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Wave scattering over an underwater trench. Experiment

1. Introduction

The work reported herein concerns the propagation of surface waves passing perpendicularly over an underwater trench. The problem has an essential practical significance. Navigation channels excavated in shallow water area influence the surface wave motion and the alongshore sediment transport. This is of interest so to the navigators, as to the harbour authorities obliged to maintain the depth along the channel. Thus the main interest concerns the rate of wave reflection and transmission, due to the presence of the trench and the water velocities in its vicinity.

Several theoretical solutions of some particular cases of wave transformation propagating over irregular bottom are described in literature. The recent works are due to Massel (1985, 1986), who presented a linear theory related to long waves equations. It enables numerical calculation of wave parameters for almost arbitrary shape of the cross-section of the channel and arbitrary angle of wave approach. The aim of this work is an experimental verification of wave reflection and transmission coefficients found in that theory.

According to the theory the reflection coefficient changes in an oscillatory manner in function of the relative wave length and the characteristics of the trench. The changes of the transmission coefficient are found as a result of the energy conservation principle:

$$|K_R|^2 + |K_T|^2 = 1 \tag{1}$$

as the solution neglects the dissipation effects.

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Only few experimental results are published on wave scattering, due to the presence of a trench (Lee and Ayer 1981, Bendykowska 1988, 1989). Experiments reported below were performed in a wave flume for two different cross sections of the trench: rectangular and trapezoidal and two relations of water depths over the trench and over its up – and downstream area.

The laboratory conditions differ essentially in several aspects from those assumed in the theory:

- The area up – and downstream of the trench is limited. The wave is reflected not only from the trench, but also from the boundaries of the flume, i.e. from the beach and from the wave generator.
- A part of the wave energy is dissipated due to the propagation over the horizontal bottom as well as to the passage over the trench.
- According to the theory, the strongest diffraction effects concern long waves (small relative water depth); the solution for this area shows fast changes of the reflection coefficient, from zero up to 10 - 15%. To avoid the influence of the surface tension and ensure a sufficient precision of measurements, the wave height in the flume should exceed ~ 1.0 cm. Such waves, generated in laboratory conditions, are strongly nonlinear, due to parasite effects connected with mechanical wave generation (Bendykowska and Massel 1988).

Another important difference between theory and experiment exists, independently from the technical laboratory conditions. Due to the passage over the trench, similarly as in the case of other obstacles (sills, bars) higher harmonics were generated in the transmitted wave motion (fig. 2 and 3). These effects are not described on the level of the linear approximation of the theoretical solution.

The author tried to take into account all the above given aspects in the experimental investigation and, particularly, in the analysis of measured results.

The wave reflection coefficient was determined basing on simultaneous measurements of water surface profiles in three cross-sections of the flume (three point method). In analysing the measured data it was assumed, following the conclusion from Fontanet (1961) theory, that the complex wave motion in the flume may be treated on the water surface as a simple superposition of free and bounded harmonic wave components propagating with their different phase speeds. Only the reflection coefficient of the first harmonic was analysed. The influence of the reflection from the beach at the end of the flume was taken into account on the reflection measured in the area upstream the trench. This influence appeared to be very essential for the case of the reflection coefficient. The experimental results confirmed the theoretical relations found by Massel (1986). The theory failed in determing the wave transmission coefficient.

2. Experimental equipment and procedure

The experiments were performed in a wave flume of 23 m length and 0.5 m width (fig. 1). The horizontal bottom of the flume was lowered on a length of 2 m, to model a trench. Two shapes of the cross section of the trench were modelled, a rectangular and a trapezoidal one, both of 0.2 m depth in relation to the bottom up– and downstream.

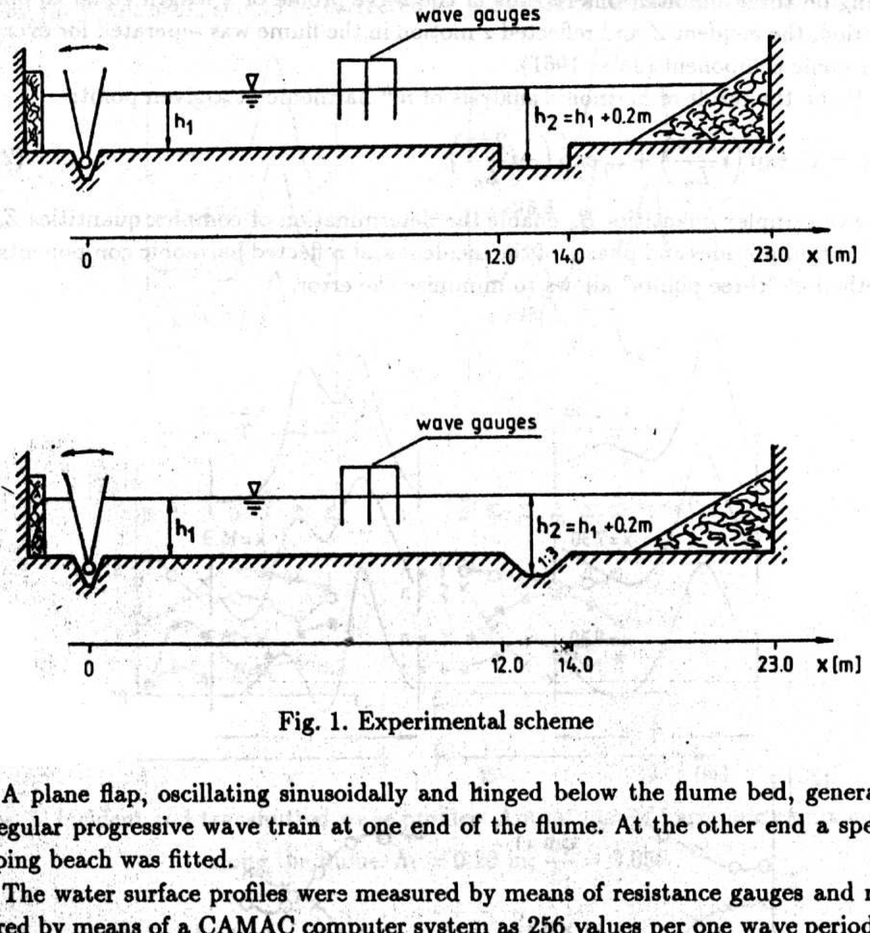

Fig. 1. Experimental scheme

A plane flap, oscillating sinusoidally and hinged below the flume bed, generated a regular progressive wave train at one end of the flume. At the other end a special sloping beach was fitted.

The water surface profiles were measured by means of resistance gauges and registered by means of a CAMAC computer system as 256 values per one wave period. In every measurement point three simultaneous records of wave profiles were made, by an array of three gauges, equally spaced at a known, small, distance. Such records were taken every 0.5 - 1.0 m, over the whole length of the flume.

About half an hour was needed to perform one set of records, so the wave motion in the flume had to be stationary. Special preparatory tests were made to ensure the stationarity, as for several wave periods a resonant motion occured.

Parameters of the tested incident waves were the following:

water depth	$h_1 = 0.10$ and 0.20 m $h_2 = 0.30$ m and 0.40 m
wave period	$T = 0.95 - 2.5$ s
relative wave length	$L/h = 5.5 - 25$
relative wave height	$H/h \approx 0.05 - 0.30$

3. Method of Analysis

Basing on three simultaneous records of the wave profile of a length equal to one wave period, the incident Z and reflected z motion in the flume was seperated for every n-th harmonic component (Jolas 1961).

Let B_n be the result of harmonic analysis of n^{th} harmonic at a given point:

$$B_n = Z_n \exp\left(i\frac{2\pi x}{L_n}\right) + z_n \exp\left(-i\frac{2\pi x}{L_n}\right) \tag{2}$$

Two known complex quantities B_n enable the determination of complex quantities Z_n and z_n – the amplitudes and phases of the incident and reflected harmonic components. The method of "three points" allows to minimize the error.

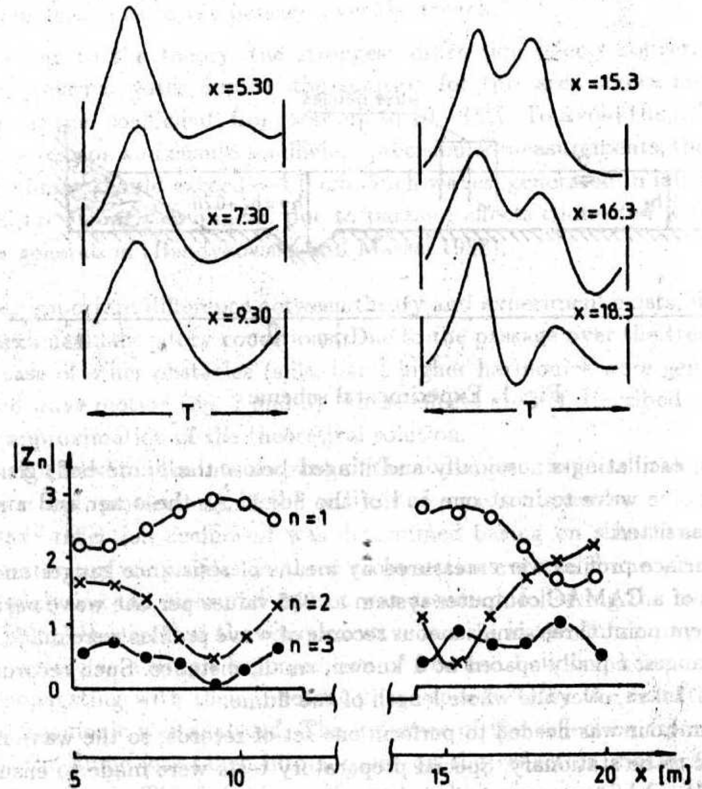

Fig. 2. Incident and transmitted wave profiles. Amplitudes of harmonic components along the flume. $h_1 = 0.2$ m; $\frac{h_1}{L_1} = 0.08$

In fig. 3 and 4 examples the results of $|Z_n|$ on the length of the flume are given. Due to parasite effects of the mechanical wave generation - the wave motion in the flume consists of free harmonic waves of fundamental (generation mode) and higher

orders, thus also of interaction waves. So the values of Z_n are a vectorial sum of wave components of the same frequency and different phase velocities. To isolate the free waves, the result of B_n should be corrected for the share of the forced wave components (B'_n).

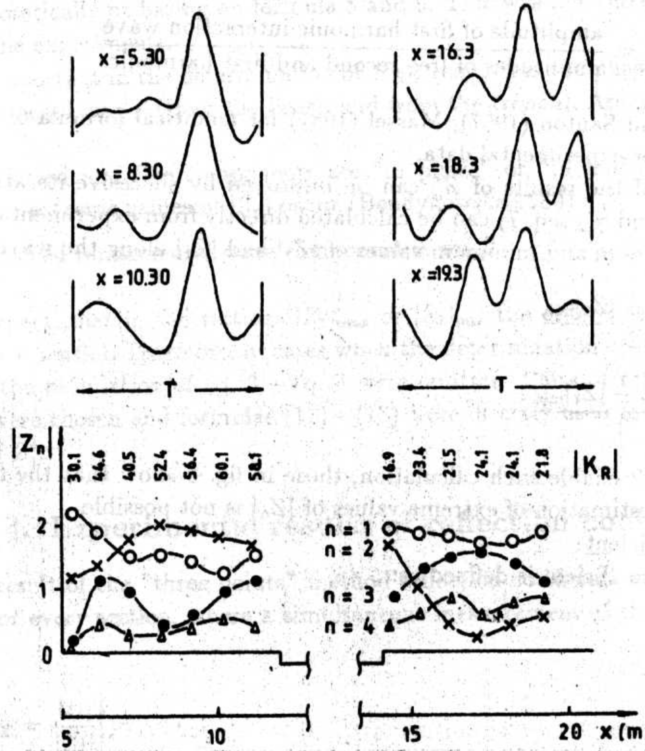

Fig. 3. Incident and transmitted wave profiles. Amplitudes of harmonic components along the flume. $h_1 = 0.20$ m; $\frac{h_1}{L_1} = 0.058$

The analysis was limited to the determination of the fundamental wave parameters (verification of a linear theory).

$$B'_2 = B_2 - MZ_1^2 \exp\left(i\frac{4\pi x}{L_1}\right) \tag{3}$$

$$B'_1 = B_1 - \gamma_{12} Z_1 \overline{Z}'_2 \exp\left(i\frac{2\pi x}{L_1}\right) \tag{4}$$

where:

$$M = \frac{A_{2s}}{A_1} = \frac{\text{amplitude of the second order forced wave}}{\text{amplitude of the first harmonic}} \tag{5}$$

or

$$M = \frac{1}{2}k\left(1 + \frac{3}{\tanh^2 kh^2}\right)\coth kh \tag{6}$$

$$\gamma_{12} = \frac{A_{12}}{A_2 \cdot A_1} = \frac{\text{amplitude of first harmonic interaction wave}}{\text{amplitudes of free second and first harmonics}} \tag{7}$$

find Kratchenko and Santon (1957), Massel (1981) for analitical formula of γ_{12}; Bendykowska (1980) for experimental data.

The accuracy of the results of Z_n' can be improved by succesive iteration. Both values M (eq. 6) and γ_{12} (eq. 7) can be calculated directly from experimental data, as a function of maximum and minimum values of $|Z_1|$ and $|Z_2|$ along the wave flume:

$$A_2 = \frac{|Z_2|_{\max} - |Z_2|_{\min}}{2} \tag{8}$$

$$A_{12} = \frac{|Z_1|_{\max} - |Z_1|_{\min}}{2} \tag{9}$$

The results in fig. 2 enable such calculation, those in fig. 3 show that the flume was too short and the estimation of extreme values of $|Z_n|$ is not possible.

Reflection coefficient

The reflection coefficient is defined as:

$$|K_R| = \frac{|z'|}{|Z'|} \tag{10}$$

Let us consider:

X_A – a given section in the upstream area,
X_B – a given section in the downstream area.

In the downstream area values z'_{XB} describe the wave reflected from the beach. Values z'_{XA} are a vectorial sum of two waves –the one reflected from the trench plus that, reflected from the beach and transmitted over the trench. To find the complex number of the wave reflected from the trench the value z'_{XA} should be corrected by the share of the wave transmitted from the downstream area. The phase shift induced by the presence of the trench and the rate of the transmission can be calculated from:

$$C_T = \frac{Z'_{XB}}{Z'_{XA}} = \frac{\bar{Z}'_{XA} \cdot Z'_{XB}}{|Z'_{XA}|^2} \tag{11}$$

Then the wave reflected from the trench is:

$$C_R = z'_{XA} - \left\{C_T \cdot z'_{XB} \exp\left[i\frac{2\pi(X_B - X_A)}{L_1}\right]\right\} \tag{12}$$

The wanted reflection coefficient from the trench is determined as:

$$|K_{RK}| = \frac{|C_R|}{|Z'_{XA}|} \tag{13}$$

The above procedure can be used when M (eq. 5 and 6) and γ_{12} (eq. 7) are given either theoretically or basing on formula 8 and 9. This was not the case for the whole range of the experiments.

Very important in the determination of K_{RK} (eq. 13) is the phase of superposition of both reflected waves (from the beach and from the trench). Assuming that:

- the forced and free components are "in phase" or "in anti-phase" in sections where extreme values of $|Z_n|$ occur (Bendykowska 1980)
- the forced fundamental wave component is small

we can expect, that in the sections $|Z_1|_{max}$ or $|Z_1|_{min}$ the differences between Z_1 and Z'_1 are not essential. Therefore, in cases when the determination of M and γ_{12} was not possible, the calculation of eq. 3 – eq. 9 were omitted. Pairs of relative sections X_A and X_B were chosen and formulae (11) - (13) were directly used for values Z_1 (eq. 2) instead of Z'_1.

4. Experimental results of reflection coefficient

As a result of the "three points" method reflection coefficient could be calculated directly for every section, where a simultaneous measurement of the wave profile was taken

$$K_{RX} = \left.\frac{|z_1|}{|Z_1|}\right|_{\text{in section } X} \tag{14}$$

In fig. 3 an example of the values of K_{RX} is given along the flume. In the upper region they vary between 30 – 60%, in the downstream area the differences are smaller.

Fig. 4a illustrates the differences between K_{RX} (eq. 14) and K_{RK} (eq. 13). The corrections for the reflection from the beach are positive or negative (arrows) and in every case diminish the scatter of the experimental results in relation to the theoretical curve. In further figures 4 and 5 only the values of K_{RK} are presented.

The agreement with the theory is relatively good. With a good precision the experiment confirms the area of theoretical maxima and zero values of the reflection coefficient. It should be stressed, that such a precision was attained although: in majority cases the tested waves were strongly nonlinear (relative wave length up to 25) and the results concern only their first harmonic component determined basing on harmonic analysis of the wave profile. This confirms the validity of the applied method of laboratory results analysis, based on the assumption that on the free surface the principle of a simple superposition of free and forced wave components can be accepted for a broad range of wave parameters.

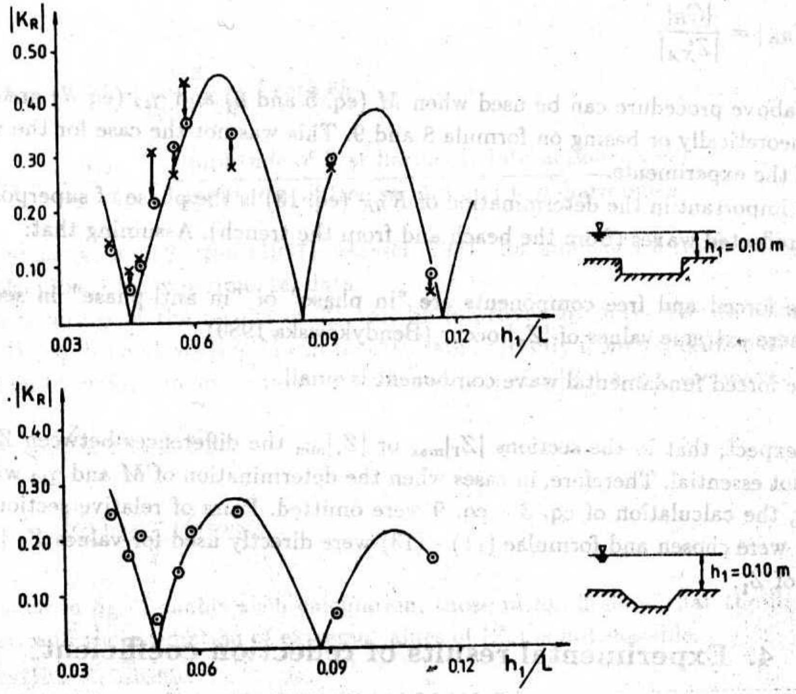

Fig. 4. Reflection coefficient.
——— theory
⊙ experimental results K_{RK} (eq. 13) × experimental results K_R (eq. 10)

5. Transmission of the motion due to the trench. Wave transmission coefficient

The frequency structure of the incident wave motion changed due to the propagation of the wave over the trench.

Fig. 2 and 3 present examples of the incident and transmitted wave profiles, as well as the amplitudes $|Z_n|$ up to the third and fourth harmonic components in the up- and downstream areas. In table 1 the relations of downstream $|Z_{nB}|$ to upstream $|Z_{nA}|$ mean amplitudes of four harmonics ($n = 1, 2, 3$ and 4) are given, for all tested cases. The results prove that strong nonlinear effects are connected with the transmission over a deeper area similarly to the transmission over a more shallow area (Bendykowska et al. 1984). The figures in table 1 show an increase of the transmitted second harmonic for shorter waves ($\frac{h}{L} = 0.08 - 0.1$). Longer waves show an essential increase of the third and fourth harmonic, while the second and first harmonics decreased. For a certain range of long waves the transmitted fundamental wave increased with a simultaneous decrease of the second harmonic. It's possible, that subharmonic components ($2T$) appeared in the transmitted motion, but only records of one period (T) length were subject to harmonic analysis.

Table 1

Transmission over the trench of harmonic wave components. Experiment.

Test	$\frac{h}{L}$	$\frac{\lvert Z_{nB}\rvert}{\lvert Z_{nA}\rvert}$			
		$n = 1$	$n = 2$	$n = 3$	$n = 4$
rectangular trench $h_1 = 0.2$ m $h_2 = 0.4$ m	0.058	0.93	0.76	0.81	0.78
	0.065	1.11	0.30	1.79	1.84
	0.070	1.15	0.49	1.16	1.55
	0.074	0.95	0.89	0.81	0.94
	0.080	0.86	1.21	1.52	1.91
	0.085	0.78	1.55	1.98	3.05
	0.094	0.88	1.10	0.78	0.75
	0.100	0.77	1.70	1.73	1.86
	0.138	0.91	1.16	–	–
	0.165	0.90	0.85	–	–
rectangular trench $h_1 = 0.1$ m $h_2 = 0.3$ m	0.0408	0.88	0.87	0.78	0.77
	0.0450	0.89	0.43	1.50	1.54
	0.0476	1.16	0.26	0.97	1.20
	0.0510	1.17	0.44	0.96	1.10
	0.0560	0.83	0.63	0.60	0.60
	0.0590	0.83	0.63	0.60	0.60
	0.0690	0.60	1.30	1.16	1.45
	0.0920	0.69	1.48	1.55	–
	0.115	0.75	1.12	–	–
trapezoidal trench $h_1 = 0.1$ m $h_2 = 0.3$ m	0.0408	0.78	1.07	1.18	1.33
	0.0450	0.94	0.57	1.62	1.50
	0.0476	0.97	0.40	1.77	1.40
	0.0510	1.09	0.49	1.23	1.67
	0.0560	0.95	0.80	1.00	1.25
	0.0590	0.88	0.81	0.89	1.50
	0.0690	0.73	2.00	–	–
	0.092	0.85	0.87	–	–
	0.115	0.83	0.71	–	–
trapezoidal trench $h_1 = 0.2$ m $h_2 = 0.4$ m	0.058	1.10	0.42	1.40	1.68
	0.065	1.14	0.58	1.01	1.47
	0.070	1.04	0.60	1.15	1.43
	0.074	0.96	0.87	0.90	0.86
	0.080	0.95	0.76	1.00	1.10
	0.085	0.91	1.71	1.85	1.79
	0.100	0.86	2.02	2.00	2.10
	0.138	0.92	1.10	–	–
	0.176	0.72	0.82	–	–

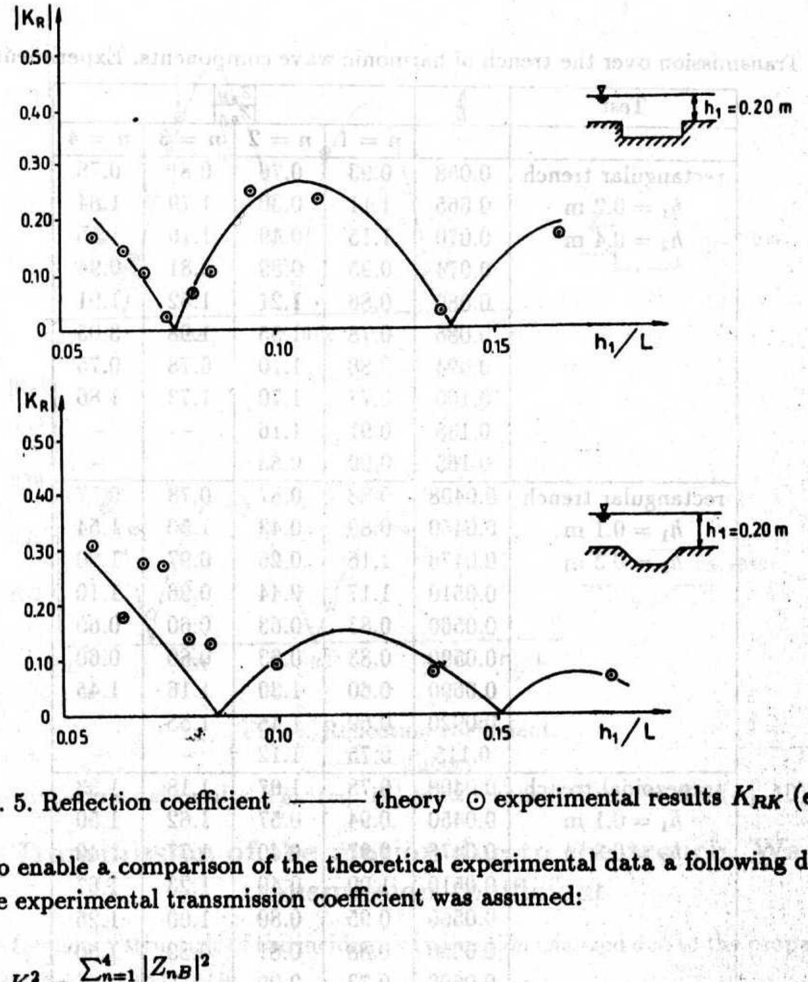

Fig. 5. Reflection coefficient ——— theory ⊙ experimental results K_{RK} (eq. 13)

To enable a comparison of the theoretical experimental data a following definition of the experimental transmission coefficient was assumed:

$$K_T^2 = \frac{\sum_{n=1}^{4} |Z_{nB}|^2}{\sum_{n=1}^{4} |Z_{nA}|^2} \tag{15}$$

The comparison is given on fig. 6.

The experimental transmission is smaller than predicted theoretically, particularly for shorter waves. The disagreement is stronger for smaller water depth ($h_1 = 0.1$ m), what is probably partly due to laminar wave damping in laboratory condition.

6. Conclusions

- **The experimental results confirm with good precision the theoretical solution of Massel (1985, 1986) of wave reflection from an underwater trench for a broad range of wave parameters and two given shapes and dimension relations of the trench.**

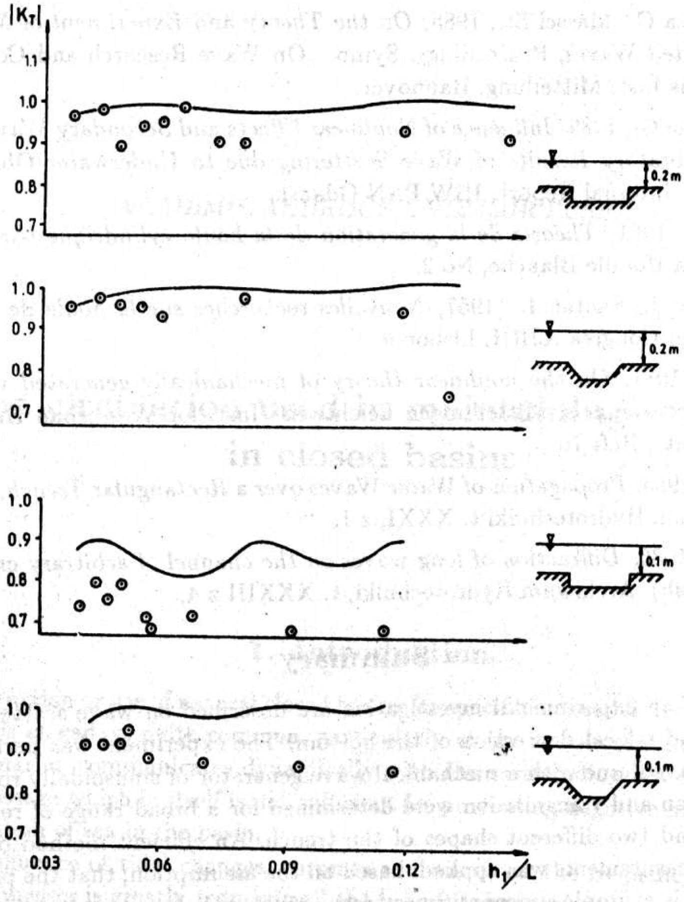

Fig. 6. Transmission coefficient. ——— theory ⊙ experimental results K_T (eq. 15)

- The theory (Massel 1985, 1986) does not describe properly the wave transmission coefficient. The reason are strong nonlinear effects occuring due to the propagation over the trench, not to be found on the level of linear theory.
- The method of analysing laboratory results applied in the experiment and described above, proved to give good results for very complex and nonlinear wave motion in the wave flume ($\frac{h}{L} = 0.0408$ and $\frac{H}{h} = 0.1$, $\frac{h}{L} = 0.058$ and $\frac{H}{h} = 0.15$).

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Summary

Results of an experimental investigation are described on wave scattering due to the presence of a local depression of the bottom. The experiment was performed in a laboratory wave flume with a mechanical wave generator of sinusoidally rotating flap. Wave reflection and transmission were determined for a broad range of relative wave parameters and two different shapes of the trench. An efficient method of analysing laboratory measurements was applied, based on the assumption, that the profile of the water surface is a simple superposition of free and bounded wave components, present in the wave flume.

The experimental data for the wave reflection coefficient confirm with good precision the theoretical results of a linear theory of Massel (1985, 1986), based on long wave equations. The wave transmitted beyond the trench showed strong nolinear effects and the experimental transmission coefficient was in disagreement with the theoretical results of Massel's theory.