solutions of the hydrodynamic dispersion equation

When constructing the approximate solutions of the hydrodynamic dispersion equation, equation (2) was made use often. The following notations were introduced:

$$u_i = u(z_i, t_j), \quad \hat{u}_i = u(z_i, t_{j+1}),$$

where: for the schemes with constant space digitisation interval $z_i = i * dz$, $t_j = j * dt$,
or $z_i = \sum_{k=1}^{i} dz_k$ for the approximation with variable space interval,
dz_i, dt – space and time digitisation interval, respectively, $i = 0, 1, \ldots, N$,
$j = 0, 1, \ldots, M$,

$$\Delta u_i = (u_{i+1} - u_i), \quad \nabla u_i = (u_i - u_{i-1}), \quad \Delta^2 u_i = \Delta(\Delta u_i),$$

$$k_i = \hat{\theta}_i D_s(\hat{\theta}_i, \hat{v}_i), \qquad \hat{k}_i = \frac{k_i + k_{i+1}}{2}, \qquad \check{k}_i = \frac{k_i + k_{i-1}}{2},$$

$$\Lambda \hat{c}_i = \frac{\hat{k}_i}{2} \frac{\Delta \hat{c}_i}{dz} - \frac{\check{k}_i}{2} \frac{\nabla \hat{c}_i}{dz},$$

$$r_i = \hat{\theta}_i + \rho k, \qquad \mu_i = \mu_w \hat{\theta}_i + \mu_s \rho, \qquad \gamma_i + \gamma_w \hat{\theta}_i + \gamma_s \rho,$$

2.1. Approximate solution of the dispersion equation, obtained using the differential scheme method

Let

$$q_i^+ = 0.5(-\hat{q}_i + |\hat{q}_i|), \quad q_i^- = -0.5(\hat{q}_i + |\hat{q}_i|),$$

$$\kappa_i = \frac{1}{1 + R_i}, \quad \text{where} \quad R_i = \frac{dz|\hat{q}_i|}{2k_i}.$$

Then, equation (2) can be approximated as follows:

$$r_i \frac{\hat{c}_i - c_i}{dt} = \frac{\kappa_i}{dz} (\Lambda \hat{c}_i) + \frac{q_i^+}{k_i} \frac{\hat{k}_i}{2} \frac{\Delta \hat{c}_i}{dz} + \frac{q_i^-}{k_i} \frac{\check{k}_i}{2} \frac{\nabla \hat{c}_i}{dz} - \mu_i \hat{c}_i + \gamma_i \tag{3}$$

for $i = 1, 2, \ldots, N-1$.

This approximation is stable regardless of the flow direction. The oscillations do not occur either. Unfortunately, should the flow values markedly exceed the dispersion coefficient, there manifests itself an influence of the numerical dispersion process in the obtained solution. This effect is exemplified in Fig. 2, compared with the above-presented analytical solution.

2.2. Approximate solution of the dispersion equation according to van Genuchten (1974)

In this case, the approximate solution of the transport equation was worked out on the grounds of the Galerkin method, utilising linear base functions. Using this method,

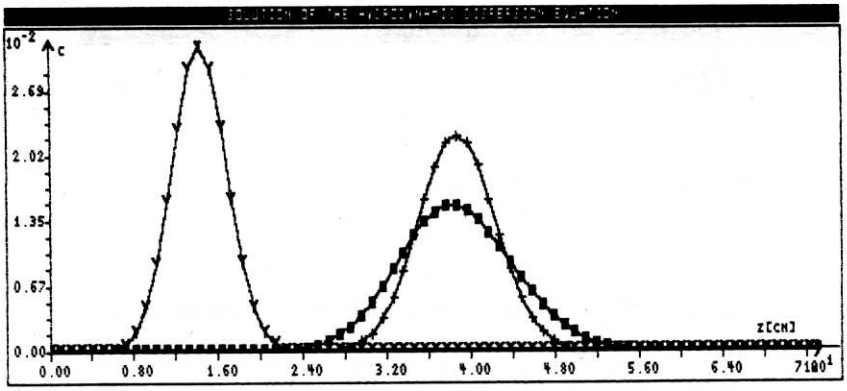

Fig. 2. Method 1 – based solution of the hydrodynamic dispersion equation
Y initial condition, + concentration after the process,
□ solution according to the scheme (3)

equation (2) might be approximated as follows:

$$r_i\left(\frac{1}{6}\,\frac{\hat{c}_{i+1}-c_{i+1}}{dt}+\frac{2}{3}\,\frac{\hat{c}_i-c_i}{dt}+\frac{1}{6}\,\frac{\hat{c}_{i-1}-c_{i-1}}{dt}\right)=$$
$$=\frac{1}{2dz}(\Lambda\hat{c}_i)+\frac{1}{2dz}(\Lambda c_i)-\frac{\nabla\hat{c}_i}{2dz}-\frac{\nabla c_i}{2dz}-\mu_i\hat{c}_i+\gamma_i, \tag{4}$$

for $i=1,2,\ldots,N-1$,

where: in the differential operator Λc_i, the dispersion coefficient's value is increased by $\frac{q^2dt}{6\theta^2R}$, where $R=r/\theta$, whereas in the operator $\Lambda\hat{c}_i$, it decreases by this number, at the respective time range. The results obtained for so-constructed algorithm do not exhibit an influence of numerical dispersion, however there occurring considerable oscillations of the approximate solution, the greatest at big values of $qdz/(\theta D_s)$. This effect is exemplified in Fig. 3, compared with the above analytical solution.

The author also analysed the case of the right term of equation (2) approximated like in scheme (3). Unfortunately, in this case the obtained results retained the features of scheme (3).

2.3. Approximate solution of the dispersion equation – the Galerkin-Pietrov method

The Galerkin method's modification Pietrov suggested consists of introducing an α – coefficient, which brings about a change of the base function form, from linear into square. Practically, in the language of differential schemes, the idea lines in a different approximation of the $\frac{\partial c}{\partial z}$ derivative and a modification to the $\frac{\partial c}{\partial t}$ derivative approximation. The necessity to avoid oscillation requires that, for $q>0.0$, the $\frac{\partial c}{\partial z}$ derivative be approximated as follows $\frac{\partial c}{\partial z}=\frac{\nabla c_i}{dz}$. Since $\frac{\nabla c_i}{dz}=\frac{c_{i+1}-c_{i-1}}{2dz}-\frac{\Delta^2 c_i}{2dz}$, the introduced α – coefficient thus corrects this approximation:

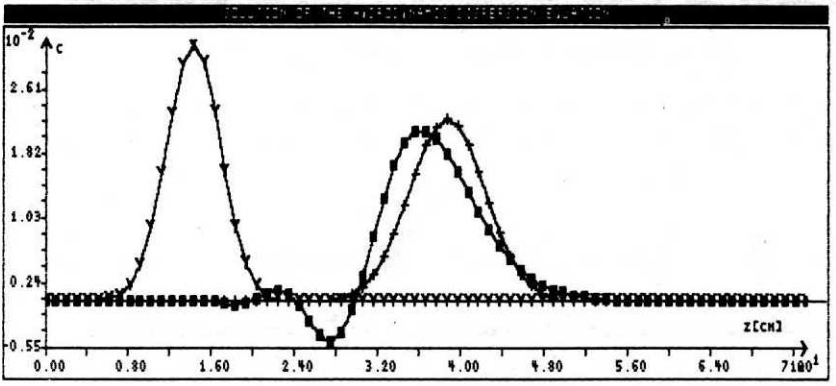

Fig. 3. Method 2 – Based solution of the hydrodynamic dispersion equation
Y initial condition, + concentration after the process,
■ solution according to the scheme (4)

$$\frac{\nabla c_i}{dz} = \frac{c_{i+1} - c_{i-1}}{2dz} - \alpha \frac{\nabla^2 c_i}{2dz}.$$

Utilising this method, equation (2) was approximated as follows:

$$r_i \left(\left(\frac{1}{6} + \frac{\alpha}{4} \right) \frac{\hat{c}_{i+1} - c_{i+1}}{dt} + \frac{2}{3} \frac{\hat{c}_i - c_i}{dt} + \left(\frac{1}{6} - \frac{\alpha}{4} \right) \frac{\hat{c}_{i-1} - c_{i-1}}{dt} \right) =$$
$$= \frac{1}{dz} (\Lambda \hat{c}_i) - q \left(\frac{\hat{c}_{i+1} - \hat{c}_{i-1}}{2dz} - \alpha \frac{\Delta^2 \hat{c}_i}{2dz} \right) - \mu_i \hat{c}_i + \gamma_i, \tag{5}$$

for $i = 1, 2, \ldots, N-1$.

It is essential here to select appropriate value of α. In paper (Fletcher 1984), the problem was subjected to an analysis. The authors suggest accepting the following value:

$\alpha = \text{cth}\beta - 1/\beta$, where $\beta = \frac{q\,dz}{2rD_s}$

or $\alpha = 1 - 1/\beta$, for great values of β.

The analysis was carried out in a steady problem situation. On the other hand, for unsteady problems, they suggest accepting $\alpha = \beta/3$. The approximate solutions of the dispersion equation obtained by means of this method exhibit the same features as approximation (3), being more accurate however. The further analyses assume the value $\alpha = \beta/3$.

2.4. Approximate solution of the dispersion equation – the Galerkin-Pietrov method modified

In paper (1979) Griffiths and Mitchell generalised the Galerkin-Pietrov method for unsteady problems. This was done by introducing three parameters, α_1, α_2, α_3, modifying the linear base functions. The application of this method to equation (2) approximation leads us to the following differential problem:

$$r_i\left(\left(\frac{\alpha_3-\alpha_2}{2}\right)\frac{\hat{c}_{i+1}-c_{i+1}}{dt}+(1-\alpha_3)\frac{\hat{c}_i-c_i}{dt}+\left(\frac{\alpha_3+\alpha_2}{2}\right)\frac{\hat{c}_{i-1}-c_{i-1}}{dt}\right)=$$

$$=\frac{1}{dz}(\Lambda\hat{c}_i)-q\left(\frac{\hat{c}_{i+1}-\hat{c}_{i-1}}{2dz}-\alpha_1\frac{\Delta^2\hat{c}_i}{2dz}\right)-\mu_i\hat{c}_i+\gamma_i, \qquad (6)$$

for $i=1,2,\ldots,N-1$.

This computational scheme is, assuming $\alpha_1=\alpha_2=0$ and $\alpha_3=1/3$, the on of Galerkin; whereas for $\alpha_2=0.5\alpha_1$ and $\alpha_3=1/3$, it is scheme (5), obtained by means of the Galerkin-Pietrov method. The numerical experiments accomplished using this scheme for α_1 – values as in the Galerkin-Pietrov method and $\alpha_2=0.5\alpha_1$ and $\alpha_3=1/3+\left(\frac{qdt}{6\theta dz}\right)^2$, which is suggested by paper's authors, allow for stating the obtained results hardly differ from the solutions attained with the help of the Galerkin-Pietrov method. The scheme's form nevertheless permits undertaking an attempt to select such values of the α_1, α_2, α_3 – parameters that the properties of the solutions obtained can be improved. The simulations accomplished for different alues of the parameters occurring in this problem made the selection of such values possible. It was accepted that

$$\alpha_1=\tanh\left(\frac{q\,dz}{6rD_s}\right),\quad \alpha_3=\tanh\left(\frac{7qdt}{6rdz}\right),\quad \alpha_2=|\alpha_3|.$$

The solution obtained using this scheme is exemplified in Fig. 4. Accepted were the same values of dt, dz as those when computing the solution displayed in Fig. 2.

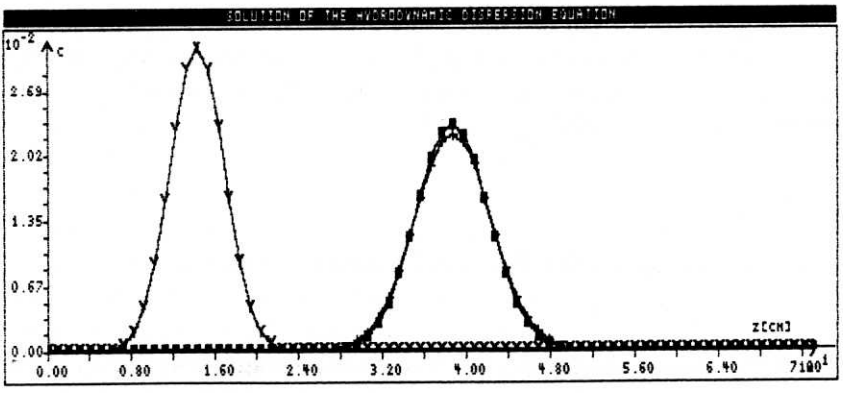

Fig. 4. Method 4 – based solution of the hydrodynamic dispersion equation
Y initial condition, + concentration after the process
■ solution according to the scheme (6)

2.5. Approximate solution of the dispersion equation –introduction of „migratory coordinates"

The above-mentioned methods for an approximate solution of the hydrodynamic dispersion equation did not bring about a change of the equations's form or the values

of the parameters it includes. Hence the problem of the influence function q exerted on the solution remained unchanged. Any operation done in the above methods was of technical character. Therefore it seems crucial to undertake an attempt which, owing to an appropriate transformation, would diminish the influence of the advection component on the equation or even eliminate it.

Let

$$C(z,t) = c^*(y,t),$$

where:

$$\bar{t} = t, \qquad y = z - \int_o^t \frac{q(z,\tau)}{\theta(z,\tau) + \rho k}\, d\tau. \tag{7}$$

Let us denote $f(z,t) = \dfrac{\partial}{\partial z} \displaystyle\int_o^t \frac{q(z,\tau)}{\theta(z,\tau) + \rho k}\, d\tau.$

Since the problem obtained after the transformation will be solved applying the method of two-ply differential schemes, it is possible to assume $|f(z,t)| < 1$, hence the transformation applied is correct. In the computational algorithm worked out, this conditionis verified and, in unfulfilled, the dt value is corrected accordingly.

It was accepted that what we looked for was the solution of equation (2) with the following initial-boundary conditions system imposed on:

$$c(z,0) = c_p(z), \qquad (0 \le z \le L),$$

$$\text{Dirichlet} - \quad c(0,t) = c_i(t), \quad (t > 0),$$

or

$$\text{Neumann} - \quad q_c(t) = \left(-D_s\theta \frac{\partial c}{\partial z} + v\theta c\right)\Big|_{z=0}$$

$$\frac{\partial c}{\partial z}(L,t) = 0,$$

and assuming the values of the flow function q meet the conditions

$$q(0,t) \ge 0, \qquad q(L,t) = 0.$$

Having applied the above-defined change of variables, equation (2) can by presented as follows:

$$r^* \frac{\partial c^*}{\partial t} = (1 - f^*)^2 \frac{\partial}{\partial y}\left(\theta^* D_s^* \frac{\partial c^*}{\partial y}\right) - \left(q^* f^* - \left(\frac{\partial f}{\partial z}\right)^* \theta^* D_s^*\right) \frac{\partial c^*}{\partial y} - \mu c^* + \gamma \tag{8}$$

where: $u^*(y,\bar{t})$ is $u(z,t)$ - dependent variable, calculated according to the new coordinates.

In equation (8), the convection component depends on the value and derivative of function f^*, hence its value does not depend on the flow but only on the fluctuations this function displays. When considering steady flows, i.e. when q = const and θ = const, equation (8) is reduced to the form

$$r^* \frac{\partial c^*}{\partial t} = \frac{\partial}{\partial y}\left(\theta^* D_s^* \frac{\partial c^*}{\partial y}\right) - \mu c^* + \gamma$$

However, solving equation (8), we are obliged to introduce differential diagrams with a variable space interval. To do this, utilised was the method submitted in 2.1.

Let

$$q_i^+ = 0.5\left(-\hat{q}_i^* \frac{dy_i(f_i^* - k_i^* f_i^{*\prime})}{\overline{dy}_i} + 2|\hat{q}_i^*|(f_i^* - k_i^* f_i^{*\prime})\right),$$

$$q_i^- = -0.5\left(\hat{q}_i^* \frac{dy_{i+1}(f_i^* - k_i^* f_i^{*\prime})}{\overline{dy}_i} + 2|\hat{q}_i^*|(f_i^* - k_i^* f_i^{*\prime})\right),$$

$$\kappa_i = \frac{(1 - f_i^*)^2}{1 + R_i}, \text{ where } R_i = \frac{\overline{dy}_i |G_i^*|}{(1 - f_i^*)^2 k_i^*}, \; G_i^* = (-\hat{q}_i^* f_i^* - k_i^* f_i^{*\prime}), \; f^{*\prime} = \left(\frac{\partial f}{\partial z}\right)^*.$$

Then, equation (8) can be approximated as follows:

$$r_i^* \frac{\hat{c}_i^* - c_i^*}{dt} = \frac{\kappa_i}{\overline{dy}_i}(\Lambda \hat{c}_i^*) + \frac{q_i^+}{k_i^*} \frac{\hat{k}_i^*}{2} \frac{\Delta \hat{c}_i^*}{dy_{i+1}} + \frac{q_i^-}{k_i^*} \frac{\check{k}_i^*}{2} \frac{\nabla \hat{c}_i^*}{dy_i} - \mu_i \hat{c}_i^* + \gamma_i \tag{9}$$

where: $\overline{dy}_i = 0.5(dy_{i+1} + dy_i)$, $\Lambda \hat{c}_i = \frac{\hat{k}_i}{2} \frac{\Delta \hat{c}_i}{dy_{i+1}} - \frac{\check{k}_i}{2} \frac{\nabla \hat{c}_i}{dy_i}$, for $i = 1, 2, \ldots, N-1$.

In this case, the solving of the system of equation (9) is the final stage of the entire algorithm, which has been divided into two parts depending on the flow value $q(0,t)$. The one, simpler case is to be dealt with when $q(0,t) = 0$. Then the number of digitising grid nodes along the spatial variable increases by one at each time range. In order not to cause an excess, in plausible cases (a cumulation of the grid's nodes in the environment $z = L$), it is possible to withdraw the last but one node of the digitising grid there form. Before starting to solve the system of equation (9) in the elaborated algorithm determined are values of z-variable appropriate for the accepted digitisation of variable y the values of variable z at a new time range ($n - 1$ equations have to be solved). This permits determining the values of the parameters which occur in equation (9) and transforming the boundary conditions.

The presented method is definitely more time-consuming referring to its computations, yet the obtained approximate solution of the dispersion equation do not depend that substantially on the flow values. This is especially visible when compared with the analytical solution with the help of which the other methods were tested. The approximate solutions obtained by means of this method are practically identical with the analytical solutions. The numerical experiments accomplished according to this algorithm, however, make us justified to say that once the diagram's space and time intervals have been chosen incorrectly, there might occur noticeable errors in the approximate solution. These errors are caused by a great variability of the space interval assumed a priori and the one determined by the algorithm during the simulation.

3. A comparison of approximate methods for integrating the dispersion equation

The five submitted methods for attaining the approximate solution of the dispersion equation have been mutually compared using the analytical solution presented in the Introduction. The following measures characterising the occurring approximation error have been taken:

- correlation coefficient r
- relative integrating error $Bc = \int_o^L \frac{|c_a - c_s|}{M_o} dx$
- mean square deviation error $Bs = \sum_{i=1}^{N} (c_a - c_s)^2$

where:

c_a – is the analytical solution,

c_s – is the approximate solution.

The analysis of the submitted methods correctness was two-staged. At a fixed process time and the interval of spatial digitisation $dz = 0.5$, changed were the values of the time interval. The cases considered referred to 6, 12, 24, 36, 72, 144 time ranges. The results are presented in the subsequent figures. Fig. 5 displays correlation coefficient values, whilst Figs 6 and 7 – the relative integrating and the mean square errors, respectively. The subsequent analysis concerned the correctness of the presented methods in case

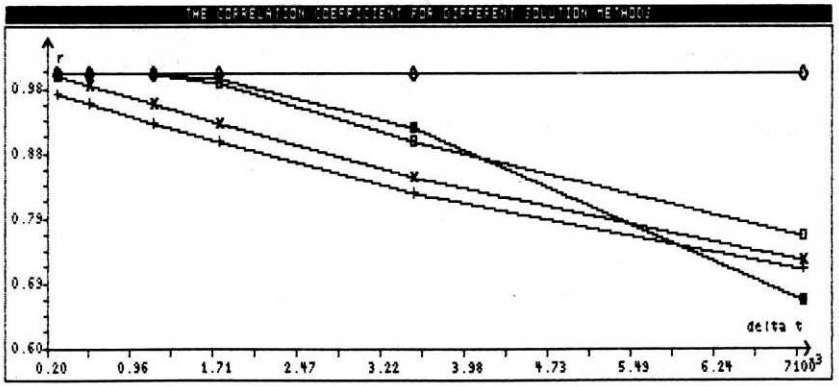

Fig. 5. The correlation coefficient for different solution methods
+ corrected diagram, ■ van Genuchten,
× Galerkin-Pietrow, ◇ migratory coordinates,
□ G-P modified

the space interval values change, at fixed process time and fixed interval of temporal digitisation, i.e. 24 time ranges. Considered were 5 cases, for $dz = 0.125$, 0.25, 0.5, 1.0, 2.0. The results are presented in the subsequent figures. Fig. 8 displays correlation coefficient values, whilst Fig. 9 – the relative integrating error. What we abandoned here was the mean square error as it is very similar to the integrating error.

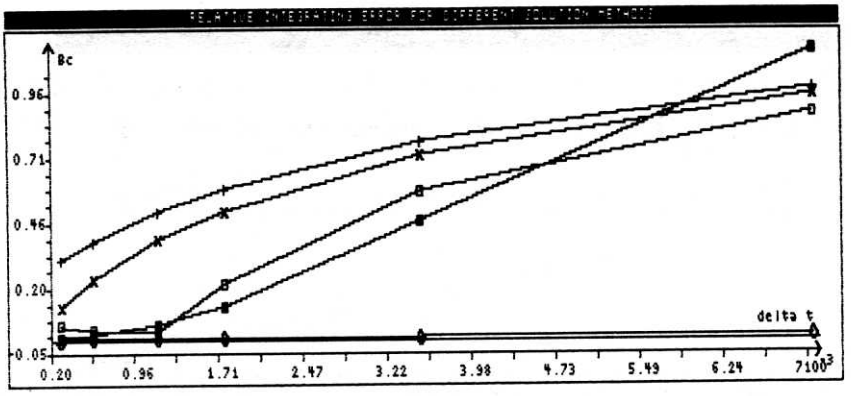

Fig. 6. Relative intergrating error for different solution methods
+ corrected diagram, ■ van Genuchten,
× Galerkin-Pietrow, ◇ migratory coordinates,
□ G-P modified

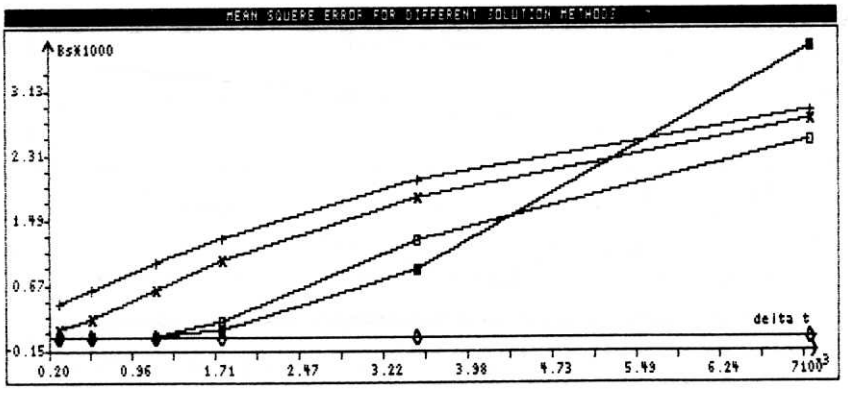

Fig. 7. Mean squere error for different solution methods
+ corrected diagram, ■ van Genuchten,
× Galerkin-Pietrow, ◇ migratory coordinates,
□ G-P modified

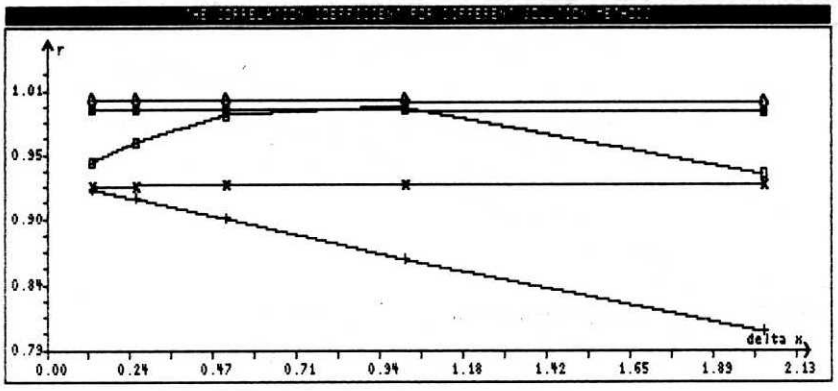

Fig. 8. The correlation coefficient for different solution methods
+ corrected diagram, ■ van Genuchten,
× Galerkin-Pietrow, ◇ migratory coordinates,
□ G-P modified

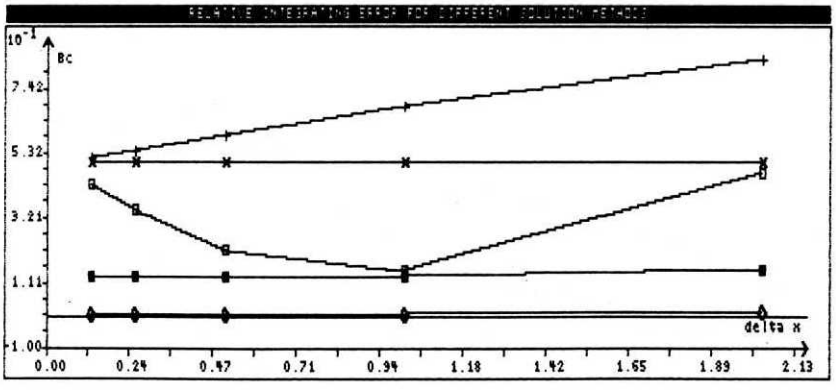

Fig. 9. Relative integrating error for different solution methods
+ corrected diagram, ■ van Genuchten,
× Galerkin-Pietrow, ◇ migratory coordinates,
□ G-P modified

4. Conclusions

The first four methods presented in the paper, more advantageous because of the computation time, permit obtaining the best results when the value $\sqrt{2D_s dt}$ is greater than the value $|v * dt|$ which occur in the process, or if a long time of contaminating is considered. In these cases, the numerical dispersion does not affect the solution that much, but it is only oscillations found out to occur in some of the methods (pt 2.2) that force us to choose a small time interval. In the analysed example, the method proposed by van Genuchten and Wierenga, the oscillations vanished only after assuming 144 time ranges for the considered process time. In such cases, the method of the so-called „migratory coordinates", of little effectiveness due to its computation time yet very accurate, becomes a tempting proposal. The Galerkin-Pietrov method, its modified version or the van Genuchten-Wierenga method, as the methods utilising correcting parameters entered a priori, always require a detailed analysis of the process's parameters. This is necessary due to the possibility of the approximate solution's oscillation, as it took place in the example presented in Fig. 3.

While carrying on computer simulations of long-lasting processes of contamination transport, it is essential to choose the proper computation method in relation to the parameters of simulated process. The factors substantially conditioning simulation effectiveness are the accuracy of the calculations and their time. The analysis of the effectiveness the five methods exhibit, carried out in this paper, permits making such a choice for the process's parameters accepted in this example.

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Summary

In the paper we apply the finite differences method for the approximate solution of one dimensional hydrodynamic dispersion equation. Five various algorithms are investigated. The accuracy analysis of the solution for different steps of the time and space are presented. Obtained solutions are compared with the analytical one of the linear dispersion equation. We accept three measures of approximate deviations:

- correlation coefficient,
- integral relative error,
- sum square deviations.

On the base of measures analysis we can accept algorithm, time and space steps for the best approximate solution of the one dimensional hydrodynamic equation.

Streszczenie

W pracy zastosowano metodę różnic skończonych do przybliżonego rozwiązania jednowymiarowego zagadnienia dyspersji hydrodynamicznej. Poddano badaniom pięć różnych algorytmów rozwiązania tego zagadnienia. Analizę dokładności przybliżonych rozwiązań przeprowadzono w zależności od przyjętej dyskretyzacji zmiennej czasowej i przestrzennej. Otrzymywane rozwiązania przybliżone porównano z rozwiązaniami analitycznymi zlinearyzowanego równania dyspersji. Przyjęto następujące miary charakteryzujące błąd przybliżenia:

- współczynnik korelacji,
- względny błąd całkowy,
- sumę odchyleń kwadratowych.

Otrzymane rezultaty i sformułowane na tej podstawie wnioski umożliwiają dobór algorytmu oraz przyjęcie dyskretyzacji zmiennej czasowej i przestrzennej w zależności od zmienności parametrów rozpatrywanych zagadnień.