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The mathematical model for the analysis of Stokes' type waves

1. Introduction

The vibrations of a pile due to harmonic water waves were considered by P. Wilde and A. Kozakiewicz [1]. A mathematical model was proposed to decompose the measured displacements into components with the multiples of the wave frequency and the natural frequency of the pile. In a second paper [2] the same authors considered the problem of beating. The waves generated in the flume for bigger heights may be described by the Stokes' approximation in which the profile does not follow the sinusoidal form. The mathematical model proposed in [1] may be used to obtain a decomposition into harmonics but in the flume additionally free waves with double frequency are generated and thus the second harmonic is not a component of the Stokes' wave. This problem was discussed by G. Bendykowska [3] and St. Massel [4]. It is useful to construct a mathematical model which is able to find the Stokes' wave in the measurements and to calculate the free wave. For random water waves in nature it is interesting to find a Stokes' type wave with slowly in time varying amplitude from the measurements.

The present paper is devoted to the construction of a mathematical model as the basis for decomposition of measurements into a Stokes' type wave and free waves. The stochastic approach is used and the decomposition is based on the filter theory. In the case of Stokes' waves the problem is nonlinear and an iterative procedure is established which enables the use of the linear theory as proposed by Kalman [5].

The mathematical model should be very simple to obtain a useful method for data processing. Thus, simplifications are introduced. The applications to the measurements in the wave flume showed that the introduced simplifications do not cause significant differences in decompositions.

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The elevation of the free surface $\rho(x, t)$ for Stokes' waves within the second approximation is described by the following relation (see for example [6]):

$$\zeta(x, t)=\frac{H}{2}\left\{\cos(kx-\omega t)+\frac{1}{2}\frac{H}{h}\gamma\cos\left[2(kx-\omega t)\right]\right\}, \tag{1.1}$$

where h – water depth, H – wave height of the basic wave, k – wave number, ω – angular frequency and

$$\gamma=\frac{kh}{2}\operatorname{ctgh}(kh)\left[1+\frac{3}{2\sinh^2(kh)}\right].$$

The dispersion relation for the second approximation is the same as in the linear theory. Thus:

$$\frac{\omega^2 h}{g}=kh\operatorname{tgh}(kh). \tag{1.2}$$

It is possible to take the third approximation for the dispersion relation, but calculations showed that for the present study the difference in results is very small and does not justify the complications in analysis.

The Stokes' theory is based on an expansion in power series of a small parameter. The starting solution is the sinusoidal wave of the linear theory. The approximation is good if the additional terms are small. Thus, the nonlinear term in H which enters the relation (1.1) must be small compared to the first one. It is possible to take more terms in the expansion but the calculations will be more complicated.

The elevation is measured at one point. Thus $x=0$ can be taken without loss of generality. The relation (1.1) may by written in complex numbers in the following form:

$$\zeta(t)=\operatorname{Re}\left[\frac{H}{2}e^{-i\omega t}+\frac{1}{4}\frac{H^2}{h}\gamma e^{-i2\omega t}\right]. \tag{1.3}$$

A generalization will be considered, that the Stokes' wave has a slowly in time vaiying amplitude and phase shift. This case can be described by the following expression:

$$\zeta(t)=\operatorname{Re}\left[C(t)e^{-i\omega t}+\frac{\gamma}{h}C^2(t)e^{-i2\omega t}\right] \tag{1.4}$$

where

$$C(t)=\frac{H(t)}{2}e^{i\Psi(t)}=A(t)+iB(t).$$

Thus, the absolute value of $C(t)$ is the amplitude of the basic wave and the argument is the phase shift.

In the proposed formulation it is assumed that changes in amplitudes and phase shifts are so small within a period of the wave that for a neighbourhood of a fixed time it is possible to assume that they are approximately constant.

Let us introduce the following notation:

$$Z(t)=X(t)+iY(t)=C(t)e^{-i\omega t}. \tag{1.5}$$

It follows in real variables:

$$X(t) = A(t)\cos\omega t + B(t)\sin\omega t$$
$$Y(t) = -A(t)\sin\omega t + B(t)\cos\omega t. \tag{1.6}$$

In these notations the expression (1.4) assumes the following form:

$$\zeta(t) = X(t) + \frac{\gamma}{h}[X^2(t) - Y^2(t)]. \tag{1.7}$$

Now let us assume that $A(t)$ and $B(t)$ are independent stationary gaussian processes with average values equal to zero and the same correlation functions $k_{aa}(\tau)$ where $\tau = t_2 - t_1$, $t_2 \geqslant t_1$. In such a case it follows that the average values of the functions $X(t)$ and $Y(t)$ defined by (1.6) are equal to zero and the correlation functions are given by the following formulae:

$$k_{XX}(\tau) = k_{YY}(\tau) = k_{aa}(\tau)\cos\omega\tau$$
$$k_{XY}(\tau) = E\{X(t_1)\,Y(t_2)\} = -k_{aa}(\tau)\sin\omega\tau. \tag{1.8}$$

The elevation of the free surface may be also expressed in terms of the function $Z(t)$ by the following relation:

$$\zeta(t) = \mathrm{Re}\left[Z(t) + \frac{\gamma}{h}Z^2(t)\right]. \tag{1.9}$$

It follows from the relation (1.6) that:

$$CC^T = X^2(t) + Y^2(t) = A^2(t) + B^2(t) \tag{1.10}$$

where C^T is the complex conjugate.

Let us discuss the relation of the functions $X(t)$, $Y(t)$ to the pair of function $X(t)$, $\check{X}(t)$ where $\check{X}(t)$ is defined as the Hilbert transformation by the following formula:

$$\check{X}(t) = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{X(s)}{t-s}\,ds \tag{1.11}$$

In the special case of (1.6) when:

$$X(t) = A(0)\cos\omega t + B(0)\sin\omega t \tag{1.12}$$

substitution and integration in the sens of the Cauchy principal value yields the following result:

$$X(t) = A(0)\sin\omega t - B(0)\cos\omega t. \tag{1.13}$$

Comparison with the relation (1.6) shows that:

$$Y(t) = -\check{X}(t). \tag{1.14}$$

The minus sign is insignificant as far as the envelope is concerned as defined by the square root of the relation (1.10).

Now it is possible to examine the general case. The relation (1.14) would be true in the mean square sense if J defined by:

$$J = E\{[Y(t) + X(t)]^2\} \tag{1.15}$$

would be equal to zero. Substitution of (1.11) and (1.6) into (1.15) with the change of order of integration assumed to be possible yields:

$$J = 2k_{aa}(0) - \frac{2}{\pi}\int_{-\infty}^{\infty} \frac{k_{aa}(|t-s|)\sin\omega(s-t)}{s-t}\,ds. \tag{1.16}$$

In the case ω goes to infinity and $k_{aa}(|t-s|)$ satisfies the Dirichlet conditions the expression for J goes to zero. The same is true for a constant correlation function, but this is the case stated in the relation (1.12). In a general situation J is not equal to zero. For example if:

$$k_{aa}(\tau) = k_{aa}(0)\,e^{-\eta\tau} \tag{1.17}$$

it follows:

$$J = 2k_{aa}(0) - \frac{4}{\pi}\operatorname{arctg}\frac{\omega}{\eta}k_{aa}(0). \tag{1.18}$$

Thus, J goes to zero if ω goes to infinity or η goes to zero.

It is assumed in this paper that $A(t)$ and $B(t)$ are slowly varying functions as measured by the period T. Thus the correlation functions are close to constant functions and the value of J is small. In the case of the example with the correlation function (1.17) it means η/ω is a small number.

The analytical signal defined by (1.5) is not obtained with the help of the Hilbert transformation as given for example in the book [7] but as described in the book [8] where the construction of the envelope is considered. For the present study the envelope is important because it defines the local amplitude of the slowly in time varying Stokes' wave.

2. The mathematical model

The measurements are given at discrete times $t = r\Delta t$, where Δt is the time interval. Let us consider a random sequence $A(0), A(1), \ldots, A(r), \ldots$ described by the following stochastic difference equation:

$$e^{\eta\Delta t}A(k+1) - 2A(k) + e^{-\eta\Delta t}A(k-1) = aV(k+1), \quad k = 1, 2, \ldots \tag{2.1}$$

where η is a parameter with dimensions s^{-1}, a is a constant and $V(2), V(3), \ldots, V(k), \ldots$ is a sequence of independent gaussian random variables with expected values equal to zero and variances equal to one, $N(0, 1)$.

When the relation (2.1) is multiplied by $A(k)$ and the expected value is calculated it follows:

$$k(1) = k(0)/\cosh\eta\Delta t$$

where

$$k(r)=E\{A(k+r)A(k)\}=E\{A(k-r)A(k)\}. \tag{2.2}$$

When the relation (2.1) is multiplied by $A(k-r)$ where $r=1, \ldots, k$ the following difference equation is obtained:

$$e^{\eta\Delta t}k(k+1-r)-2k(k-r)+e^{-\eta\Delta t}k(k-1-r)=0. \tag{2.3}$$

It is easy to verify that the solution of (2.3) which satisfies (2.2) is given by the following formula:

$$k(s)=k(0)e^{-\eta s\Delta t}[1+s\,\mathrm{tgh}\,\eta\Delta t], \quad s=0, 1, \ldots \tag{2.4}$$

When $t=s\Delta t$ is substituted and Δt goes to zero it follows:

$$k(t)=k(0)e^{-\eta t}[1+\eta t]. \tag{2.5}$$

This is the corresponding correlation function in continuum. When $k(t)$ is differentiated and $t=0$ substituted it follows $k'(0)=0$. Thus the process is differentiable in the mean square sense.

When both sides of equation (2.1) are squared and the expected values are calculated it follows:

$$a=2\sqrt{k(0)}\sqrt{\mathrm{tgh}\,\eta\Delta t}\sinh\eta\Delta t. \tag{2.6}$$

To start the simulation of the sequence the first two random variables $A(0)$, $A(1)$ have to be calculated. They should have the covariances $k(0)$ and $k(1)$ according to the relation (2.2). Thus:

$$\begin{aligned} A(0)&=\sqrt{k(0)}\,V(0)\\ A(1)&=A(0)/\cosh\eta\Delta t+\sqrt{k(0)}\,\mathrm{tgh}\,\eta\Delta t\,V(1). \end{aligned} \tag{2.7}$$

It is easy to show that the sequence $A(0), \ldots, A(r), \ldots$ corresponds to a stationary process.

The second independent random sequence $B(0), \ldots, B(k), \ldots$ is given by identical relations.

Thus, as in the relations (1.6) the random sequences $X(k)$ and $Y(k)$ may be calculated according to the formulae:

$$\begin{aligned} X(k)&=A(k)\cos\omega k\Delta t+B(k)\sin\omega k\Delta t\\ Y(k)&=-A(k)\sin\omega k\Delta t+B(k)\cos\omega k\Delta t. \end{aligned} \tag{2.8}$$

The calculation of covariance matrices corresponding to the correlation functions (2.2) is straightforward. It follows:

$$\begin{aligned} k_{xx}(r\Delta t)&=k_{yy}(r\Delta t)=k(0)e^{-\eta r\Delta t}(1+r\,\mathrm{tgh}\,\eta\Delta t)\cos\omega r\Delta t\\ k_{xy}(r\Delta t)&=E\{X(k\Delta t)\,Y[(k+r)\Delta t]\}=-k(0)e^{-\eta r\Delta t}\sin\omega r\Delta t. \end{aligned} \tag{2.9}$$

Let us discuss the influence of the value of the parameter η. If η is equal to zero then according to (2.7) $A(1)=A(0)$ and the relation (2.6) yields $a=0$. Thus, from the difference equation (2.1) it follows that all $A(k)$ are equal to $A(0)$. In the same way it follows that $B(k)=B(0)$ and $A(0)$ and $B(0)$ are two independent random variables with variances equal to $k(0)$. The sequences $X(k)$ and $Y(k)$ represent two trigonometric functions. Thus, when the values of $\zeta(x, t)$ are substituted into (1.7) one obtains from (1.4) that $C(k\Delta t)$ is a constant complex number othe same for all k. The values H and Ψ are constants and the result is a regular Stokes' wave.

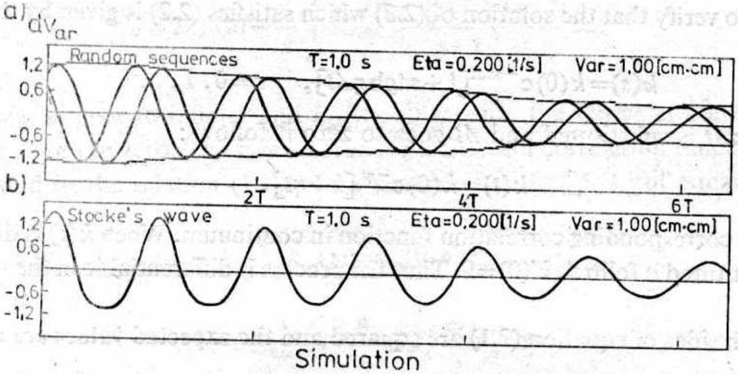

Fig. 1. a) first component with varying amplitude and phase shift, b) the corresponding Stoke's type wave

If η is small, there will be a slight modification of the Stokes' wave. The absolute value of C as calculated from (1.10) gives the amplitude of $X(t)$ and $Y(t)$. The situation is ilustrated in Fig. 1a. The function of amplitudes corresponds to the envelope of the stochastic functions $X(t)$ and $Y(t)$. The graph shows a sinusoidal wave with a slowly varying amplitude and phase shift. On Fig. 1b the corresponding Stokes' wave is shown.

The relations (2.8) may be written in the following matrix notation:

$$\mathbf{X}(k)=\boldsymbol{\varphi}_r(k)\,\mathbf{A}(k) \tag{2.10}$$

where

$$\mathbf{X}(k)=\begin{bmatrix} X(k) \\ Y(k) \end{bmatrix}, \quad \boldsymbol{\varphi}_r(k)=\begin{bmatrix} \cos\omega k\Delta t & \sin\omega k\Delta t \\ -\sin\omega k\Delta t & \cos\omega k\Delta t \end{bmatrix}, \quad \mathbf{A}(k)=\begin{bmatrix} A(k) \\ B(k) \end{bmatrix}.$$

It is easy to verify that:

$$\boldsymbol{\varphi}_r(k)=\boldsymbol{\varphi}_r^k \tag{2.11}$$

where

$$\boldsymbol{\varphi}_r=\begin{bmatrix} \cos\omega\Delta t & \sin\omega\Delta t \\ -\sin\omega\Delta t & \cos\omega\Delta t \end{bmatrix}$$

is an orthogonal matrix and thus

$$\mathbf{A}(k)=(\boldsymbol{\varphi}_r^T)^k\mathbf{X}(k). \tag{2.12}$$

The stochastic difference equation (2.1) may be written in the following block matrix notation:

$$\begin{bmatrix} \mathbf{A}(k+1) \\ \mathbf{A}(k) \end{bmatrix}=\begin{bmatrix} 2e^{-\eta\Delta t}\mathbf{I} & -e^{-2\eta\Delta t} \\ \mathbf{I} & \mathbf{0} \end{bmatrix}\begin{bmatrix} \mathbf{A}(k) \\ \mathbf{A}(k-1) \end{bmatrix}+a\,e^{-\eta\Delta t}\begin{bmatrix} \mathbf{I} \\ \mathbf{0} \end{bmatrix}\mathbf{V}(k+1) \tag{2.13}$$

where $\mathbf{I}$ is the (2×2) identity matrix, $\mathbf{0}$, (2×2) zero matrix $\mathbf{A}(k)$ is the (2×1) matrix defined in (2.10) and $\mathbf{V}^T(k)=[V(k),\ U(k)]$.

Substitution of expressions (2.13) and premultiplication by the following (4×4) matrix:

$$\begin{bmatrix} (\boldsymbol{\varphi}_r^T)^{k+1} & \mathbf{0} \\ \mathbf{0} & (\boldsymbol{\varphi}_r^T)^k \end{bmatrix} \tag{2.14}$$

yields finally the following difference equation:

$$\begin{bmatrix} \mathbf{X}(k+1) \\ \mathbf{X}(k) \end{bmatrix} = \begin{bmatrix} 2e^{-\eta\Delta t}\boldsymbol{\varphi}_r & -e^{-2\eta\Delta t}\boldsymbol{\varphi}_r^2 \\ \mathbf{I} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{X}(k) \\ \mathbf{X}(k-1) \end{bmatrix} + ae^{-\eta\Delta t} \begin{bmatrix} \boldsymbol{\varphi}_r^{k+1} \\ \mathbf{0} \end{bmatrix} \mathbf{V}(k+1) \tag{2.15}$$

The last term in the relation (2.15) needs discussion. The elements of the matrix $\mathbf{V}(k+1)$ correspond to two independent random variables $V(k+1)$ and $U(k+1)$ with distributions $N(0,1)$. The multiplication by the (4×2) matrix corresponds to the following transformation:

$$\mathbf{V}^*(k+1) = \begin{bmatrix} \boldsymbol{\varphi}^{k+1} \\ \mathbf{0} \end{bmatrix} \mathbf{V}(k+1). \tag{2.16}$$

It is easy to verify that the random variables $V^*(k+1)$ and $U^*(k+1)$ have distributions $N(0,1)$ and are independent. Thus, the matrix $\mathbf{V}^*(k+1)$ has the same properties as the matrix $\mathbf{V}(k+1)$ and they may be interchanged. Finally the relations (2.15) assume the following form:

$$\begin{bmatrix} \mathbf{X}(k+1) \\ \mathbf{X}(k) \end{bmatrix} = \begin{bmatrix} 2e^{-\eta\Delta t}\boldsymbol{\varphi}_r & -e^{-2\eta\Delta t}\boldsymbol{\varphi}_r^2 \\ \mathbf{I} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{X}(k) \\ \mathbf{X}(k-1) \end{bmatrix} + ae^{-\eta\Delta t} \begin{bmatrix} \mathbf{I} \\ \mathbf{0} \end{bmatrix} \tag{2.17}$$

Simple calculations yield the following expressions for the first two terms of the sequence:

$$\begin{aligned} \mathbf{X}(0) &= \sqrt{k(0)}\,\mathbf{V}(0) \\ \mathbf{X}(1) &= (\cosh\eta\Delta t)^{-1}\boldsymbol{\varphi}_r\mathbf{X}(0) + \sqrt{k(0)}\,\mathrm{tgh}\,\eta\Delta t\,\mathbf{V}(1). \end{aligned} \tag{2.18}$$

By the relations (2.17) and (2.18) the simulation of the random sequence $\mathbf{X}(0), \ldots$ $\ldots, \mathbf{X}(k), \ldots$ is completely described.

The relation (1.7) gives the elevation of the Stokes' wave in terms of the functions $X(t)$ and $Y(t)$. Let us assume that there is an observation noise added to the true values. Thus, the observation model is given by the following relation:

$$\zeta(k\Delta t) = X(k) + \frac{\gamma}{h}\left[X(k)^2 - Y(k)^2\right] + \sigma_0 W(k), \tag{2.19}$$

where $W(k)$ is a white noise sequence with distributions $N(0,1)$ and σ_0 is the standard deviation of observation noise.

One can generate the sequences $A(k)$ and $B(k)$ and then calculate the corresponding values of $X(k)$ and $Y(k)$ from the relation (1.6). Substitution into the relation (2.19) leads to an expression for the observation model in terms of the terms of the sequences $A(k)$ and $B(k)$. It should be stressed however that in such a formulation the coefficients in the observation model depend upon the step k and the data processing involves computer time.

3. The iterative solution

The mathematical model given by the equations (2.17) is a linear one, but the observation model is nonlinear. Thus the problem is formulated in a nonlinear theory.

One may consider the nonlinear term as a correction to the observations and rewrite the relation (2.19) in the following form:

$$\zeta(k\Delta t)-\zeta_c(k\Delta t)=\mathbf{h}\mathbf{X}(k)+\sigma_0 W(k) \tag{3.1}$$

where

$$\zeta_c(k\Delta t)=\frac{\gamma}{h}[X(k)^2-Y(k)^2], \quad \mathbf{h}=[1,0].$$

Now the observation model is written in the standard linear form and the standard Kalman filter method may be applied.

The estimator of $\mathbf{X}(k)$ based on the observation at times 1, ..., k denoted by $\hat{\mathbf{X}}(k \mid k)$ is given by the following relation:

$$\hat{\mathbf{X}}(k|k)=\hat{\mathbf{X}}(k|k-1)+\mathbf{K}[\zeta(k)-\zeta_c(k)-\mathbf{h}\hat{\mathbf{X}}(k|k-1)] \tag{3.2}$$

where $\hat{\mathbf{X}}(k \mid k-1)$ is the estimate based on the data at times 1, ..., $k-1$, $\mathbf{K}$ is the Kalman matrix.

To calculate $\zeta_c(k)$ according to (3.1) the values of $X(k)$ and $Y(k)$ are needed and they should be approximated by the estimates $\hat{X}(k \mid k)$, $\hat{Y}(k \mid k)$ which are not known at that time. An iterative solution is used. As the first approximation the predicted estimates $\hat{X}(k \mid k-1)$, $\hat{Y}(k \mid k-1)$ are used and the first approximation denoted by $\hat{\mathbf{X}}^{(1)}(k \mid k)$ is calculated. In the second approximation these values are substituted in the formula for the correction term and the second approximation for $\hat{\mathbf{X}}^{(2)}(k \mid k)$ is calculated. Thus, on this way a sequence of approximations is obtained. When from the n-th approximation the $n-1$ is substructed it follows:

$$\hat{\mathbf{X}}^{(n)}-\hat{\mathbf{X}}^{(n-1)}=\mathbf{K}\frac{\gamma}{h}[-(X^{(n-1)})^2+(X^{(n-2)})^2+(Y^{(n-1)})^2-(Y^{(n-2)})^2] \tag{3.3}$$

The expression in the square brackets on the right side includes differences of squares. One can write then as products of sums and differences. Then the expression may be considered as the scalar product of two vectors one corresponding to the differences denoted by $\Delta X^{(n-1)}$ and the second to $2\mathbf{X}(k)$. (It is assumed that the sum may be approximated by the second expression). The product of two vectors is smaller than the product of their absolute values. The length of the second vector is equal to $H(k)$ that is the wave height at that moment. The result is the following inequility:

$$\begin{bmatrix}|\Delta\hat{X}^{(n)}|\\|\Delta\hat{Y}^{(n)}|\end{bmatrix}\leqslant\frac{\gamma}{h}H(k)\begin{bmatrix}K_1\\K_2\end{bmatrix}\sqrt{(\Delta X^{(n-1)})^2+(\Delta Y^{(n-1)})^2}. \tag{3.4}$$

If the Euclidean measure is used it follows:

$$|\Delta\mathbf{X}^{(n)}|\leqslant\frac{\gamma}{h}H(k)\sqrt{K_1^2+K_2^2}\,|\Delta\mathbf{X}^{(n-1)}|. \tag{3.5}$$

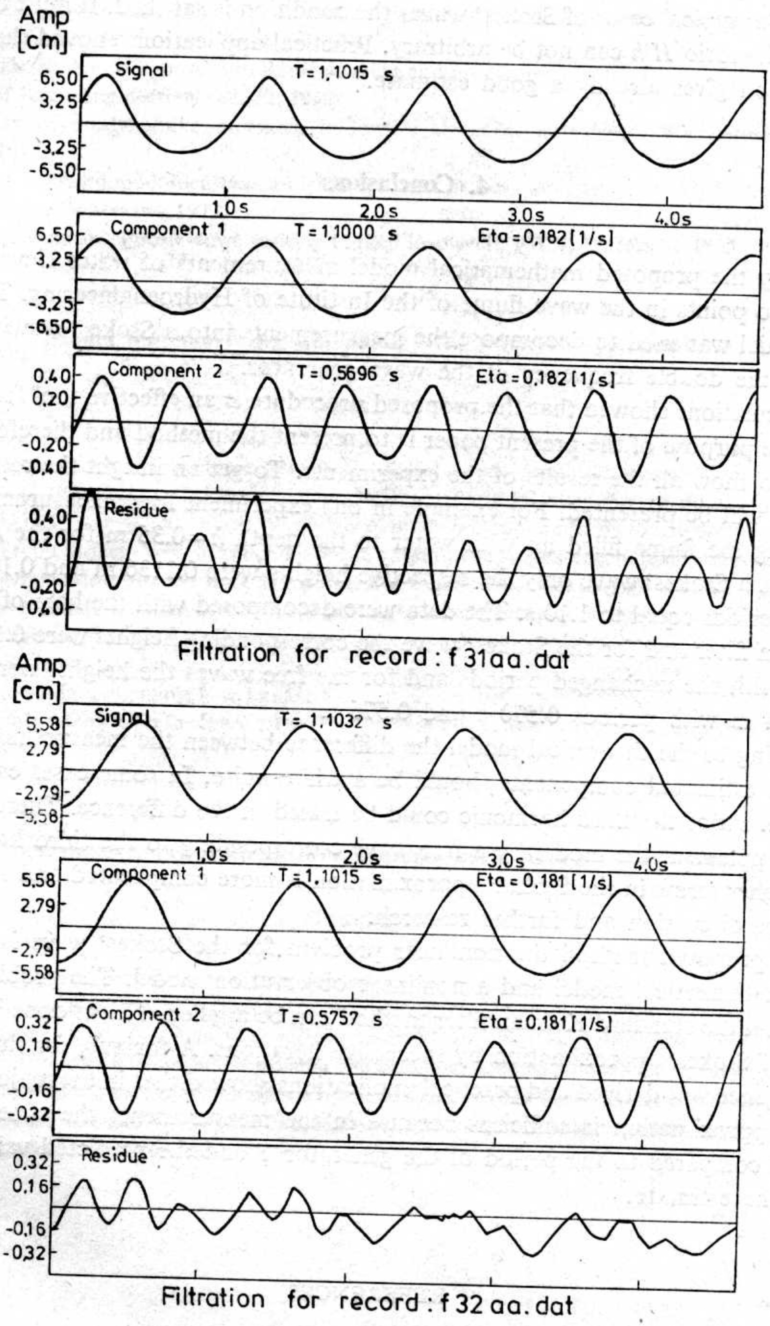

Fig. 2. Decomposition by the standard Kalman Filter

For convergence the coefficient should be smaller than one, which gives the sufficient condition. For typical cases of Stokes' waves the condition is satisfied. It must be remembered that the ratio H/h can not be arbitrary. Practical applications shoved that the first approximation gives already a good estimate.

4. Conclusions

To verify the proposed mathematical model measurements of water elevations were taken at two points in the wave flume of the Institute of Hydroengineering. The mathematical model was used to decompose the measurements into a Stokes' wave and a free wave with the double frequency of the wave generator.

The calculations showed that the proposed procedure is an effective tool for data processing. The purpose of the present paper is to present the method and therefore it is not necessary to show all the results of the experiments. To get an insight the results of one experiment will be presented. For example in one experiment from measurements a two points along the flume filled up with water to the depth $h=0.35$ m for the assumption that there is a Stokes' wave only the respective heights were 0.1136 m and 0.1087 m with the same periods equal to 1.10 s. The data were decomposed with the help of the proposed Kalman filter and for the Stokes' wave the corresponding heights were 0.1061 m and 0.1070 m with the unchanged periods and for the free waves the heights were 0.0058 m and 0.0064 m with periods 0.570 s and 0.576 s.

According to the theoretical model the difference between the measurements and the sum of the estimated components should be a white noise. In some cases especially for greater H/h ratios the third harmonic could be trated in the difference. There is no difficulty to supplement the model by a free wave corresponding to the third harmonic. To include higher terms in the Stokes' approximation is more complicated and the necessity is open for discussion and further research.

In the proposed method the nonlinear problem for the Stokes' wave is reduced to a linear mathematical model and a nonlinear observation model. The problem is linearized so that the standard Kalman filter method can be applied. This is done by following the idea of Stokes' approximation by successive iterations. A formula for the estimation of convergence was derived and practical applications showed that in the majority of cases the first approximation is sufficient because in the measurements the time interval Δt was small compared to the period of the generator T and the predicted value was very close to the estimate.

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Model matematyczny dla analizy fal wodnych typu Stokesa

Streszczenie

Liniowa teoria falowania wodnego jest słuszna dla fal o małej amplitudzie w porównaniu z długością fali oraz głębokością akwenu. Stokes podał przybliżoną teorię fal nieliniowych opracowaną przy wykorzystaniu metody małego parametru. W niniejszej pracy podano model matematyczny pozwalający na symulację fali typu Stokesa o mało zmieniającej się amplitudzie i przesunięciu fazowym w czasie jednego okresu. Obwiednia falowania jest funkcją losową. Dla zagadnienia dyskretnego w czasie opis sprowadza się, do liniowego modelu matematycznego w postaci stochastycznego macierzowego równania różnicowego oraz nieliniowego modelu obserwacji. Analizę danych pomiarowych dokonuje się metodą kolejnych przybliżeń stosując w każdym kroku metodę filtracji Kalmana. Metodę zastosowano do dekompozycji pomiarów falowania wykonanych w kanale hydraulicznym Instytutu Budownictwa Wodnego na falę Stokesa oraz falę wolną o podwojonej częstości. Badania modelowe potwierdziły przydatność proponowanej metody.

Математическая модель для анализа волн на воде типа Стокса

Содержание

В работе представлена модель позволяющая симулировать волну типа Стокса с мало меняющейся амплитудой и фазовым сдвигом на протяжении одного периода. Для задачи дискретной по времени описание сводится к линейной математической модели в виде стохастического матричного разностного уравнения и нелинейной модели наблюдения. Анализ данный измерений производится методом последовательных приближений с применением метода фильтрации Кальмана на каждом шагу. Экспериментальные модельные исследования в гидравлическом лотке подтвердили пригодность разработанного метода.