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Wind shear stress influence on river flow

1. Introduction

The flow at the river mouth is determined by several factors. The most important of them are:
- gravitational forces and tides,
- wind shear stress at the water surface,
- bottom stress,
- Coriollis forces, and
- mixing processes.

Those factors cause the motion of water and depending on the boundary conditions the flow can be stable or unstable. At the Maritime Institute, Szczecin Branch, research has been carried out for several last years in order to select the main factors of flow at the Odra river mouth. According to those research, the main factors are: rapid sea tides and winds, especially from the north. During the strong north wind (>6°B) the river depth is of about 1 m greater even when the flow along the river is not high, and the extent of the backwater curve reaches over 150 km upstream. The vertical velocity profiles were also measured. Certain phenomena were recorded during steady flow, at the water surface the velocity was much smaller then the mean velocity and sometimes surface backward current was created depending on the wind. This backward current upstream pollutants toward water intake. Certain contaminats were detected. Those observations leads to a conclusion – the wind shear stress opposing the river flow changes the relation: flow, depth, slope, and distorts the vertical velocity profile herein called tachoida.

Wide literature analysis [3] shows, that there does not exist explicit formulae, which can be used for the beckwater curve calculation in case of wind action. A method of including wind action in calculating river discharge was given by Robert Reid [10]. This method, however based on turbulent momentum transfer does not give acceptable results for practical estimation. Exact analysis shows, that the increase of wind velocity of about 10% causes the increase of depth of the same order. This disagrees with the results from experiments. Assuming that, another method was developed in order to include wind effect on the backwater curve.

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Notation

a – parameter characterizing eddy viscosity,
A – cross section area,
B – river width,
C – Chezy's constant,
C_0, C_1 – constants in eddy viscosity formula,
D_b, D_w – functions in equation of vertical velocity distribution,
g – acceleration due to gravity,
H – river depth,
I – slope of water surface,
I_d – slope of bottom,
k – Prandtl constant,
K_x, K_z – eddy viscosity coefficient in x and z direction,
K_0 – reference value of eddy viscosity,
n – Manning roughness coefficient,
p – pressure,
q – flow per unit width,
Q – total flow,
R_H – hydraulic radius,
V – velocity,
V_0 – depth averaged velocity,
V_p – surface velocity of water,
V_ξ – dimensionless velocity,
V_1 – dimensionless surface velocity,
W – wind velocity,
x, y, z – main system of coordinates,
η – dimensionless coordinate in vertical direction,
$\kappa_0, \kappa_2, \kappa_3$ – coefficients,
ε – channel roughness,
λ – ratio of wind stress to bottom stress,
ν – kinematic viscosity coefficient of water,
ρ – water density,
ξ – dimensionless coordinate in horizontal direction,
τ – shear stress,
τ_w – wind shear stress at the water surface,
τ_b – bottom shear stress,
χ – weted perimeter.

2. Mathematical description of problem

The proposed here model of flow is based on the equations of turbulent motion of water. Following simplifications were made in order to satisfy the physical nature of flow:

– the problem is plane in vertical direction,
– the turbulent momentum transfer is limited to the vertical direction,
– at the water surface wind shear stress exists, and the waves were neglected, however those results are included indirectly by correcting wind effect,
– at the bottom shear stress exists,
– to determine the turbulent shear stress, the Boussinesq approximation was taken, and the eddy viscosity coefficient varies upwards,
– density of water is assumed to be constant,
– the flow is generated by the mass forces (component parallel to the flow direction) and the wind action,
– the model assumes uniform flow i.e. the bottom slope I_d, to be equal to the water surface slope I ($I=I_d$).

The main system of coordinates x, z was chosen in such a way that: z-axis is directed vertically upward, and x-axis – horizontally. The bottom slope I_d is the angle between the x-axis and the bottom line. It has been also asumed, that flow q is positive when it follows the x-axis. The governing equations of motion can be written as [2]

$$\frac{dV_x}{dt}=X-\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{\partial}{\partial x}(\tau_{xx})+\frac{\partial}{\partial z}(\tau_{xz})$$
$$\frac{dV_z}{dt}=Z-\frac{1}{\rho}\frac{\partial p}{\partial z}+\frac{\partial}{\partial x}(\tau_{zx})+\frac{\partial}{\partial z}(\tau_{zz}) \tag{1}$$

where:

$$\tau_{xx}=K_x\frac{\partial V_x}{\partial x} \quad \tau_{xz}=K_z\frac{\partial V_x}{\partial z} \quad K_{ij}=\frac{-\rho\overline{V_i V_j}}{\overline{\frac{\partial V_i}{\partial x_j}}}$$

Assuming the earlier mentioned simplifications and after cross differentiation the set of equations (1) can be expressed as [7]

$$\frac{d^2}{dz^2}(\tau_{xz})=0. \tag{2}$$

As a basic equation for further analysis relationship (2) was chosen, and it is differential equation of one order higher then formula (1). This approach is justified by two factors:

– in the case of wind backwater curve the water surface slope and the bottom slope

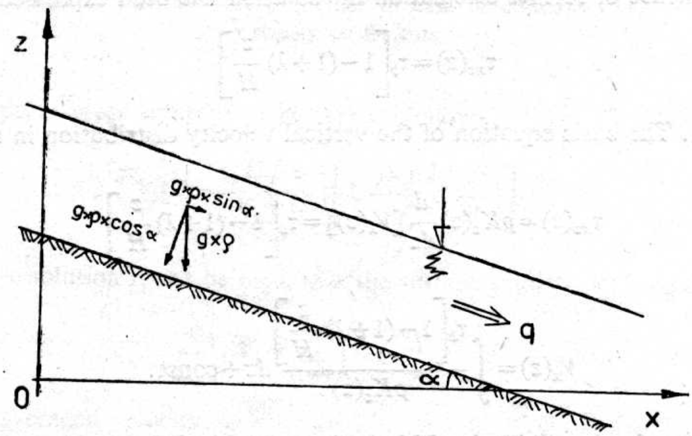

Fig. 1. Coordinates system

are very small ($10^{-5}\div10^{-6}$), it makes the component of force in flow direction very small too. It is very inconvenience to operate with the slope,

– the second reason is the fact that the goal of the present research is to formulate the relationship between wind shear stress and bottom shear stress. This relation is easier to obtain for equation (2).

We are looking for the solution of the above relationship fulfiling following boundary conditions:

$$\text{for: } z=H; \quad \tau_{xz}=-\tau_w, \qquad \text{for: } z=0; \quad \tau_{xz}=\tau_b. \tag{3}$$

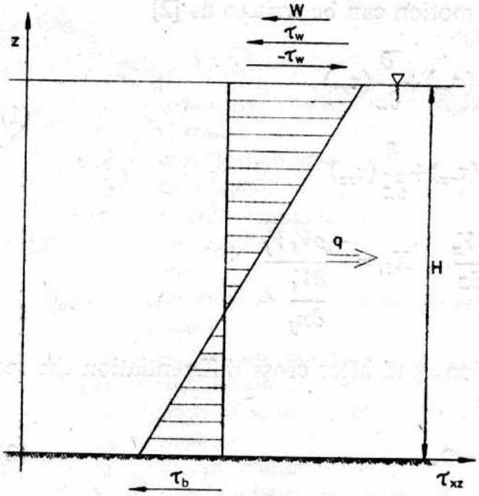

Fig. 2. Vertical distribution of turbulent shear stress

The above boundary conditions leads to the solution of a linear form (see Fig. 2)

$$\tau_{xz}(z)=\tau_b-(\tau_w+\tau_b)\frac{z}{H}. \tag{4}$$

For the convenience of further calculation the solution has been expressed as

$$\tau_{xz}(z)=\tau_b\left[1-(1+\lambda)\frac{z}{H}\right] \tag{5}$$

where $\lambda=\tau_w/\tau_b$. The basic equation of the vertical velocity distribution in this case takes form

$$\tau_{xz}(z)=\rho K_z(z)\frac{d}{dz}[V_x(z)]=\tau_b\left[1-(1+\lambda)\frac{z}{H}\right] \tag{6}$$

and so

$$V_x(z)=\int\frac{\tau_b\left[1-(1+\lambda)\frac{z}{H}\right]}{\rho K_z(z)}dz+\text{const}. \tag{7}$$

The constant value in eq. (7) should be estimated from following boundary condition: for: $z=0$ it must be $V_x(z)=0$, and so it gives that const. $=0$ (8)

3. Analysis of various type of curves

3.1. Case of constant eddy viscosity coefficient

Many of authors suggest, that for practical calculation allowable is assumption, that eddy viscosity coefficient is constant along all the river depth [6, 8]. So

$${}_z(z)=\text{const}=K_0. \tag{9}$$

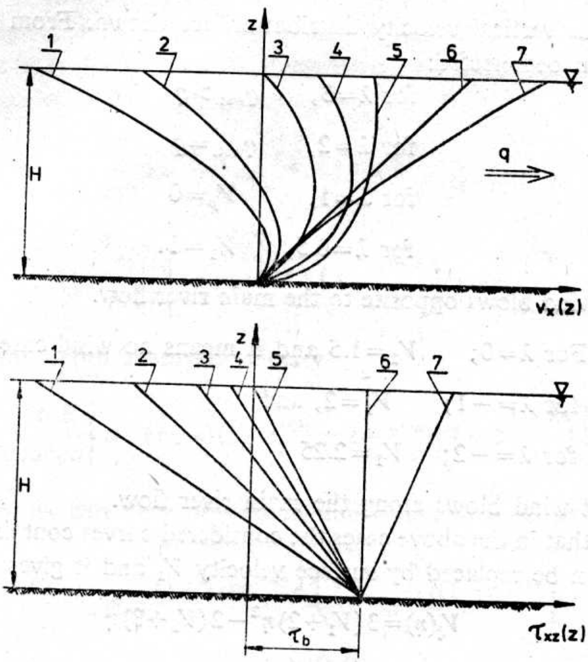

Fig. 3. Vertical velocity profiles in case of constant eddy viscosity coefficients

That assumption gives solution of eq. (7) as follows

$$V_x(z)=\frac{\tau_b H}{\rho K_0}\left(\frac{z}{H}\right)\left[1-\frac{1+\lambda}{2}\left(\frac{z}{H}\right)\right]. \tag{10}$$

Taking the above solution it can be seen, that the surface velocity V_p is equal to

$$V_p=\frac{\tau_b H}{\rho K_0}\left(\frac{1-\lambda}{2}\right) \tag{11}$$

and the depth averaged velocity

$$V_0=\frac{1}{2}\frac{\tau_b H}{\rho K_0}\left(\frac{2-\lambda}{3}\right). \tag{12}$$

From the obtained relationships it comes, that when $\lambda=1$ then $V_p=0$. In order to make the solution more general dimensionless variable can be introduced $\eta=z/H$ and then

$$V_\xi(\eta)=3\eta\left[\frac{2-\eta(1+\lambda)}{2-\lambda}\right] \quad \text{and} \quad V_1=3\frac{1-\lambda}{2-\lambda} \tag{13}$$

where

$$V_\xi(\eta)=\frac{V_x}{V_0} \quad \text{while } V_1=\frac{V_p}{V_0}.$$

In Fig. 3 various vertical velocity distribution are shown. From that figure several particular cases ban be selected:

$$\text{for } \lambda=3, \quad q_{\text{wst.}}>q$$

$$\text{for } \lambda=2, \quad q_{\text{wst.}}=q$$

$$\text{for } \lambda=1, \quad V_p=0$$

$$\text{for } \lambda=0.5, \quad V_1=1.$$

In all those cases wind blows opposite to the main river flow.

For $\lambda=0$; $V_1=1.5$ and it means no wind case

for $\lambda=-1$; $V_1=2$, and

for $\lambda=-2$; $V_1=2.25$

and it means, that wind blows along the main river flow.

It can be seen, that in the above cases the considered curves contains one parameter λ. That parameter can be replaced by surface velocity V_1 and it gives

$$V_\xi(\eta)=3(V_1+2)\eta^2-2(V_1+3)\eta \tag{14}$$

and furthermore

$$\lambda=\frac{3-2V_1}{3-V_1}. \tag{15}$$

From eq. (15) it comes, that when $V_1=3$, then $\lambda\to-\infty$. And it means, that $V_1=3$ is the terminal surface velocity, which can not be exceded. The obtained relationships enable to formulate the following equations relating wind shear stress τ_w to the bed shear stress τ_b. We have then

$$\frac{\tau_b H}{\rho V_0 K_0}=2(3-V_1) \quad \text{and} \quad \frac{(-\tau_w)H}{\rho V_0 K_0}=4\left(V_1-\frac{3}{2}\right) \tag{16}$$

and so

$$-\tau_w+2\tau_b=6\rho\frac{V_0 K_0}{H}. \tag{17}$$

The formula (17) means, that there exists direct functional relation between wind and bed shear stress. It means also, that the wind shear stress and bed shear stress can not be chosen arbitrarly. They are not an independant boundary conditions. Bed shear stress is the result of wind action and can be calculated according to the flow conditions.

3.2. Case of expotentially varying eddy viscosity coefficient

Several authors [7, 9] suggest expotential function as the vertical distribution of the eddy viscosity coefficient. We have then

$$K_z(z)=c_1\exp\left(a\frac{z}{H}\right). \tag{18}$$

This formula includes fact, that two factors produces turbulence: wind and bottom friction. The value c_1 is the eddy viscosity coefficient at the bottom. The parameter a can be positive or negative. After integration eq. (7) we obtain

$$V_x(z)=\frac{\tau_b H}{\rho c_1 a}\left[(1+\lambda)\frac{z}{H}e^{-a\frac{z}{H}}+\left(1-\frac{1+\lambda}{a}\right)\left(1-e^{-a\frac{z}{H}}\right)\right] \tag{19}$$

and then

$$V_p=\frac{\tau_b H}{\rho c_1 a}\left[(1+\lambda)e^{-a}+\left(1-\frac{1+\lambda}{a}\right)(1-e^{-a})\right] \tag{20}$$

and furthermoıe the depth averaged velocity

$$V_0=\frac{\tau_b H}{\rho c_1 a}\left[a-(1+\lambda)(1+e^{-a})-(1-e^{-a})\left(1-2\frac{1+\lambda}{a}\right)\right]. \tag{21}$$

The above solution can be expressed in dimensionless form as follows

$$V_\xi(\eta)=\frac{a}{E_0}\left[(1+\lambda)\eta e^{-a\eta}+\left(1-\frac{1+\lambda}{a}\right)(1-e^{-a\eta})\right] \tag{22}$$

$$V_0=\frac{\tau_b H}{\rho c_1}\frac{E_0}{a^2} \tag{23}$$

where

$$E_0=a-(1+\lambda)(1+e^{-a})-(1-e^{-a})\left(1-2\frac{1+\lambda}{a}\right). \tag{24}$$

It can be seen, that

$$\lim_{a\to 0}\frac{E_0}{a^2}=\frac{2-\lambda}{6}+a\frac{\lambda-1}{12} \tag{25}$$

$$\lim_{a\to 0}V_x(\eta)=\frac{\tau_b H}{\rho c_1}\eta\left[1-\frac{1}{2}\eta(1+\lambda)\right] \tag{26}$$

$$\lim_{a\to 0}V_0(\eta)=\frac{\tau_b H}{\rho c_1}\frac{2-\lambda}{6} \tag{27}$$

$$\lim_{a\to 0}V_\xi(\eta)=\frac{6\eta-3\eta^2(1+\lambda)}{2-\lambda}. \tag{28}$$

It easy to notice, that the asymptotic cases (eqs 25 - 58) represent the solution with constant eddy viscosity coefficient.

Considering eq. (20), conditions can be determined when the surface velocity is opposite to the main flow. We have then $V_p \geqslant 0$ when

$$(1+\lambda)e^{-a}+\left(1-\frac{1+\lambda}{a}\right)(1-e^{-a})\leqslant 0. \tag{29}$$

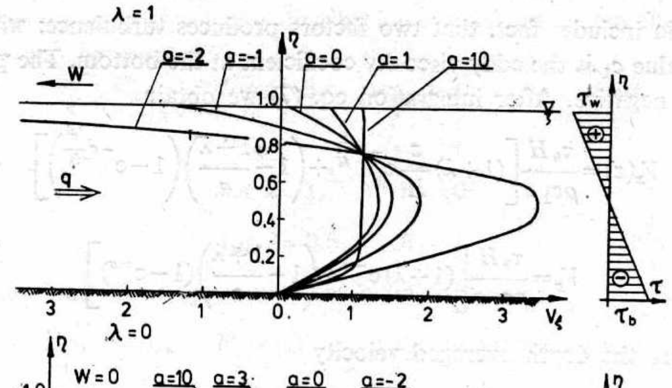

Fig. 4. Vertical velocity profiles in case of exponentially varying eddy viscosity coefficient

In Fig. 4, the vertical velocity distribution curves are shown for the exponential eddy viscosity coefficient. From that figure it comes, that big $K_z(z)$ at the water surface produces small velocity changes at that region. Similar to the previous case we have

$\lambda<0$ when wind blows along the river flow,

$\lambda=0$ when no wind, and

$\lambda>0$ when wind blows opposite to the main flow.

Following the idea from the previous chapter relationship between τ_w and τ_b was estimated. In that case it gives

$$\frac{V_0 \rho c_1 a^2}{H} = \tau_b D_b(a) + \tau_w D_w(a) \tag{30}$$

where

$$D_w(a) = 2\frac{1-e^{-a}}{a} - (1+e^{-a}), \quad D_b(a) = a - (1-e^{-a}) + D_w(a) \tag{31}$$

The asymptotic formulae are as follows:

$$\lim_{a\to 0}\frac{D_w(a)}{a^2}=-\frac{1}{6},\quad \lim_{a\to 0}\frac{D_b(a)}{a^2}=\frac{1}{3},\quad \lim_{a\to\infty}D_w(a)=-1,$$
$$\lim_{a\to\infty}D_b(a)=\infty,\quad \lim_{a\to 0}\frac{D_w(a)}{D_b(a)}=-\frac{1}{2}. \tag{32}$$

And following conditions come from those formulae

$$-1\leqslant D_w\leqslant 0,\quad 0\leqslant D_b\leqslant\infty,\quad -\frac{1}{2}\leqslant\frac{D_w}{D_b}\leqslant 0,\quad -\frac{1}{6}\leqslant\frac{D_w}{a^2}\leqslant 0,\quad 0\leqslant\frac{D_b}{a^2}\leqslant\frac{1}{3}. \tag{33}$$

It is easy to prove, that in the case when $a\to 0$ the resulting relationships reduce to the case $K_z=\text{const}$. By comparison to the previous case the present solution includes two parameters: a and λ. It enables wide verification of the theoretical investigations.

Linear Prandtl formula for the eddy viscosity coefficient was also tested

$$K_z(z)=\begin{cases}0{,}4\sqrt{\dfrac{\tau_b}{\rho}}\,z & \text{when } \delta\leqslant z<H\\[2ex] \nu & \text{when } 0\leqslant z<\delta\end{cases} \tag{34}$$

where δ is the thickness of the laminar sublayer. As a result of the calculations it comes, that such a sort of K_z distribution implies wind blowing along the main flow. It arises from the fact, that $\tau_{xz}=\text{const}$ was assummed by Prandtl in his theory. So this case can not be emplayed when the wind effect is considered.

4. Estimation of parameters of solution

The functions describing vertical velocity distribution obtained in the previous chapters contains several parameters:

- K_0, C and a representing eddy viscosity coefficient
- wind shear stress τ_w, and
- bed shear stress τ_b.

It must be stressed, that there exists direct relationship between τ_w and τ_b, and the appropriate formulae were derived earlier. Another relationship between τ_w and τ_b was obtained by considering balance of the elementary volume of water in the river [5] in the case of the uniform flow, and it gives

$$\rho g R_H I_d=\tau_b+\tau_w\frac{B}{\chi} \tag{35}$$

where χ denotes weted perimeter. The most uncertain parameter in the obtained solution is the eddy viscosity coefficient. Most of the authors among them: Rosby and Montgomery, Bowden, Van Veen, Kent and Pritchard, Bengtson relates this value to the main depth averaged velocity in the river. It leads to the conclusion, that there exists linear relationship: $K_z \sim V_0 H$. Such assumption was taken also in the present paper. Hence

$$K_0 = \kappa_0 V_0 H \tag{36}$$

$$C_1 = \kappa_2 V_0 H \tag{37}$$

Putting the above relationship to the equations describing vertical velocity distribution, the following formula for V_0 can be obtained in the case of uniform flow when wind action exists at the free water surface:

$$V_0 = \sqrt{\frac{g}{3\kappa_0}} \sqrt{R_H I_d - \frac{\tau_w}{2\rho g}\left(1 + 2\frac{B}{\chi}\right)} \tag{38}$$

By comparison it to the oryginal Chezy formula one can easy prove, that

$$C = \sqrt{\frac{g}{3\kappa_0}}, \quad \kappa_0 = \frac{g}{3C^2} \tag{39}$$

where C denotes Chézy constant. Furthermore, if C is taken according to Manning, it gives

$$\kappa_0 = \tfrac{1}{3} g n^2 R_H^{-1/3} \tag{40}$$

Under average conditions κ_0 satisfies following inequality

$$10^{-3} \leqslant \kappa_0 \leqslant 10^{-2}$$

and it agrees with the results obtained by the earlier mentioned autors.

Similarly in the case when K_z varies exponentially the relevant formula takes form

$$V_0 = \sqrt{\frac{g}{\kappa_2} \frac{D_b(a)}{a^2}} \sqrt{R_H I_d - \frac{\tau_w}{\rho g}\left[\frac{D_w(a)}{D_b(a)} + \frac{B}{\chi}\right]} \tag{41}$$

Similar to the previous case we obtain

$$C = \sqrt{\frac{g}{\kappa_2} \frac{D_b(a)}{a^2}}, \quad \kappa_2 = \frac{g}{C^2} \frac{D_b(a)}{a^2}. \tag{42}$$

In the case $a \to 0$ the equations (41) and (38) are identical. It can be seen, that wind action in equations (38) and (41) reduces the effect of water level slope. If we denote the wind effect by I_w i.e.

$$I_w = \frac{1}{R_w} \frac{\tau_w}{\rho g} \frac{1}{2} \frac{B}{\chi}$$

for K_z=const, and

$$I_w = \frac{1}{R_H} \frac{\tau_w}{\rho g}\left[\frac{D_w(a)}{D_b(a)} + \frac{B}{\chi}\right] \tag{43}$$

for exponential formula K_z, then the modified Chézy formula for the case of uniform flow including wind be expressed as

$$V_0 = C\sqrt{R_H(I_\alpha \pm I_w)} \tag{44}$$

The sign (+) in the eq. (44) means the wind blowing according to the main river flow, and sign (−) concerns case, when wind acts opposite to the river flow. In the case $W=0$, i.e. $\tau_w=0$, eq. (44) reduces to the oryginal Chezy's form. To ilustrate the method, following average conditions were chosen:

$$H=2.0\,\text{m}=R_H\,; \quad \frac{B}{\chi}=1\,; \quad I=0.00025\,; \quad C=30$$

$$W=10\,\text{m/s}\,; \quad a=0.864\,, \quad D_w/D_b=-0.405\,; \quad \frac{D_b}{a}=0.272$$

and then the wind effect gives

− for the case K_z=const; $V_0=0.64$ m/s,

− for the exponential K_z; $V_0=0.57$ m/s.

In case of very small slope (river mouth) i.e. $I=10^{-5}\div 10^{-6}$ wind of velocity 1.5 m/s reduces slope by 10%. And it means, that, for areas, where water surface slope is very small, wind action causes significant changes.

More precise estimation needs wind shear stress. This term is usually expressed as:

$$\tau_w = \kappa_3\, \rho_a\, W_{10}^2 \tag{45}$$

where

$$10^{-6} < \kappa_3 \leq 3\cdot 10^{-6}$$

The coefficient κ_3 enables to include water surface roughness due to waves. Arguczincew [1] proposes the following formula:

$$\kappa_{ij} = \kappa_3 + 0.001\,(h_b)_{ij} \tag{46}$$

where h_b is the wave height.

In open channel flow often the shear velocity is needed, in order to calculate the bed sediment transport, and it is defined as:

$$u_*^2 = \frac{\tau_b}{\rho} \tag{47}$$

In the case of uniform flow with wind action it takes form:

$$u_*^2 = gR_H I - \frac{B}{\chi}\frac{\tau_w}{\rho}\,. \tag{48}$$

5. Conclusions

The method, which enable to estimate the influence of wind action on river flow was described. The presented here method concerns simple case of flow: steady and uniform, but it can be extended to the general backwater curve assumping water surface slope instead of bed slope.

From the research it comes out, that wind shear stress and bed stress should not be treated as independent variable, unless bed stress is a matter of calculation as unknown. The obtained results indicate, that bed shear stress depends on wind action.

The shape of the vertical velocity distribution shows, that wind can create a sort of backwater curve. In this case wind blowing opposite to the river main flow makes the surface velocity weaker, and when wind velocity exceeds certain value, water at the surface starts flowing opposite to the main flow – backward current. In order to keep the discharge constant, the river depth growth.

The presented results explain vertical momentum transfer and creation of the vertical profile of velocity in the case of wind action.

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Wpływ naprężeń ścinających wywołanych przez wiatr na nurt rzeki

Streszczenie

Praca przedstawia metodę umożliwiającą określenie wpływu wiatru na nurt rzeczny w przypadku przepływu jednolitego. Metoda ta umożliwia także zastosowanie jej wyników do zjawiska cofki przy uwzględnieniu nachylenia powierzchni wody zamiast nachylenia dna. Przeprowadzone badania wykazały, iż naprężenia ścinające dna zależą od działania wiatru.

Z ukazanego rozkładu pionowego prędkości wynika, że wiatr o kierunku przeciwnym do kierunku nurtu rzeki powoduje zmniejszenie się prędkości powierzchni wody, a po przekroczeniu określonej wartości wiatru powierzchniowa warstwa rzeki zaczyna płynąć wstecz – tworzy się cofka. Jednocześnie rośnie głębokość rzeki. Przedstawiane wyniki wyjaśniają pionowe przenoszenie pędu oraz tworzenie się pionowego przekroju powierzchni w wyniku działania wiatru.