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A mathematical model of pollutant dispersion in bidirectional flow and its empirical verification

1. Introduction

This paper presents a mathematical model and results of calculation and measurements of the flat tracer concentration field in conditions of bidirectional flow in laboratory channel. Bidirectional flows occur in mouth regions of river-beds. Run-off in the lower part of river-bed is the result of action of gravity force component parallel to levelled bottom slope. A backwater stream, which occurs in the upper part of river-bed with reverse direction to the run-off, is generated by wind action. In Polish conditions, bidirectional flows occur in mouth regions of the Vistula and the Oder River, and some other rivers on the Baltic sea-shore.

The wastes disposed of into rivers during bidirectional flow are transported in backwater "upstream" thus acting on places of water utilization and intaking situated beyond their extent in conditions of one-way flow.

Pollutant concentrations at points situated at an equal distance from the waste water outlet are greater than those, which occur in one-way flows.

Bidirectional flows occur often enough, that can not be neglected in elaborations referring to the present state and water quality prediction in rivers. For example the mean year frequency of northern winds in Świnoujście is 23%, and the mean number of days with wind velocity above 10 m/s is 36.5.

The measurements of flow velocity and tracer concentration were carried out in a laboratory channel. The bidirectional flow in the channel was generated by an adequate

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location of the inflows and outflows (Fig. 1) with applying stream-guides for run-off and backwater streams.

The calculation of tracer concentration field in the laboratory channel was carried out basing on presented mathematical model. The estimation of vertical dispersion coefficient was carried out basing on vertical flow velocity distribution measured during the laboratory experiments.

Notation

The following symbols are used in this paper:

$A(S_j)$	$[mg \cdot m^{-2}]$	– concentration field of tracer in a limited flat stream,
$D^{(z)}$	$[m^2 s^{-1}]$	– vertical dispersion coefficient,
H	$[m]$	– depth in bidirectional flow,
J	$[mg \cdot m^{-2} s^{-1}]$	– dispersion flux intensity,
K	$[m^2 s^{-1}]$	– coefficient of eddy viscosity,
$L(S_j)$	$[mg \cdot m^{-2}]$	– tracer concentration field in an unlimited flat stream,
l	$[m]$	– scale of turbulence,
S	$[mg \cdot dm^{-3}]$	– pollutant concentration,
t_r	$[s]$	– time of polutant propagation in run-of stream,
Δt	$[s]$	– time step,
v	$[m \cdot s^{-1}]$	– mean stream velosity,
κ		– von Karman's constant,
κ_j		– tracer reception coefficient,
σ		– Prandtl - Schmidt number.

2. Mathematical model of pollutant propagation.

The pollutant propagation is defined as a process occurring in a river channel, resulting from advection, turbulent diffusion and also from mesoscale and microscale turbulent fluctuations. The pollutant advection is defined as a mass transfer with an average velocity of water. The turbulent diffusion is the result of microscale pulsative displacements of water volume elements and pollutant concentration gradients. Mesoscale fluctuations are of channel depth size and smaller. Macroscale turbulent fluctuations are structures of dimensions close to channel width.

Mathematical model of pollutant propagation in bidirectional flow is elaborated basing on the rule of superposition of component pollutant concentration fields. It is possible, if we include the linearity of constitutive advection-dispersion equations. Mathematical descriptions of component pollutant concentration fields were elaborated in recurrence formula forms, including successive different conditions of pollutant propagation. We include the following component concentration fields: background, supplementary and recessive concentration field. The background concentration field is defined as the one which is translocating during one–way flow from the outlet downstream and during bidirectional flow is transported in backwater stream in reverse direction. The background concentration field in backwater stream cross-section is unsteady. Maximum concentration scalars occur at the moment of bidirectional flow forming. With passage of advection time, scalar values are tending to zero.

The supplementary concentration field is defined as a field generated by pollutant outlet to backwater flow. Considering the occurrence of significant vertical gradient of flow velocity, supplementary concentration field is unsteady. Values of supplementary concentration scalars in stream filament cross-section are varying to time t_j, which is equal to the time of flow of stream filament j from channel cross-section where pollutant oulet is situated.

Recessive concentration field is defined as the field occuring in run-off stream. This field is formed as a result of action of added up background and supplementary concentration fields, which occur at the surface interfacing backwater and run-off streams.

Apart from the above mentioned main components of the mathematical model, in bidirectional flow occur the auxiliary components: an external concentration field and a concentration field in one-way flow. The external concentration field is generated by a pollutant source situated at a distance greater or equal than necessary to full equalizing pollutant concentration in river water.

The order of fields calculation is as follows: a supplementary concentration field, with the employment of an auxiliary concentration field in one-way flow and a recessive concentration field. The calculation of tracer concentration field on the laboratory model was carried out in two stages. In the first stage the supplementary and recessive concentration fields were calculated and estimation of vertical dispersion coefficient was carried out. In the second stage on the ground of separate input data, calculations of supplementary recessive, background concentrations and concentrations in one-way flow were done. The mentioned division and the sequence of algorithms are motivated by possibility of carrying out measurements to estimate dispersion coefficient and to verify mathematical model on the laboratory model of a limited length (4.0 m).

In the laboratory model conditions flat backwater and run-off streams occured. Hence the constitutive equation for quantifying supplementary concentration field in the backwater flow is an advection - dispersion equation.

$$v\frac{\partial S}{\partial x} = D^{(z)}\frac{\partial^2 S}{\partial z^2} \tag{1}$$

in which x and z – coordinates with x parallel to the main backwater flow direction, z positively downward and with the origin on the axis of tracer outlet (Fig. 1). Analytical solution of equation (1) for the tracer outlet with practical dimensions and boundary conditions

$$\begin{aligned} x = 0;\quad -l_z \le z \le l_z;&\quad S = S_p \\ z \to \infty&\quad S \to O \end{aligned}$$

is as follows (Crank 1956):

$$S(x,z) = \frac{S_p}{2}\left[\operatorname{erf}\left(\frac{l_z + z}{2\sqrt{D^{(z)}\frac{x}{v}}}\right) + \operatorname{erf}\left(\frac{l_z - z}{2\sqrt{D^{(z)}\frac{x}{v}}}\right)\right] \tag{2}$$

where $2l_z$ [m] is vertical and only dimension of the initial surface in case of flat flow in the laboratory channel. The initial surface is a physical model tracer outlet with practical dimension. It is situated perpendicularly to the main flow direction, directly behind the tracer outlet.

After introducing discrete division of backwater stream to filaments j, instead of (2) we obtain vertical distribution of concentration of the laboratory model.

$$S_j^{(0)} = S_0^{(0)} \kappa_j(D_0^{(z)}, t_0); \quad j = 0, -1', \pm 1, \pm 2, \pm 3, \pm p \tag{3}$$

$$S_o^{(0)} = S_p \mathrm{erf}\left(\frac{l_z}{2\sqrt{D_o^{(z)} t_o}}\right) \tag{4}$$

$$\kappa_j\left(D_o^{(z)}, t_o\right) = \frac{\mathrm{erf}\left[\frac{l_z+z_j}{2\sqrt{D_o^{(z)} t_o}}\right] + \mathrm{erf}\left[\frac{l_z-z_j}{2\sqrt{D_o^{(Z)} t_o}}\right]}{2\mathrm{erf}\left[\frac{l_z}{2\sqrt{D_o^{(z)} t_o}}\right]} \tag{5}$$

Superscript (0) means rectangular distribution of velocity and dispersion coefficient in backwater stream and κ_j is tracer mass reception coefficient by stream filament j. Applying reception coefficient κ_j enables inclusion gradient of flow velocity in the next description, and then a variation of tracer advection time t_j and gradient of dispersion coefficient $D_j^{(z)}$. Calculation of concentration distribution $S_j^{(1)}$ within boundaries from $j = -p$ to $j = p$ is motivated by penetration of vertical dispersion flux from backwater stream to run–off stream, where it is transported in the direction reverse to backwater stream. In formula (3) and the next ones a real backwater stream in boundaries from interface to water surface $(j \pm p)$ and fictitious backwater stream which is a mirror reflexion of the real stream was included.

Taking into account vertically equal values of $D_j^{(z)}$ and $t_o = x/v_o$, the intensity of vertical dispersion flux is

$$J_j^{(0)} = -D_o^{(z)} \frac{\partial S_j^{(0)}}{\partial z_j} = = -\frac{D_o^{(z)} S_o^{(0)}}{\sqrt{\Pi D_o^{(z)} t_o}\ \mathrm{erf}\left(\frac{l_z}{2\sqrt{D_o^{(z)} t_o}}\right)} \exp\left[-\frac{(l_z + z_j)^2 - (l_z - z_j)^2}{4D_o^{(z)} t_o}\right] \tag{6}$$

In case of occurring vertical gradient of flow velocity, tracer advection time in stream filament j is $t_j = x/v_j$. Intensity of dispersion flux becomes

$$J_j^{(1)} = \frac{D_o^{(z)}\ S_o^{(0)}}{\sqrt{\Pi D_o^{(z)} t_o}\ \mathrm{erf}\left(\frac{l_z}{2\sqrt{D_o^{(z)} t_o}}\right)} \exp\left[-\frac{(l_z + z_j)^2 - (l_z - z_j)^2}{4D_o^{(z)} t_o}\right]. \tag{7}$$

Reception coefficient κ_j for stream filament j is

$$\kappa_j(D_o^{(z)}, t_j) = \frac{\operatorname{erf}\left(\frac{l_z+z_j}{2\sqrt{D_o^{(z)}t_j}}\right) + \operatorname{erf}\left(\frac{l_z-z_j}{2\sqrt{D_o^{(z)}t_j}}\right)}{2\operatorname{erf}\left(\frac{l_z}{2\sqrt{D_o^{(z)}t_j}}\right)} \tag{8}$$

and vertical distribution of concentration including velocity gradient is

$$S_j^{(1)} = S_o^{(o)}\kappa_j(D_o^{(z)}, t_j); \quad j = 0, -1', \pm1, \pm2, \pm3, \pm p \tag{9}$$

Including gradient of vertical dispersion coefficient as well as flow velocity gradient we obtain the following recurrence formulas

$$S_j^{(2)} = S_o^{(o)}\kappa_j(D_j^{(z)}, t_j); \qquad j - \text{ as above} \tag{10}$$

$$\kappa_j(D_j^{(z)}, t_j) = \frac{\operatorname{erf}\left(\frac{l_z+z_j}{2\sqrt{D_j^{(z)}t_j}}\right) + \operatorname{erf}\left(\frac{l_z-z_j}{2\sqrt{D_j^{(z)}t_j}}\right)}{2\operatorname{erf}\left(\frac{l_z}{2\sqrt{D_j^{(z)}t_j}}\right)} \tag{11}$$

The study of mathematical model sensitivity for variation of advection time and dispersion coefficient has shown that velocity gradient causes a change of concentration field.

$$A(S_j^{(1)} <> A(S_j^{(o)}) \tag{12}$$

Coefficient gradient causes the change of concentration distribution, but doesn't change the quantity of concentration field. Including the above formula we obtain

$$S_j^{(3)} = S_j^{(2)}\frac{A(S_j^{(1)})}{A(S_j^{(2)})} \tag{13}$$

$$A(S_j^{(3)}) = A(S_j^{(2)}) \tag{14}$$

Formulas (8 ÷ 14) include vertical distributions of velocity in backwater stream, measured on the laboratory model, and distributions of dispersion coefficient $D_j^{(z)}$, estimated basing on model measurements. Tracer concentration distribution $S_j^{(3)}$ includes real and fictitious backwater stream. After including water surface action we obtain

$$S_j^{(4)} = S_j^{(3)}\frac{A(S_j^{(3)})}{A_1(S_j^{(3)})} \tag{15}$$

where $A_1(S_j^{(3)})$, $S_j^{(4)}$ and $S_j^{(3)}$ are calculated with $j = 0, -1', 1, 2, 3, p$.

Supplementary concentration field is also influenced by the action of bottom of the laboratory model. Then

$$S_j^{(5)} = S_j^{(4)}\beta_1; \qquad j = 0, -1', 1, 2, 3, p. \tag{16}$$

Value of coefficient $\beta_1 > 1$ we calculate with quantifying of recessive concentration field in run-off stream (21).

To calculate recessive concentration field, a simplified model of slotted source was employed. This source is translocating with run-off flow average velocity along the surface interfacing backwater and run-off streams. Tracer concentration in the source is equal to supplementary concentration occurring at the interface. Vertical distribution of recessive concentration $S_j^{(6)}$ is found as

$$S_j^{(6)} = S_0^{(6)} \exp\left(-\frac{z_j^2}{4D_s^{(z)}t_r}\right); \qquad j = 0, 1, 2, 3, s...m \tag{17}$$

$$t_r = \frac{k\Delta t}{1 + \frac{v_s^{(o)}}{v_o}} \tag{18}$$

where:

- $S_0^{(6)} = S_p^{(4)}$ – is supplementary concentration in backwater stream for filament $j = p$, calculated using formula (15),
- $D_s^{(z)}$ – average coefficient of vertical dispersion in run-off stream,
- v_o – velocity of backwater stream filament $j = 0$,
- $v_s^{(0)}$ – mean velocity in run-off stream,
- $j = s$ – stream filament situated at the channel bottom,
- $j = m$ – stream filament in which concentration is negligible small.

Recessive concentration field in an unlimited flat stream can be found analytically

$$L(S_j^{(6)}) = S_0^{(6)} \int_0^\infty \exp\left(-\frac{z_j^2}{4D_0^{(z)}t_r}\right) dz = S_0^{(6)}\sqrt{\Pi D_o^{(z)} t_r} \tag{19}$$

Including the influence of the channel bottom, vertical distribution of concentration was calculated based on the following formulas

$$S_j^{(7)} = S_j^{(6)}\beta_1; \qquad j = 0, +1, +2, +3, +s \tag{20}$$

$$\beta_1 = \frac{A_1(S_j^{(4)}) + L(S_j^{(6)})}{A_1(S_j^{(4)}) + A(S_j^{(6)})} \tag{21}$$

where:

- $A_1(S_j^{(4)})$ – the supplementary concentration field calculated using formula (15) in backwater stream limited by stream filaments $j = p$ and $j = -1'$;
- $A(S_j^{(6)})$ – recessive concentration field in run-off stream limited by stream filaments $j = 0$ and $j = s$.

3. Verification of mathematical model and estimation of vertical dispersion coefficient

The input data for calculation of supplementary concentration field on the laboratory model are presented in Table 1. The distributions of the supplementary concentration were calculated in two cross-sections of backwater stream at distances x=180 and x=220 cm from the tracer outlet. Taking into account the unsteady values of supplementary concentrations, the calculations were carried out for the time coordinates $t_o + k\Delta t$, $k = 0, 1, 3, 6$ (Tab. 1). The input data used for calculation of recessive concentration field are presented in Table 1.

Table 1.
Input data for calculation of supplementary and recessive concentrations. Measurement series 1 and 2. Cross-sections x = 180 and 220 cm.

Supplementary concentration field										
	0	1	2	3	p	$-1'$	-1	-2	-3	$-p$
z_j [cm]	0.0	5.0	10.0	15.0	19.3	−3.2	−5.0	−10.0	−15.0	−19.3
v_j [cm^{-1}]	12.8	8.4	5.2	2.2	0.0	13.0	12.8	8.6	5.2	2.8
D_j^z [cm^{2}s^{-1}]	5.3	16.8	7.8	3.6	1.9	0.0	4.4	17.0	8.4	4.2
Times t_j [s] in cross-section x = 180 [cm]										
$t_0 = 20$ s	14	20	20	20	20	14	14	20	20	20
$t_0 + \Delta t = 40$ s	14	21	35	40	40	14	14	21	35	40
$t_0 + 3\Delta t = 80$ s	14	21	35	80	80	14	14	21	35	64
$t_0 + 6\Delta t = 140$ s	14	21	35	82	140	14	14	21	35	64
Times t_j [s] in cross-section x = 220 [cm]										
$t_0 = 20$ s	17	20	20	20	20	17	17	20	20	20
$t_0 + \Delta t = 40$ s	17	26	40	40	40	17	17	26	40	40
$t_0 + 3\Delta t = 80$ s	17	26	42	80	80	17	17	26	42	78
$t_0 + 6\Delta t = 140$ s	17	26	42	100	140	17	17	26	42	78
Recessive concentration field										
	0	1	2	3	s	4	5	6	7	
z_j [cm]	0.0	5.0	10.0	15.0	20.0	25.0	30.0	35.0	40.0	
v_j [cm^{-1}]	0.0	2.0	4.4	8.4	0.0	6.4	4.4	2.0	0.0	
D_j^z [cm^{2}s^{-1}]	1.9	0.8	0.25	0.2	0.1	0.2	0.25	0.8	1.9	
$v_s^{(0)} = 3.20$ [cm·s^{-1}]; $D_s^{(z)} = 0.56$ [cm^{2}s^{-1}]; $2l_z = 1.0$ [cm]										

The calculation of concentration field was carried out for cross-sections situated at distances x=180 and x=220 cm from the tracer outlet and for the time-coordinates $t = t_o + k\Delta t$; $k = 0, 1, 3, 6$. The results of calculation of supplementary and recessive concentrations on the laboratory model are presented in Tab. 2 and Tab. 3. The calculation was done on an IBM PC/XT using program SPD-1.

An estimation of vertical dispersion coefficient $D^{(z)}$ was carried out on the laboratory model Fig. 1. The estimation was based on the results of velocity measurement in bidirectional flow. The measured vertical distribution of velocity was steady. Basing on Reynolds assumption we can express the turbulent diffusion coefficient depending upon the eddy viscosity coefficient in parameterized form.

Table 2. Results of calculation of supplementary concentrations. $S_j \cdot 10^{-3}$ [mg dm^{-3}]
Cross-sections x = 180 [cm] / x = 220 [cm]

j	$S_j^{(0)}$	$S_j^{(1)}$				$S_j^{(2)}$				$S_j^{(3)}$			
		t_0	$t_0+\Delta t$	$t_0+3\Delta t$	$t_0+6\Delta t$	t_0	$t_0+\Delta t$	$t_0+3\Delta t$	$t_0+6\Delta t$	t_0	$t_0+\Delta t$	$t_0+3\Delta t$	$t_0+6\Delta t$
-p	9.34	13.61	21.10	24.88	24.88	10.81	18.81	23.16	23.16	10.88	19.02	23.12	22.99
	10.58	12.35	19.15	23.72	23.72	9.81	17.07	22.36	22.36	9.85	17.33	22.42	22.27
-3	15.35	19.26	24.18	24.18	24.18	23.43	27.04	27.04	27.04	23.57	27.33	27.00	26.85
	15.92	17.48	22.79	23.08	23.08	21.26	25.13	25.33	25.33	21.34	25.51	25.40	25.23
-2	23.38	25.86	26.16	26.16	26.16	30.42	30.52	30.52	30.52	30.61	30.86	30.48	30.31
	22.52	23.47	24.78	24.78	24.78	27.60	28.08	28.08	28.08	27.71	28.49	28.15	27.97
-1	30.09	30.09	30.09	30.09	30.09	29.58	29.58	29.58	29.58	29.77	29.90	29.54	29.38
	27.72	27.72	27.72	27.72	27.72	27.33	27.33	27.33	27.33	27.43	27.74	27.40	27.22
-1'	31.63	31.63	31.63	31.63	31.63	26.07	26.07	26.07	26.07	26.24	26.36	26.03	25.89
	28.88	28.88	28.88	28.88	28.88	24.63	24.63	24.63	24.63	24.72	24.99	24.69	24.53
0	32.73	32.73	32.73	32.73	32.73	32.73	32.73	32.73	32.73	32.94	33.09	32.68	32.50
	29.71	29.71	29.71	29.71	29.71	29.71	29.71	29.71	29.71	29.82	30.15	29.78	29.59
1	30.09	30.86	30.95	30.95	30.95	32.13	32.16	32.16	32.16	32.33	32.51	32.11	31.94
	27.72	28.01	28.39	28.39	28.39	29.16	29.29	29.29	29.29	29.27	29.72	29.36	29.17
2	23.38	25.86	28.61	28.61	28.61	27.89	29.87	29.87	29.87	28.07	30.20	29.83	29.67
	22.52	23.47	26.41	26.56	26.56	25.31	27.42	27.53	27.53	25.41	27.83	27.60	27.42
3	15.35	19.26	25.11	28.67	28.76	15.00	22.16	26.93	27.06	15.09	22.40	26.89	26.87
	15.92	17.48	22.79	26.02	26.72	13.61	20.11	24.44	25.41	13.66	20.41	24.50	25.32
p	9.34	13.61	21.10	26.28	28.88	2.83	9.62	17.75	23.07	2.85	9.73	17.72	22.91
	10.58	12.35	19.15	23.85	26.21	2.57	8.73	16.11	20.94	2.58	8.86	16.15	20.86
$A(S_j)$ [mg cm]		966.20	1064.12	1099.94	1105.94	960.18	1052.64	1101.65	1113.70	966.20	1064.12	1099.94	1105.94
		878.68	980.45	1017.49	1025.00	875.45	966.06	1014.59	1029.81	848.68	980.45	1017.49	1025.80

j	$S_j^{(4)}$				$S_j^{(5)}$			
	t_0	$t_0+\Delta t$	$t_0+3\Delta t$	$t_0+6\Delta t$	t_0	$t_0+\Delta t$	$t_0+3\Delta t$	$t_0+6\Delta t$
-p								
-3								
-2								
-1								
-1'	45.65	45.50	44.15	43.59	45.65	45.50	44.16	43.84
	43.07	43.35	42.22	41.52	43.07	43.35	42.23	41.75
0	57.31	57.12	55.44	54.72	57.31	57.12	55.45	55.04
	51.96	52.29	50.92	50.08	51.96	52.29	50.93	50.36
1	56.26	56.12	54.47	53.77	56.26	56.12	54.48	54.07
	51.00	51.55	50.20	49.37	51.00	51.55	50.21	49.65
2	48.83	52.13	50.59	49.94	48.83	52.13	50.60	50.23
	44.27	48.27	47.19	46.41	44.27	48.27	47.20	46.67
3	26.26	38.66	45.61	45.24	26.26	38.66	45.62	45.50
	23.81	35.39	41.90	42.85	23.81	35.39	41.91	43.09
p	4.96	16.79	30.05	38.57	4.96	16.79	30.06	38.79
	4.49	15.37	27.61	35.30	4.49	15.37	27.61	35.50
$A(S_j)$ [mg cm]	966.20	1064.12	1089.91	1105.94				
	848.68	980.45	1017.49	1025.80				

Table 3

Results of calculation of recessive concentrations $S_j \times 10^{-3}$ [mg·dm^{-3}]. Cross-section x = 180 and 220 [cm]

j	$S_j^{(6)}$			$S_j^{(7)}$		
	Δt	$3\Delta t$	$6\Delta t$	Δt	$3\Delta t$	$6\Delta t$
0	16.79	30.05	38.57	16.79	30.06	38.79
	15.37	27.73	35.30	15.37	27.61	35.50
1	8.38	23.84	34.36	8.38	23.85	34.55
	7.68	21.90	31.44	7.68	21.91	31.62
2	1.04	11.91	24.28	1.04	11.91	24.42
	0.96	10.94	22.22	0.96	10.94	22.34
3	0.03	3.74	13.61	0.03	3.74	13.69
	0.03	3.44	12.46	0.03	3.44	12.53
s	0.00	0.74	6.05	0.00	0.74	6.09
	0.00	0.68	5.54	0.00	0.68	5.57
4	0.00	0.09	2.14	0.00	0.00	0.00
	0.00	0.08	1.95	0.00	0.00	0.00
5	0.00	0.01	0.60	0.00	0.00	0.00
	0.00	0.01	0.55	0.00	0.00	0.00
6	0.00	0.00	0.13	0.00	0.00	0.00
	0.00	0.00	0.12	0.00	0.00	0.00
$L(S_j^{(6)})$	89.28	276.80	502.38			
	81.73	254.28	459.77			
β_1	1.000000	1.000195	1.005708			
	1.000000	1.000194	1.005655			

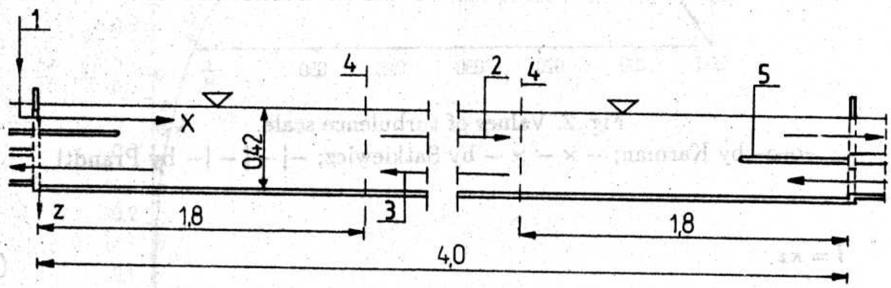

Fig. 1. Diagram of laboratory model.
1 – outlet of tracer solution; 2 – backwater stream; 3 – run-off stream; 4 – location of tracer concentration measurement cross-sections; 5 – stream-guide

$$D^{(z)} = \frac{K}{\sigma} \qquad (22)$$

Value of Prandtl -Schmidt number for pollutant dispersion in an open channel was used $\sigma = 0,5$ (Fisher et al. 1979, Luszczak et al. 1986, Yokusutra et al. 1970).

Value of vertical eddy viscosity coefficient may be found basing on Prandtl formula

$$K = l^2 \left| \frac{dv}{dz} \right| \qquad (23)$$

The scale of turbulence l is defined as a characteristic size of fluctuative translocation of volume fluid elements in turbulent stream.

Vertical distribution of velocity and pollutant dispersion are generated mostly by mesoscale and microscale turbulent fluctuations. The scale of turbulence corresponding to the mentioned fluctuations can be found (Grinwald 1974) in case of one-way flow basing on Prandl (24), Karman (25) and Satkiewicz (26) formulas (Fig.2).

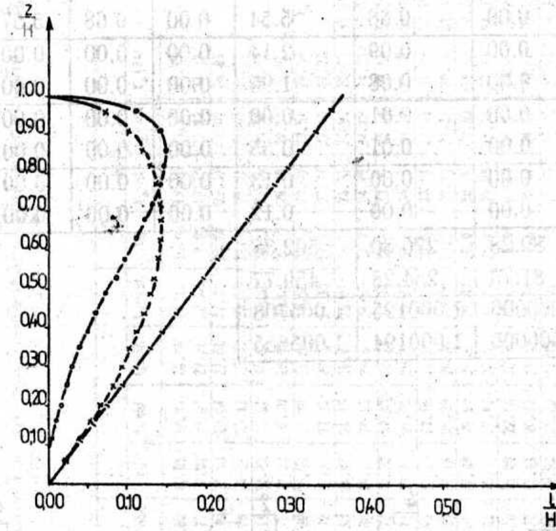

Fig. 2. Values of turbulence scale.
-o-o- by Karman; – × – × – by Satkiewicz; –|–|–|– by Prandtl

$$l = \kappa z \qquad (24)$$

$$l = 2\kappa z \left(\sqrt{1 - \frac{z}{H}} + \frac{z}{H} - 1 \right) \qquad (25)$$

$$l = \kappa z \sqrt{1 - \frac{z}{H}} \qquad (26)$$

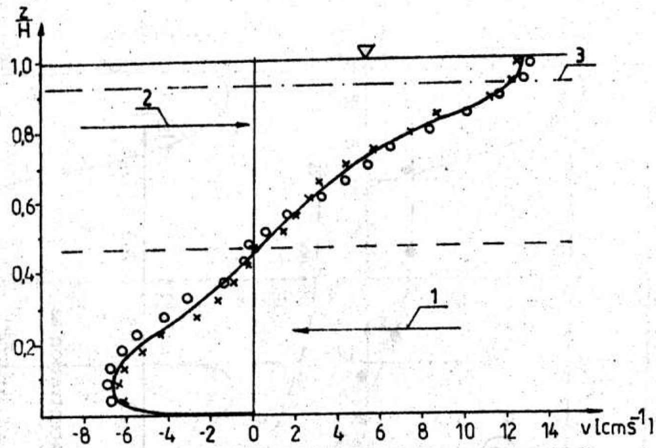

Fig. 3. Results of vertical distribution of flow velocity measurements
× × × × × first measurement series; ooooooo second series;
1 – run-off stream; 2 – backwater stream; 3 – tracer outlet axis

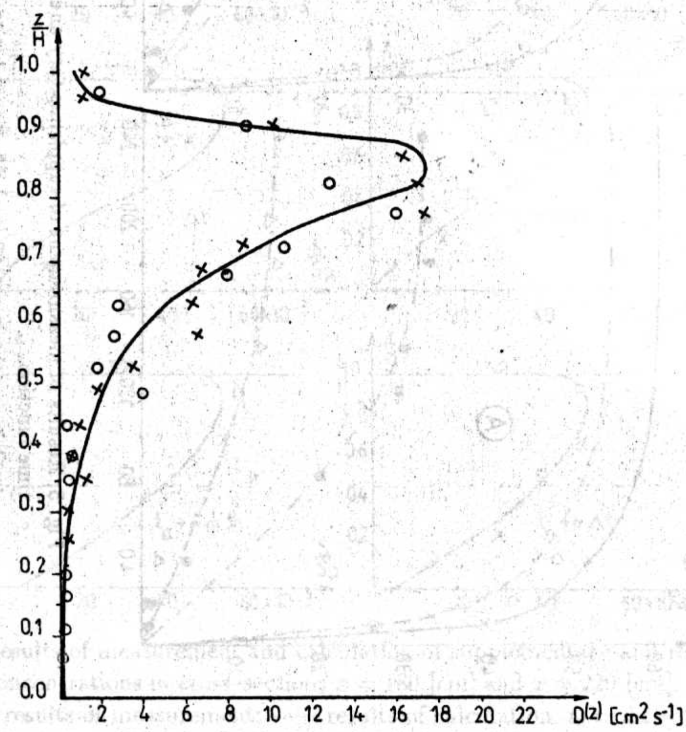

Fig. 4. Results of estimation of vertical distribution of coefficient of dispersion
× × × × × first measurement series; ooooooo second series

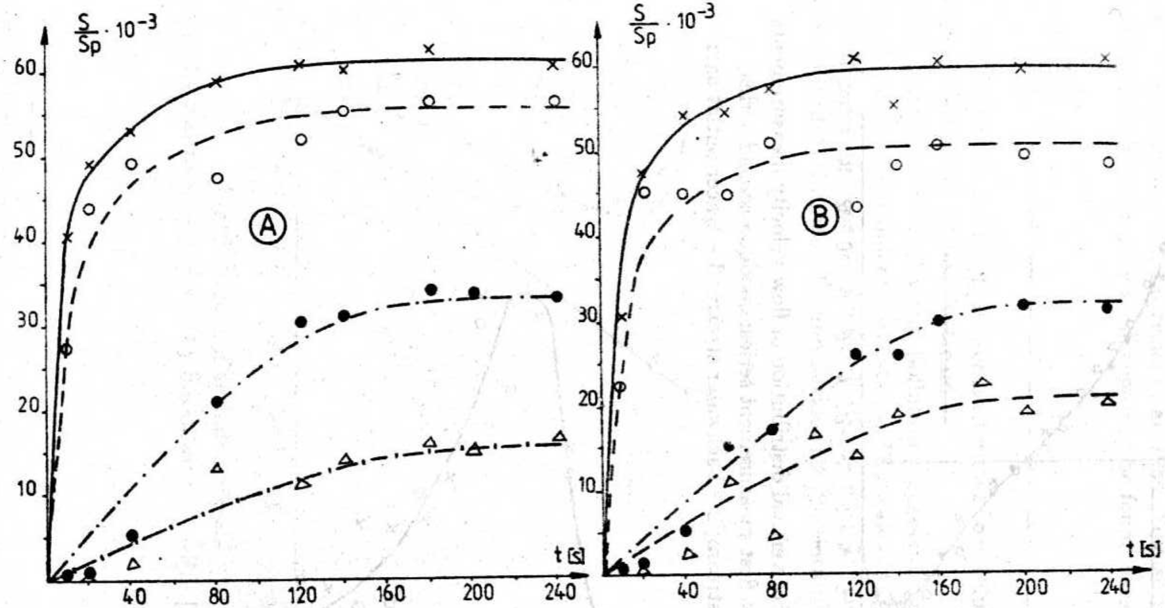

Fig. 5. Results of measurement of supplementary and recessive concentration as time function

A. Cross-section x = 180 [cm]. Measurement series 3; B. Cross-section x = 220 [cm]. Measurement series 4; × – measuring point at height 35.7 [cm] ○ – measuring point at height 29.7 [cm] • – measuring point at height 11.7 [cm] Δ – measuring point at height 5.7 [cm]

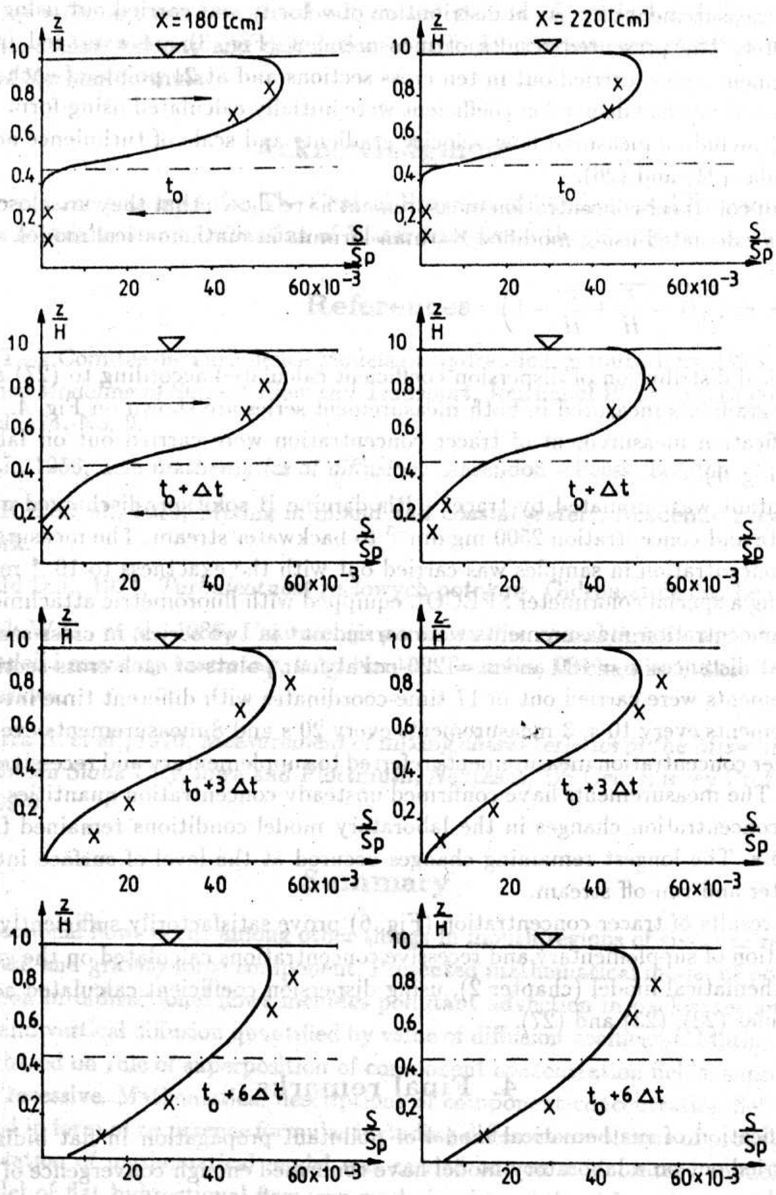

Fig. 6. Results of measurement and calculation of supplementary and recessive concentrations in cross-sections $x = 180$ [cm] and $x = 220$ [cm].
x —— results of measurement; —— results of calculation. $t_0 = \Delta t = 20$ [s]

The measurement of vertical distribution of velocity was carried out using current flow meter. The presented results of measurement (Fig. 3) are averaged from two measurement series, carried out in ten cross-sections and at 21 points of each section.

Values of vertical dispersion coefficient were initially calculated using formulas (22) and (23) including measured flow velocity gradients and scale of turbulence according to formulas (24) and (26).

Results of tracer concentration measurement have shown that they are close enough to values calculated using modified Karman formula in mathematical model.

$$l = \kappa z \left(\sqrt{1 - \frac{z}{H}} + \frac{z}{H} - 1 \right) \tag{27}$$

Vertical distribution of dispersion coefficient calculated according to (27) and flow velocity gradients measured in both measurement series are shown on Fig. 4.

Verification measurement of tracer concentration were carried out on laboratory model (Fig. 1).

Pollutant was simulated by tracer - Rhodamine B solution, discharged at a constant rate and concentration 2500 mg dm^{-3} to backwater stream. The measurement of tracer concentration in samples was carried out with the exactness to 10^{-4} mg dm^{-3}, employing a special colorimeter SPECOL equipped with fluorometric attachment. The tracer concentration measurements were carried out in two series, in cross-sections situated at distances $x = 180$ and $x = 220$ cm at four points of each cross-section. The measurements were carried out in 17 time-coordinates with different time intervals (6 measurements every 10 s, 3 measurements every 20 s and 8 measurements every 60 s).

Tracer concentration measurements referred to supplementary and recessive concentration. The measurements have confirmed unsteady concentration quantities (Fig. 5). Tracer concentration changes in the laboratory model conditions remained from 200 s to 300 s. The longest remaining changes occured at the level of surface interfacing backwater and run-off stream.

The results of tracer concentration (Fig. 6) prove satisfactorily sufficiently vertical distribution of supplementary and recessive concentrations calculated on the ground of the mathematical model (chapter 2), using dispersion coefficient calculated according to formulas (22), (23) and (27).

4. Final remarks

Verification of mathematical model of pollutant propagation in flat bidirectional flow carried out on a laboratory model have confirmed enough convergence of calculation and measurement results.

Vertical distributions of supplementary concentration in backwater and recessive concentration in run-off stream, calculated basing on mathematical model have simulated enough exactly results of tracer concentration measurement.

Results of measurement of tracer concentration in backwater and run-off stream on laboratory model have confirmed results of model analysis, that concentrations are unsteady.

The results of investigation have shown that estimation of vertical dispersion coefficient of pollutant can be carried out on the ground of vertical distribution measurements of flow velocity and calculation of turbulence scale, according to modified empirical Karman formula.

Acknowledgment

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Summary

Bidirectional flows occur among other things in mouth regions of rivers as result of wind action and gravity force component. Presented mathematical model of pollutant propagation in bidirectional flows includes pollutant advection in backwater and run--off flow and vertical diffusion quantified by value of diffusion coefficient. Mathematical model is based on rule of superposition of component concentration fields: supplementary and recessive. Mathematical descriptions of component concentration fields were elaborated in form of recurrence formulas including different conditions of propagation.

Verification of mathematical model was carried out on a laboratory model. Physical model of flat bidirectional flow was made in glass channel by adequate location of backwater and run-off stream. Pollutant was simulated by Rhodamine B solution discharged to backwater stream. Results of tracer concentration measurement on laboratory model have satisfactorily confirmed vertical distribution of supplementary and recessive concentration calculated according to mathematical model. Also fact that tracer concentrations in bidirectional flow are unsteady has been confirmed too.

Estimation of dispersion coefficient was carried out basing on measurements of vertical distribution of velocity, done on physical model of bidirectional flow, and calculation of turbulence scale basing on modified Karman formula. Enough convergence of results of calculation and laboratory measurements of tracer concentration on model is confirmation of elaborated method of coefficient estimation.