ARCHIWUM HYDROTECHNIKI
Tom XXXVII 1990 Zeszyt 1–2

*WŁADYSŁAW KRÓL, ALOJZY SZYMAŃSKI**

Prediction of large – strain consolidation in organic subsoil

1. Introduction

Earth structures in some areas cannot omit an organic subsoil. In such a case the prediction of subsoil behaviour and selection of the proper method of subsoil treatment is an engineering task so important as difficult. Difficulties result mainly from extremely high compressibility and low shear strength of such soils, which cause a large deformation of subsoil often associated with failure. Therefore, the correct estimation of deformation performance and excess pore pressure dissipation in subsoil is an important part of design. Data concerning the current state of elevation of each subsoil layer and the effective stress distribution make it possible to determine the shear strength increase and to evaluate the subsoil stability.

The paper gives a numerical scheme for the prediction of consolidation in loaded organic subsoil. The presented approach takes into account large deformation analysis based on the convective coordinate system and nonlinear variability of parameters. Application of the large - strain analysis to the prediction of consolidation in layered subsoil requires taking into consideration special boundary problems. The problems appear at level between compressible layers, because of differences in pore pressure and pore pressure gradient values obtained from classical numerical calculations for each different layer at the boundary level.

The presented scheme gives a numerical procedure for solving the boundary problems, which depend on time and loading schedule. A comparison of predicted and

---

*Dr inż. Władysław Król, Katedra Informatyki, dr inż. Alojzy Szymański, Katedra Geotechniki Szkoły Głównej Gospodarstwa Wiejskiego Akademii Rolniczej w Warszawie, ul. Nowoursynowska 166, 02-766 Warszawa observed displacements as well as excess pore pressure dissipation shows that the presented scheme can be applied to the consolidation analysis in multi-layers soft subsoil.

2. Large - strain analysis of consolidation

Prediction of deformations and consolidation course are carried out with different degrees sophistication depending on the importance of designed structures and the nature of subsoil behaviour.

The consolidation process with time has been calculated assuming one, two or three-dimensional water flow, with constant or varying permeability and with constant or changing compressibility. This process is usually described in one-dimensional state of strain by the following equations:

– stress-strain relationship for the soil skeleton expressed in terms of void ratio,

$$\frac{\partial \varepsilon_o}{\partial t} = \frac{1}{1+e}\frac{\partial e}{\partial t} = \frac{1}{1+e}\frac{de}{d\delta'}\frac{d\delta'}{dt} \tag{1}$$

where:

ε_o – volume strain,
e – void ratio,
t – time,
δ' – effective stress.

– Darcy's law for the pore water flow,

$$q_x = \frac{k_x}{\gamma_w}\frac{\partial u}{\partial x} \tag{2}$$

where:

q_x – the component of the pore water discharge,
k_x – coefficient of permeability,
γ_w – the unit weight of water,
u – excess pore pressure,
x – vertical space coordinate.

– continuity equation,

$$\frac{\partial \varepsilon_o}{\partial t} = -\frac{\partial q_x}{\partial x} \tag{3}$$

– Terzaghi's formula for the relation between total and effective stress,

$$\delta = \delta' + u \tag{4}$$

where: δ – total stress.

In the simplest forms of settlement analysis the subsoil conditions are simplified to one or several layers with invariable constant deformation and permeability properties. The deformations are then calculated as purely elastic shear strain such as

D'Appolonia's approach (1971) for determining the initial settlement or as elastic vertical compression such as in Terzaghi's consolidation theory (1924).

The assumption adopted for Terzaghi's theory, linear compressibility characteristic, constant permeability and small strains (Fig. 1), were such that its applicability was effectively limited to the consideration of relatively stiff thin layers at large depths.

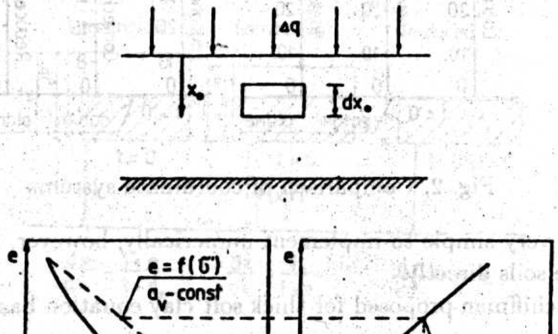

Fig. 1. Assumptions of Terzaghi's theory

Application of one-dimensional theory to the prediction of deep soft subsoil settlement requires to take into account the large strains in consolidation analysis (Larsson 1986). In this case the consolidation prediction using equations (1), (2), (3), (4), involves the application of reduced or convective coordinate system (Fig. 2). These approaches to the consolidation prediction with large strain analysis for one-dimensional state of strain and pore water flow as well as for non-linear relationships: void ratio-effective stress and void ratio-permeability, were presented by Somogyi (1979), Gibson – Schiffman (1981) and by Yong and Ludwig (1984). Somogyi proposed an equation on the basis of reduced coordinate system as:

$$\frac{\partial}{\partial z}\left[\frac{k}{\gamma_w(1+e)}\frac{\partial u}{\partial z}\right]+\frac{de}{d\delta'}\,\frac{Du}{Dt}\bigg|_x=0 \qquad (5)$$

where:

e – void ratio,
γ_w – the unit weight of water,
k – permeability,
δ' – effective stress,
u – excess pore pressure,
x – vertical space coordinate,
z – reduced coordinate,
t – time.

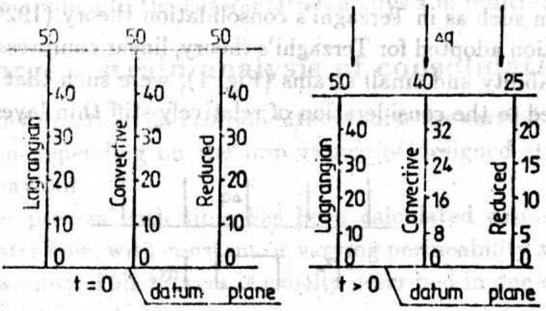

Fig. 2. Comparison of coordinate systems

This equation is very simple to implement numerically, however, it does not give the settlement of the soils directly.

Gibson and Schiffman proposed for thick soft clay equation based on reduced coordinate system as:

$$-\left[\frac{\gamma_s}{\gamma_w}-1\right]\frac{d}{de}\left[\frac{k}{1+e}\right]\frac{\partial e}{\partial z}+\frac{\partial}{\partial z}\left[\frac{k}{\gamma_w(1+e)}\,\frac{d\delta'}{de}\,\frac{\partial e}{\partial z}\right]+\frac{De}{Dt}\bigg|_x=0 \tag{6}$$

where: γ_s – the unit weight of solids.

Yong and Ludwig offered consolidation equation for deep soft subsoil with application convective coordinate system as:

$$\frac{\partial}{\partial\zeta}\left[\frac{k}{\gamma_w}\,\frac{\partial u}{\partial\zeta}\right]+\frac{1}{(1+e)}\,\frac{de}{d\delta'}\,\frac{Du}{Dt}\bigg|_x=0 \tag{7}$$

where: ζ – convective coordinate.

Comparison of computed results (McVay et al. 1986); excess pore pressure dissipation and surface pond settlements obtained by Somogyi, Gibson as well as by Yong indicate insignificant differences in these results. Verification of the numerical predictions on the basis of centrifuge simulation test results shows great differences between measured and calculated values. It was shown that most existing theoretical formulations which characterize one-dimensional consolidation are in fact the same if expressed in the same coordinates or dependent variable.

Adequate evaluation of the variability of void ratio and permeability during consolidation process is more important than selection of theoretical methods.

3. Numerical solution of the governing equation

The application of differential equations to predict the settlement of loaded subsoil and excess pore pressure dissipation requires numerical solution because of coefficients nonlinearity. The numerical solution can be based on finite difference scheme (Fig. 3) with the application of the finite-strain consolidation analysis or the piece-wise linear approach.

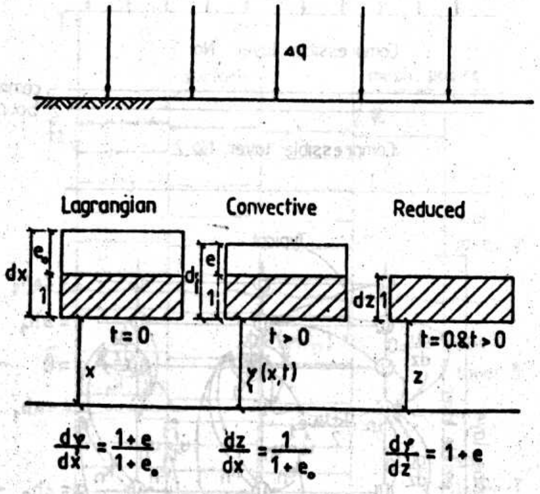

Fig. 3. Differential soil elements

The finite consolidation analysis developed by Gibson et al. (1981) is based on the reduced coordinate system. The governing equation solved in terms of reduced coordinates system required recalculation at each step for the current void ratio value and the boundary conditions defined in terms of void ratio.

In the piece-wise linear iterative analysis developed by Yong and Ludwig (1984), the derivation for finite difference consolidation is performed with respect to a convective coordinate system. This approach is based on updating the excess pore pressure explicitly.

The application of these approaches to the prediction of consolidation in layered subsoil requires taking into account the boundary problems which appear at the level between compressible layers by means of common imagined boundary with imaginary mesh points (Cargill 1982, Szymański and Lechowicz 1987). This iterative numerical procedure is complicated in design calculations of consolidation course in soft subsoil which consist of several thin layers (Fig. 4).

The adaptation of more sophisticated method without the necessity of solving boundary problems between compressible layers in the numerical calculation for layered subsoil can be done by using the implicit scheme in finite consolidation analysis. Then, the governing equation (3) can be written as:

$$\frac{\partial}{\partial\zeta}\left[C\frac{\partial u}{\partial\zeta}\right]+\left.\frac{\partial u}{\partial t}\right|_{z}=0 \tag{8}$$

where: $C=\frac{(1+e)k}{\gamma_w}\frac{d\delta'}{de}$

According to this approach the ζ-th plane is subdivided into sets of rectangles of the sides $\Delta\zeta_j$ and Δt (Fig. 5). The representative mesh point P described by coordinates (ζ_i, t_n) can be expressed as:

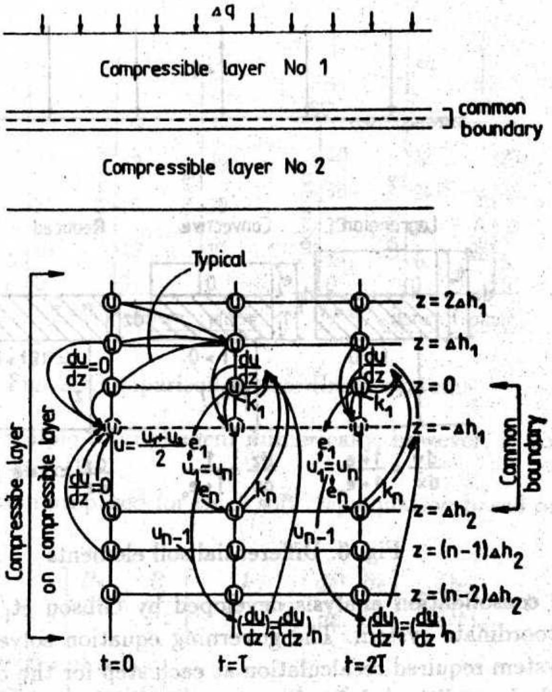

Fig. 4. Designation for numerical solution of the pore pressure dissipation of the boundary level

$$\zeta_i = \sum_{j=1}^{i} \Delta\zeta_j \tag{9}$$

and

$$t_n = n\Delta t \tag{10}$$

where: i and n are integers.

Denoting the value of u at point P by

$$u_p = u(\zeta_i, t_n) = u_{i,n} \tag{11}$$

the original differential problem (eq. 8) can be approximated by:

$$\frac{u_{i,n+1} - u_{i,n}}{\Delta t} = \frac{C_{i+\frac{1}{2},n}\dfrac{u_{i+1,n+1} - u_{i,n+1}}{\zeta_{i+1,n} - \zeta_{i,n}} - C_{i-\frac{1}{2},n}\dfrac{u_{i,n+1} - u_{i-1,n+1}}{\zeta_{i,n} - \zeta_{i-1,n}}}{\zeta_{i+\frac{1}{2},n} - \zeta_{i-\frac{1}{2},n}}. \tag{12}$$

where: $n = 0, 1, \ldots N$ and $i = 1, 2, \ldots I$

Numerical solution of this equation from $u_{i,n}$ to $u_{i,n+1}$ involves three unknown values of $u_{i,n+1}$. Thus equation (8) defines $u_{i,n+1}$ implicitly, and a tridiagonal system of

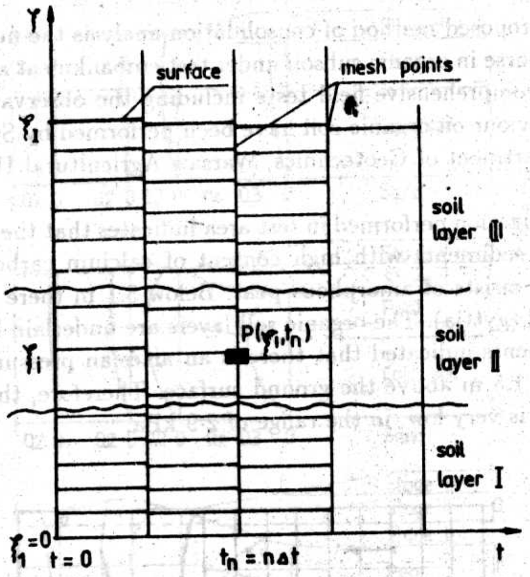

Fig. 5. Graphical scheme of the subsoil discretisation

simultaneus equations must be solved at each time step with using a straighforward Gaussian elimination method.

The equation (8) has to satisfy:
the initial condition

$$u(\zeta, t = o) = 0; \quad \text{for all} \quad \zeta \tag{13}$$

and the boundary conditions

$$u(\zeta_{\max}, t) = 0; \quad \text{for all} \quad t \tag{14}$$

$$u(\zeta = 0, t) = 0 \quad \text{or} \quad \frac{\partial u}{\partial \zeta}(\zeta = 0, t) = 0; \quad \text{for all} \quad t \tag{15}$$

These conditions are approximated and added to equation (12) and next are solved together. During simulation the value of $u(\zeta_i, t_n)$ is modified according to loading schedule:

$$u_{i,n} := u_{i,n} + \Delta\delta_n \tag{16}$$

where: $\Delta\delta_n$ – increase in total stress in subsoil, : = substitution operator.

The variation in value of $\Delta\zeta_i$ during consolidation process is described by relation:

$$\Delta\zeta_{i,n+1} = \Delta\zeta_{i,n} \frac{1 + e_{i,n+1}}{1 + e_{i,n}} \tag{17}$$

The total thickness of compressible subsoil for each t is calculated as a sum of all $\Delta\zeta$.

4. Prediction of subsoil consolidation

To verify the proposed method of consolidation analysis the numerical calculations of consolidation course in organic subsoil under test embankment at Antoniny site were carried out. The comprehensive field tests including the observation and analysis of embankment behaviour on organic soil have been performed by Swedish Geotechnical Institute and Department of Geotechnics, Warsaw Agricultural University (Wolski et al 1988, 1989).

The field investigation performed in test area indicates that the soft subsoil consists mainly of organic sediments with high content of calcium carbonate (Fig. 6). The upper 3.1 meters consists of amorphous peat. Below 3.1 m there is a layer of almost pure carbonate soil (gyttja). The organic soil layers are underlain by dense sand. Pore pressure measurements indicated that there is an artesian pressure in the sand layer with a water head 1.5 m above the ground surface. Therefore, the effective stress in the natural subsoil is very low, in the range of 2-9 kPa.

Fig. 6. Initial soil properties at Antoniny site

The test embankment have been performed at the Antoniny site located in the vicinity of the Noteć River. Due to very low shear strength of subsoil (8-10 kPa) a construction by stages was decided.

Observation of the vertical displacements in the subsoil was performed by means of settlement gauges of 4 types: hose, plate, screw and magnetic. The horizontal displacements in organic subsoil have been calculated from inclinometer readings. The

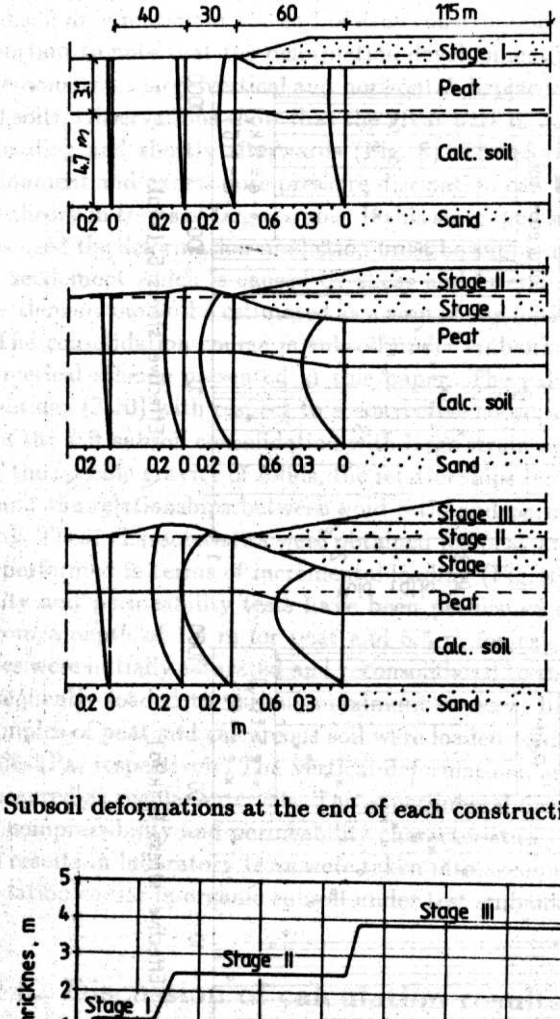

Fig. 7. Subsoil deformations at the end of each construction stage

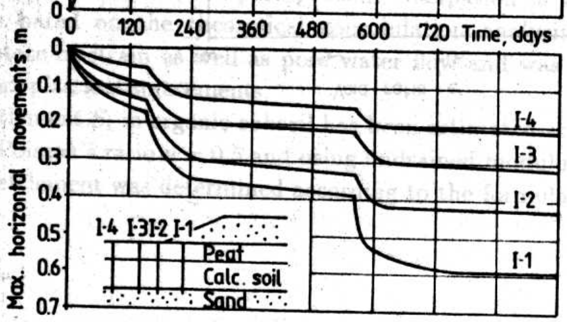

Fig. 8. Development of horizontal movements

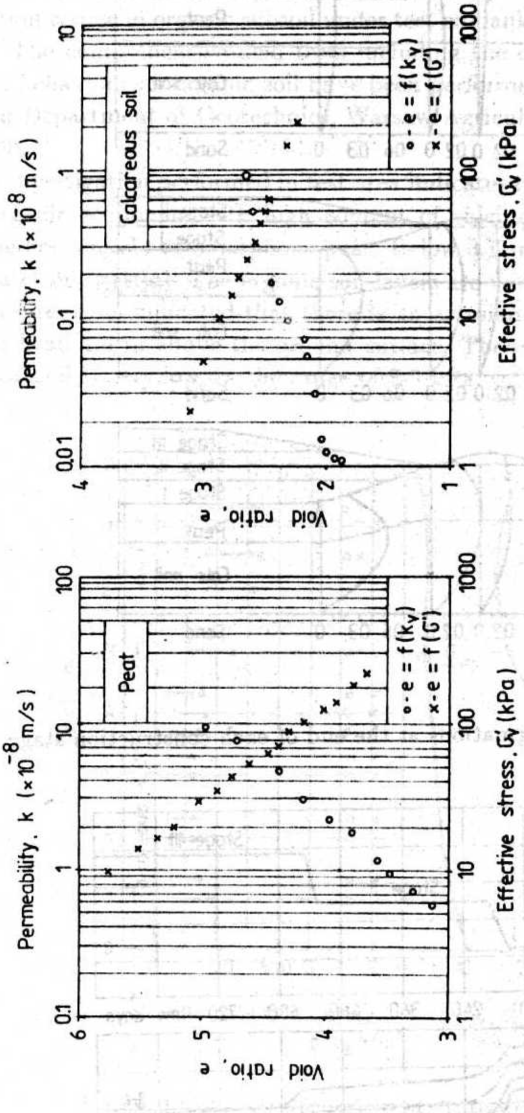

Fig. 9. Consolidation characteristics of organic soils

magnitude of subsoil deformation at the end of each construction stage is presented in Fig. 7. It is important to note that the deformation development in organic soil under embankments demonstrates large vertical and horizontal displacements rarely encountered in mineral soils. Observations show that the great part of horizontal movements appear during loading and shortly afterwards (Fig. 8). Therefore, the calculation of settlement development and excess pore pressure dissipation can be done by means of one-dimensional theory in terms of large strains. If this simplified solution of the consolidation theory is used the deformation prediction must be supplemented by evaluation of initial-plastic settlement which is caused by shear and lateral deformation in subsoil. The total settlement should be estimated as a sum of the initial and consolidation displacements. The consolidation course in subsoil under embankment was calculated by means of numerical scheme presented in this paper. The calculations were done according to equations (3-10) with respect to a convective coordinate system.

Calculation of the soft subsoil consolidation with large strain approach requires the determination of the specific gravity of solids, the relationships between void ratio and effective stress, and the relationships between void ratio and permeability for each of compressible layer. These characteristics were obtained for peat and calcareous soil in oedometer tests performed in terms of incremental loading (Fig. 9).

Compressibility and permeability tests have been performed on the undisturbed samples taken from a depth of 1.5 m for peat and 5.5 m for calcareous soil. All the individual samples were initially saturated and reconsolidated to the in-situ stress conditions and subsequently loaded to various maximum stresses. In each investigation the individual samples of peat and calcareous soil were loaded to maximum stresses of 10, 20, 40, 80, 160 kPa, respectively. The vertical deformations and volume of water released were measured at regular intervals. The experimental data were used for the determination of compressibility and permeability characteristics.

The obtained results in laboratory tests were taken into account to numerical prediction of consolidation course in organic subsoil under test embankments at Antoniny site.

5. Discussion of calculation results

The prediction of settlements and pore pressure dissipation in subsoil under test embankment was based on the theoretical consolidation analysis considering the one-dimensional state of strain as well as pore water flow and was supplemented by evaluation of initial-plastic displacements.

The initial settlement S_i in organic subsoil has been estimated according to elastic theory, assuming Poisson's ratio $\nu = 0.5$ and using undrained modulus of elasticity E_u. Then the initial settlement was determined according to the formula (D'Appolonia et al 1971):

$$S_i = IqB/E_u \tag{18}$$

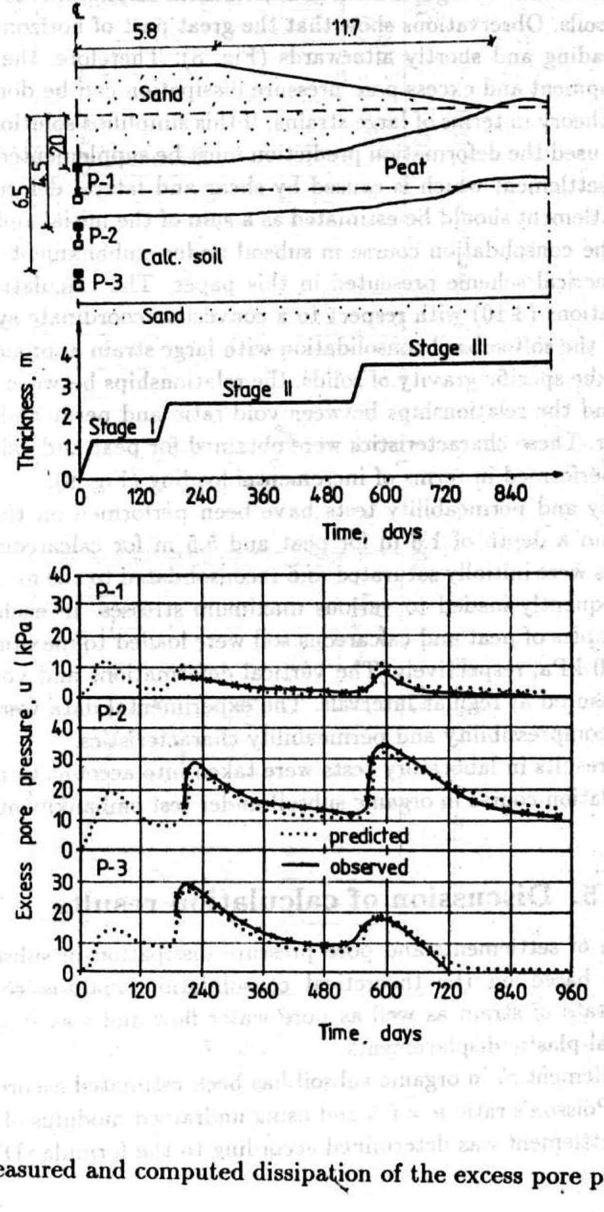

Fig. 10. Measured and computed dissipation of the excess pore pressure

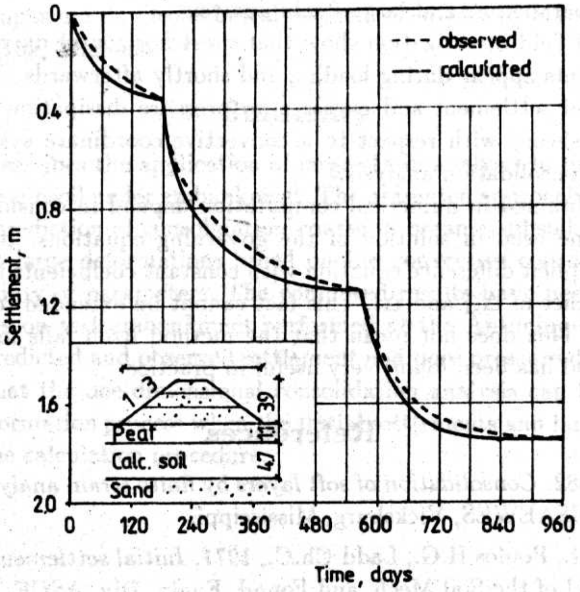

Fig. 11. Measured and computed settlement of embankment subsoil

where:

- I – the influence factor,
- q – the stress applied to the subsoil,
- B – the width of the loaded area.

The total initial settlement calculated according to eq. (18) with Foott and Ladd's (1981) empirical relations to obtain E_u value for the three stages is $S_i = 0.40$ m.

The calculation of the subsoil consolidation was carried out by numerical procedure presented above, using the compression curves and relationship between void ratio and permeability coefficient obtained in oedometer tests.

A comparison of the computation results with the field observations has shown quite good coincidence in pore pressure and vertical displacements when the numerical calculation of consolidation has been done with using the implicit scheme in consolidation analysis (Fig. 10 and 11). It is worth mentioning that in this case differences between measured and computed values are in the range of 5-10%.

An application of the implicit scheme in numerical computation makes it possible to obtain in practice proper results for any $\Delta\zeta$ and Δt in spite of nonlinear governing equation and soil characteristics.

6. Conclusions

Observations of the consolidation process in organic soils demonstrate large values and nonlinear character of deformation. Therefore, the prediction of consolidation performance in organic subsoil should be carried out by methods taking into account the variation of soil parameters and large-strain analysis.

The results of field investigation show that most horizontal movements in subsoil under embankments appear during loading and shortly afterwards. Therefore, the design calculation of settlement and excess pore pressure dissipation may be done by one-dimensional theory with respect to a convective coordinate system and implicit scheme in finite consolidation analysis.

The results obtained in numerical computation depend in considerable extend on calculation scheme used in solution of the governing equations. It is important to know that the implicit difference equation with constant coefficients is unconditionally stable for any choice of $\Delta\zeta_i$ and Δt. This fact cannot be extended to cases where the coefficients vary. This does not mean that the method itself fails for such problems; indeed the method has been found very useful in practice.

References

Cargill K.W., 1982, *Consolidation of soft layers by finite strain analysis*, MP-GL-82-3, Geot. lab. USAEWES, Vicksburg, Mississippi.

D'Appolonia D.J., Poulos H.G., Ladd Ch.C., 1971, *Initial settlement of structures on clay*, Journal of the Soil Mech. and Found. Engin. Div. ASCE. SM. 10.

Foott R., Ladd Ch., 1981, *Undrained settlement of plastic and organic clays*, Geotechnical Engineering Division. ASCE GT8.

Gibson R.E., Schiffman R.L. and Cargill K.W., 1981, *The theory of one-dimensional consolidation of saturated clays*, Canadian Geotechnical Journal 18.

Larsson R., 1986, *Consolidation of soft soils*, Report No 29. Swedish Geotechnical Institute.

Mc Vay., Townsed F. and Bloomquist D., 1986, *Quiescent consolidation of phosphatic clays*, University of Florida.

Somogyi F., 1979, *Analysis and prediction of phosphatic clay consolidation*, Bromwell Carrier Engineering Inc. Lakeland Florida.

Szymański A. and Lechowicz Z., 1987, *Numeryczna prognoza konsolidacji uwarstwionego podłoża słabonośnego*, Materiały na konferencję „Komputery w Geotechnice", Rydzyna.

Terzaghi K., 1924, *Die Theorie der Hydrodynamischen Spanungserscheinungen und ihr erdbautechnisches Answendungsgebeit*, Proc. First Inter. Congress of Applied Mech. Vol. 1. Delft, Netherlands.

Wolski W., Szymański A., Mirecki J., Lechowicz Z., Larsson R., Hartlen J., Garbulewski K., Bergdahl U., 1988, *Two stage-constructed embankments on organic soils*, Report No 32. Swedish Geotechnical Institute, Linkoping.

Wolski W., Szymański A., Lechowicz Z., Larsson R., Hartlen J., Bergdahl U., 1989, *Full-scale failure test on a stage-constructed test fill on organic soil*, Report No 36. Swedish Geotechnical Institute, Linkoping.

Yong R.N., Ludwig C.A., 1984, *Large-strain consolidation modelling of land subsidence*, Symposium on Geotechnical Aspects of Mass and Materials Transportation, Bangkok, Thailand.

Summary

This paper describes the application of large-strain analysis to predict the consolidation of organic subsoil under embankment. The presented approach gives a numerical scheme for the prediction of consolidation course in organic subsoil. This method takes into account large deformations based on the convective coordinate system and nonlinear variability of parameters. The computed results have been compared with field observations on test embankment performed at the Antoniny site in Poland. A comparison of predicted and observed settlement and pore pressure dissipation for soft soils indicates that the one-dimensional consolidation analysis can be applied to the prediction of deformation process when the initial settlements and large-strain analysis are inluded in the calculation procedure.