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Analysis of spatial models of turbulence

1. Introduction

The term "model of turbulence" describes a set of informations, which enables one to close the system of Reynolds equations with respect to the tensor Π (ρ=const):

$$\operatorname{div} \mathbf{u}=0, \tag{1}$$

$$\rho \frac{d\mathbf{u}}{dt}=\rho \mathbf{f}-\operatorname{grad} p+\mu \Delta \mathbf{u}+\operatorname{div} \Pi . \tag{2}$$

Symbol $\mathbf{u}$ denotes mean velocity, $\mathbf{u}'$ — velocity fluctuation, ρ — fluid density, $\mathbf{f}$ — unit mass force, p — pressure, μ — dynamic coefficient of viscosity, Π — Reynolds tensor:

$$\Pi_{i,j}=-\rho \overline{u_i' u_j'} . \tag{3}$$

In the bibliography of the problem under consideration one can distinguish two main classes of turbulence models, viz. "diffusive" and "differential".

The first one is based on the assumption that turbulent fluctuations of fluid elements as well as chaotic fluctuations of molecules are of the same character. This assumption, according to the classical Boussinesq-Prandtl hypothesis [3], leads to the conclusion that the Reynolds tensor is a linear function of the mean strain rate tensor. The proportionality factor in this constitutive equation is called "coefficient of turbulent viscosity" (kinematic ν_T or dynamic $\mu_T=\rho\nu_T$). There are many specific models of turbulence which give us opportunity to describe this coefficient [3, 10] — beginning with the simplest assumption that μ_T=const, through the classical Prandtl "mixing length hypothesis", up to more elaborated ones. An example of the latter is given by Prandtl-Kolmogoroff proposal, according to

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which the scalar coefficient of eddy viscosity is given by the following formula:

$$\mu_T = \rho \sqrt{k}\, l_m, \tag{4}$$

where k is a kinematic energy of turbulence, related to the mass unit, defined as:

$$k = \tfrac{1}{2}\overline{(u_x'^2 + u_y'^2 + u_z'^2)} \tag{5}$$

and can be calculated from Eq. (26). Symbol l_m in turn denotes the mixing length (or scale of turbulence).

The models belonging to the second class ("differential") consist of the system of equations of turbulent stress conservation. These equations have the following general form [3, 5]:

$$\frac{d\Pi_{ij}}{dt} + \overline{(u'\nabla)\,\Pi'_{ij}} = -\Pi_{im}\frac{\partial u_j}{\partial x_m} - \Pi_{jm}\frac{\partial u_i}{\partial x_m} + \overline{u_{i'}\frac{\partial p'}{\partial x_j}} + \overline{u_{j'}\frac{\partial p'}{\partial x_i}} - \overline{\mu u_{i'}\,\Delta u_{j'}} - \overline{\mu u_{j'}\,\Delta u_{i'}}. \tag{6}$$

Models of turbulence which one can find in the bibliography are properly worked out and well studied for simple cases, especially for one-dimensional flows. But practical needs and development of computational techniques provoke that recently investigators more often reach for two- or even three-dimensional models of turbulence.

It turns out then that constitutive equations for spatial models of turbulence have some important formal features which have essential influence on the final effect (no matter which particular model has been applied). An analysis of these problems is presented below.

2. "Diffusive" models of turbulence

According to the Prandtl-Kolmogoroff model, which has been mentioned above, the Reynolds tensor Π is a linear function of the mean strain rate tensor S. In the bibliography concerning this problem one can find different particular versions of constitutive equation [1, 3]. Let us analyse these proposals, from the simplest one, up to the most general, using some experimental data obtained for one-dimensional mean flow:

$$\mathbf{u} = u_x(y)\,\mathbf{i}. \tag{7}$$

A typical velocity profile is shown in Fig. 1a, and normalized components of the tensor Π – in Fig. 1b.

2.1. Scalar coefficient of eddy viscosity, homogeneous constitutive equation

In the simplest case we have the following relation:

$$\Pi_{ij} = \mu_T S_{ij}. \tag{8}$$

The analysis of the equation leads to the conclusion on the following defects:

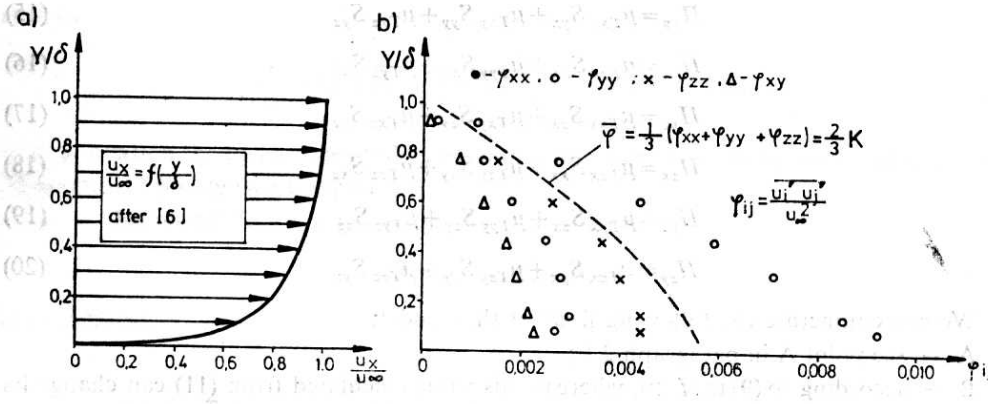

Fig. 1

A – when $S_{ij} \to 0$ (but $u \neq 0$, i.e. the flow is uniform) then $\Pi_{ij} \to 0$, what is in contradiction with observations (isotropy, [3]);

B – each component of the tensor Π in (8) is a function of only one component of the tensor S; this fact disables us to describe spatial structure of turbulence; for instance in one-dimensional case (7) only $\Pi_{xy} = \Pi_{yx} \neq 0$ and other components are equal zero what is in discrepancy with experimental data (Fig. 1);

C – traces of both sides in (8) are inconsistent:

$$\operatorname{tr} \Pi = -2\rho k < 0 \tag{9}$$

whereas:

$$\left.\begin{aligned} &\operatorname{tr}(\mu_T S) = \mu_T \operatorname{div} u = 0 && \text{– for } \rho = \text{const.} \\ &\operatorname{tr}(\mu_T S) = \mu_T \operatorname{div} u = -\frac{1}{\rho}\frac{d\rho}{dt} \gtrless 0 && \text{– for } \rho = \text{var.} \end{aligned}\right\} \tag{10}$$

D – turbulent normal stress Π_{ii} is a non-positive value (from the definition, see (3)), whereas in (8) it depends on the sign of S_{ii} which can be positive.
Reassuming we can state that these faults disquality (8).

2.2. Coefficient of eddy viscosity as a second-rank tensor, homogeneous constitutive equation

Relation $\pi(S)$ in this version is written as follows:

$$\Pi_{ij} = \mu_{Tik} S_{jk} \tag{11}$$

In Cartesian coordinates we have the following system of equations:

$$\Pi_{xx} = \mu_{Txx} S_{xx} + \mu_{Txy} S_{xy} + \mu_{Txz} S_{xz} \tag{12}$$

$$\Pi_{yy} = \mu_{Tyx} S_{yx} + \mu_{Tyy} S_{yy} + \mu_{Tyz} S_{yz} \tag{13}$$

$$\Pi_{zz} = \mu_{Tzx} S_{zx} + \mu_{Tzy} S_{zy} + \mu_{Tzz} S_{zz} \tag{14}$$

$$\Pi_{xy}=\mu_{Txx}S_{yx}+\mu_{Txy}S_{yy}+\mu_{Txz}S_{yz} \quad (15)$$

$$\Pi_{yx}=\mu_{Tyx}S_{xx}+\mu_{Tyy}S_{xy}+\mu_{Tyz}S_{xz} \quad (16)$$

$$\Pi_{xz}=\mu_{Txx}S_{zx}+\mu_{Txy}S_{zy}+\mu_{Txz}S_{zz} \quad (17)$$

$$\Pi_{zx}=\mu_{Tzx}S_{xx}+\mu_{Tzy}S_{xy}+\mu_{Tzz}S_{xz} \quad (18)$$

$$\Pi_{yz}=\mu_{Tyx}S_{zx}+\mu_{Tyy}S_{zy}+\mu_{Tyz}S_{zz} \quad (19)$$

$$\Pi_{zy}=\mu_{Tzx}S_{yx}+\mu_{Tzy}S_{yy}+\mu_{Tzz}S_{yz} \quad (20)$$

We can enumerate the following flaws of this model:

A – see point A in paragraph 2.1.;

B – according to (9) tr $\Pi<0$, whereas this value calculated from (11) can change its sign:

$$\operatorname{tr}\Pi=\mu_{Tij}S_{ij}\lesseqgtr 0 \quad (21)$$

which depends on the combination of signs of tensor S components;

C – for one-dimensional velocity field (Eq. 7) we have $\Pi_{zz}=\Pi_{zy}=\Pi_{yz}=0$ what is in contradiction with experiments (Fig. 1);

D – representation of the tensor Π in (11) is asymmetric – e.g. according to (15) and (16) $\Pi_{xy}\neq\Pi_{yx}$, whereas these values must be equal from the definition (3);

E – see point D in paragraph 2.1.

As it is seen, displacing constitutive equation (8) by (11) worsens the situation and the second version has more defects than the previous one.

2.3. Scalar coefficient of eddy viscosity, non-homogeneous constitutive equation

In this case it is assumed that the relation $\pi(S)$ contains a term which does not depend on tensor S:

$$\Pi_{ij}=\mu_{To}E_{ij}+\mu_T S_{ij} \quad (22)$$

where E_{ij} – unit tensor. Comparing the traces of this equation we have:

$$\operatorname{tr}\Pi=-2\rho k=3\mu_{To}+\mu_T \operatorname{div} u \quad (23)$$

hence:

$$\mu_{To}=-\tfrac{2}{3}\rho k-\tfrac{1}{3}\mu_T \operatorname{div} u \quad (24)$$

and we obtain instead of (22):

$$\Pi_{ij}=-(\tfrac{2}{3}\rho k+\tfrac{1}{3}\mu_T \operatorname{div} u)E_{ij}+\mu_T S_{ij}. \quad (25)$$

Thus defined model enables us to remove defects described in points A and C of paragraph 2.1, but is subject to other negative features:

A – see point B in paragraph 2.1.;

B – constitutive equation $\Pi(S)$ contains a new parameter "k"(kinetic energy of turbulence, (5); in order to determine this value we have to use the equation of the energy conservation, which can be obtained from (6) as a sum of three equations for normal stresses:

$$\frac{dk}{dt}+\overline{(u'\nabla)\,k'}=-\overline{u_i'u_j'}\,\frac{\partial u_j}{\partial x_i}-\frac{1}{\rho}\,\overline{u'\nabla p'}+\nu\overline{u'\Delta u'}. \tag{26}$$

In this situation the turbulence model becomes more complex and takes over defects of the "differential" family (see Chapter 3);

C – this model does not guarantee that the condition

$$\Pi_{ii}=-\mu_{To}+\mu_T S_{ii}\leqslant 0 \tag{27}$$

to be fulfilled.

2.4. Coefficient μ_T as a second-rank tensor, inhomogeneous constitutive equation

Ideas more complex then the previously presented have been proposed by IBRAGIMOV [4] who assumed that:

$$\Pi_{ij}=-2\rho\alpha_{ij}k+\tfrac{1}{2}(\mu_{Til}S_{lj}+\mu_{Tjl}S_{li}) \tag{28}$$

where:

$$\left.\begin{array}{ll}\alpha_{ij}=0 & \text{for } i\neq j\\ \alpha_{ij}\neq 0 & \text{for } i=j\end{array}\right\} \tag{29}$$

Analysis of Eq. (28) leads to the following conclusions:

A – the set of constitutive variables is not complete (e.g. relation which describes Π_{xy} does not contain S_{zz} what is not justified);

B – traces of both sides of Eq. (28) are not equal in general case;

C – The sign of normal turbulent stress Π_{ii} according to (28) can be positive;

D – see point B in paragraph 2.3.;

E – coefficient α must be defined as follows:

$$\alpha_{ij}=\alpha E_{ij} \tag{30}$$

otherwise α_{ij} is not a tensor which can fulfil (29).

2.5. Coefficient μ_T as a fourth-rank tensor, inhomogeneous constitutive equation

The most general linear relation between tensors Π and S has the following form [13]

$$\Pi_{ij}=\mu_{To}E_{ij}+\mu_{Tijlm}S_{lm}. \tag{31}$$

By analogy to Eqs. (23), (24) we have:

$$\operatorname{tr}\Pi=-2\rho k=3\mu_{To}+\operatorname{tr}(\mu_T S) \tag{32}$$

hence:

$$\mu_{To}=-\tfrac{2}{3}\rho k-\tfrac{1}{3}\operatorname{tr}(\mu_T S) \tag{33}$$

and:

$$\Pi_{ij} = -[\tfrac{2}{3}\rho k + \tfrac{1}{3}\,\mathrm{tr}(\mu_T S)]\, E_{ij} + \mu_{Tijlm} S_{lm}. \tag{34}$$

This form of (31) makes it possible to avoid many defects listed in previous paragraphs, but still some flaws remain:

A — the sign of turbulent normal stress depends to a certain degree on signs of tensor S components, so it is not excluded that in the course of calculations we can obtain positive values of Π_{ii};

B — see point B in paragraph 2.3.;

C — the function k in Eq. (34) has been introduced formally what has negative consequences during experimental identification of the parameters occuring in the model. In order to prove this statement let us consider a one-dimensional flow as an example (Eq. (7)). According to (34) we have:

$$\Pi_{ii} = -\tfrac{2}{3}\rho k - \tfrac{2}{3} S_{xy}(\mu_{Txxxy} + \mu_{Tyyxy} + \mu_{Tzzxy}) + 2\mu_{Tiixy} S_{xy}. \tag{35}$$

It is a system of three equations ($i = x, y, z$) with three unknown values. viz. μ_{Txxxy}, μ_{Tyyxy}, μ_{Tzzxy}. Determining Π_{xx}, Π_{yy}, Π_{zz}, k and S_{xy} from experimental data (see Fig. 1) we can try to solve this system with respect to eddy viscosity coefficients. The system can be written as follows:

$$\begin{aligned} 2\mu_{Txxxy} - \mu_{Tyyxy} - \mu_{Tzzxy} &= \beta_1 \\ -\mu_{Txxxy} + 2\mu_{Tyyxy} - \mu_{Tzzxy} &= \beta_2 \\ -\mu_{Txxxy} - \mu_{Tyyxy} + 2\mu_{Tzzxy} &= \beta_3 \end{aligned} \tag{36}$$

The determinant of this system is equal zero which means that it is impossible to compute coefficients of eddy viscosity from experimental data;

D — the model contains a considerable amount of parameters; even if we take into account the symmetry of the tensors Π and S the total number of independent coefficients as equal to 37 (36 μ_T coefficients and the kinetic energy of turbulence k).

2.6. Some corrections of diffusive model

As it is seen from our considerations, the family of diffusive models of turbulence possesses many essential defects, which practically disable us to apply this concept for spatial flows with full confidence, no matter in which way coefficients of eddy viscosity have been described.

Let us note that these defects are related in general to normal turbulent stress. Hence one can come to the conclusion that it can be profitable to neglect diagonal components of the tensor Π. There are two possibilities — we can neglect them partly or totally.

In the first case [2, 10] we can combine the term μ_{T_0} with the pressure p, writting Eq. (2) as follows:

$$\rho \frac{du}{dt} = \rho f - \mathrm{grad}\, p_k + \mu \Delta u + \mathrm{div}(\mu_T S) \tag{37}$$

where:

$$p_k = p + \mu_{T_0} = p + \tfrac{2}{3}\rho k + \tfrac{1}{3}\,\mathrm{tr}(\mu_T S) \tag{38}$$

Such a procedure (analogical to Pascal's assumption about isotropy of static pressure [7]) is partly justified because of the fact that each pressure pick-up reacts on two factors — static pressure p and dynamic pressure which includes the factor μ_{T_0}. However it is rather impossible to separate these two factors analytically.

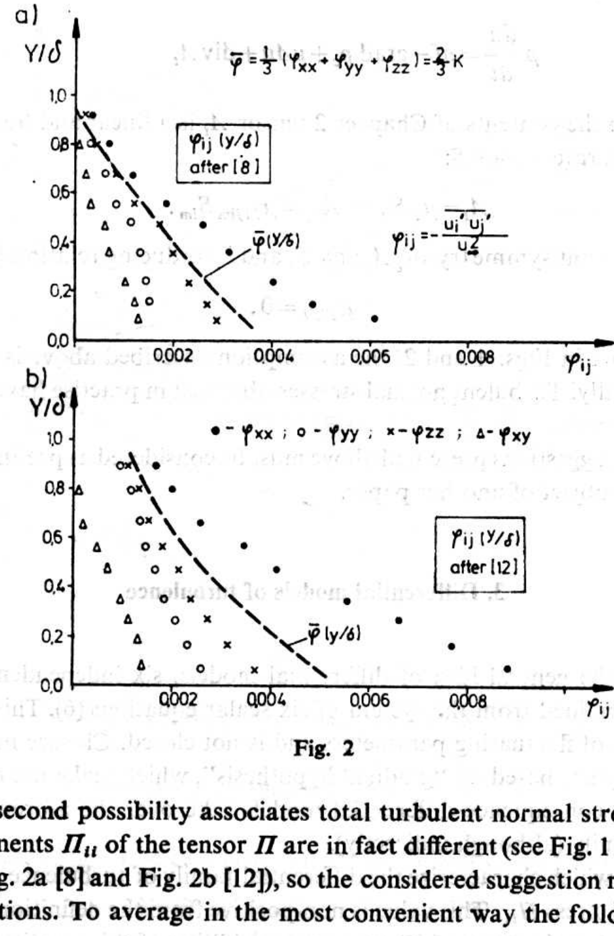

Fig. 2

The basis of second possibility associates total turbulent normal stress with the static pressure. Components Π_{ii} of the tensor Π are in fact different (see Fig. 1 and other experimental data in Fig. 2a [8] and Fig. 2b [12]), so the considered suggestion requires averaging of adequate functions. To average in the most convenient way the following assumption can be used:

$$-\rho\overline{u_x'^2}\cong-\rho\overline{u_y'^2}\cong-\rho\overline{u_z'^2}\cong-\tfrac{2}{3}\rho k. \tag{39}$$

In this manner we can replace the Reynolds tensor Π (Eq. (3)) by a certain tensor Λ:

$$\Pi\cong\Lambda\cong\begin{bmatrix}-\frac{2}{3}\rho k & -\rho\overline{u_x'u_y'} & -\rho\overline{u_x'u_z'}\\ -\rho\overline{u_x'u_y'} & -\frac{2}{3}\rho k & -\rho\overline{u_y'u_z'}\\ -\rho\overline{u_x'u_z'} & -\rho\overline{u_y'u_z'} & -\frac{2}{3}\rho k\end{bmatrix} \tag{40}$$

Now we can divide Λ into diagonal part Λ_n and non-diagonal part Λ_t:

$$\begin{aligned}\Lambda&=\Lambda_n+\Lambda_t\\ \Lambda_n&=-\tfrac{2}{3}\rho kE\\ \Lambda_t&=\Lambda-\tfrac{2}{3}\rho kE.\end{aligned} \tag{41}$$

We can combine the term Λ_n with static pressure defining the total normal stress p_c:

$$p_c = p + \tfrac{2}{3}\rho k \tag{42}$$

and the Reynolds equation assumes the form:

$$\rho \frac{du}{dt} = \rho f - \text{grad}\, p_c + \mu \Delta u + \text{div}\, \Lambda_t \tag{43}$$

where according to the contents of Chapter 2 tensor Λ_t is a linear and homogeneous function of mean strain rate tensor S:

$$\Lambda_t = \mu_T S, \quad \Lambda_{tij} = \mu_{Tijlm} S_{lm}. \tag{44}$$

Taking into account symmetry of Λ_t and S, and introducing relation (41) we can write

$$\mu_{Ttll} = 0. \tag{45}$$

As it can be seen in Figs. 1 and 2, the assumption described above is very poorly confirmed experimentally. Turbulent normal stresses observed in practise have different values, especially near the wall.

Obviously, the suggestions presented above must be considered as preliminary proposals, which will be the subject of another paper.

3. Differential models of turbulence

According to the general idea of differential models, six independent components of tensor Π are determined from the system of six scalar equations (6). This system contains averaged products of fluctuating parameters and is not closed. Closure methods presented in the bibliography are based on "gradient hypothesis", which make use of relations similar to Eq. (31). In consequence differential models take over the aforementioned defects of the diffusive family (although indirectly).

The main fault which characterize the differential family of turbulence models relates to the sign of normal stress Π_{tt}. This value is non-positive from the definition (Eq. (3)), but it is not proved so far that the form of (6) guarantees fulfilling of this condition.

In order to demonstrate this menace let us consider the equation of kinetic energy of turbulence conservation (26). For one-dimensional case (Fig. 1) we can write [5]:

$$\frac{d}{dy}\left(1.18\sqrt{k}\, l_m \frac{dk}{dy}\right) + \sqrt{k}\, l_m \left(\frac{du}{dy}\right)^2 - \frac{0.08 k^{3/2}}{l_m} = 0 \tag{46}$$

with the scale of turbulence $l_m = 0.41y$. This equation has been solved numerically, by the Runge-Kutta method, for the boundary layer, with the following boundary conditions:

$$y = 0 \rightarrow k = 0 \quad \text{and} \quad \frac{dk}{dy} = p \text{ (gived value)} \tag{47}$$

for velocity profile $u(y)$ like in Fig. 1.

The results obtained are shown in Fig. 3 (in non-dimensional system $Y=y/\delta$, $K=k/u_\infty^2$, $P=p\delta/u_\infty$, δ – boundary layer thickness, u_∞ – velocity for $y=\delta$), for $P=22-30$. As we can see there, the equation under consideration is very sensitive to the boundary conditions, and formally can give negative values of k what is inadmissible physically.

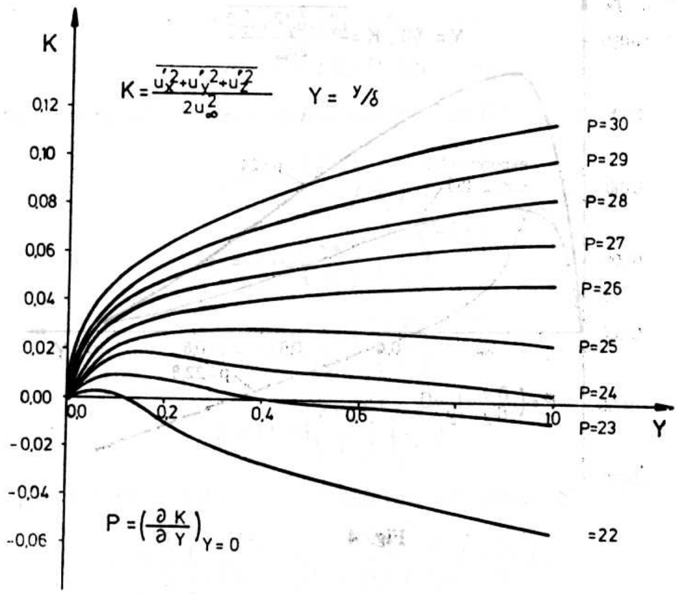

Fig. 3

As the boundary conditions (except Eqs. (47)) the following informations can be used:

$$y=\delta \rightarrow k=k_s\text{(given value) and } \frac{dk}{dy}=0. \tag{48}$$

Using empirical data (Fig. 1) we can write for the case under consideration:

$$\begin{aligned} &Y=0 \quad - \quad K=0 && \text{(a)}\\ &\quad - \quad \frac{dK}{dY}=P=22.8 && \text{(b)}\\ &Y=1 \quad - \quad K=0.0006 && \text{(c)}\\ &\quad - \quad \frac{dK}{dY}=0 && \text{(d)} \end{aligned} \tag{49}$$

Analysing curves presented in Fig. 3 we can state that:

– taking as a boundary condition the set (49 a, b) we have the solution $K(Y)$ which is partly negative (Fig. 4);
– for Eqs. (49) a, c we have calculated value $P=24$, different from the measured value (Eq. (49 b)), and the calculated profile $K(Y)$ differs from the observed curve (Fig. 4);

— for the third possibility (boundary conditions described by Eqs. (49 a, d) we have even greater difference between calculated and measured results than in the previous case (Fig. 3).

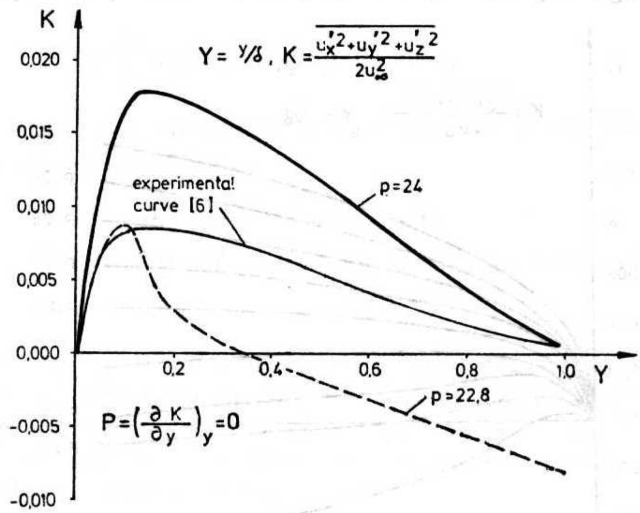

Fig. 4

In consequence we must state that similarly to the diffusive family of turbulence models differential concept does not provide a complete tool, which can fully describe mean parameters of spatial turbulent flows.

4. Convective model of turbulence

Reassuming hitherto presented conclusions we must say that it is very purposeful to maintain research works on improving existing and constructing new models of turbulence.

In this chapter a suggestion of such a new, "convective", model will be presented [9, 11]. Let us start with the statement that pulsatory velocity u' and mean velocity related to the boundary v are connected by some transformation. Basing on empirical data we can assume that this transformation is linear, so we have:

$$u' = Gv. \tag{50}$$

Substituting Eq. (50) into (3) we have:

$$\Pi_{ij} = -\rho\overline{u_i' u_j'} = -\rho A_{ijlm} v_l v_m. \tag{51}$$

The affinor G is very complex, so it is impossible to determine its components. In this situation components of the affinor A (called "coefficients of turbulence") were identified empirically.

Making use of experimental results [11] the following values of these coefficients can be proposed:

$$\begin{aligned} A_{iiii} &= \alpha_1 \\ A_{1122} &= A_{2233} = A_{3311} = \alpha_2 \\ A_{1133} &= A_{2211} = A_{3322} = \alpha_3 \\ A_{ijll} &= -\beta T_{ij} \text{ (when } i \neq j) \end{aligned} \tag{53}$$

Empirical parameters α_1, α_2, α_3 and β are defined as follows:

$$\begin{aligned} \alpha_1 &= 6.45\left(\frac{v_*}{V}\right)^2\left(\frac{\delta}{L}\right)^{-0.77} \\ \alpha_2 &= 2.62\left(\frac{v_*}{V}\right)^2\left(\frac{\delta}{L}\right)^{-0.54} \\ \alpha_3 &= 4.05\left(\frac{v_*}{V}\right)^2\left(\frac{\delta}{L}\right)^{-0.52} \\ \beta &= 11.43\left(\frac{v_*}{V}\right)^2\left(\frac{\delta}{L}\right)^{0.5} \end{aligned} \tag{54}$$

The term T_{ij} is a normalized i, j -component of the tensor S:

$$T_{ij} = \frac{1}{2}\left[\frac{\partial(v_i/V)}{\partial(H_j x_j/L)} + \frac{\partial(v_j/V)}{\partial(H_i x_i/L)}\right] = \frac{L}{V} S_{ij}. \tag{55}$$

The product $A_{ijlm} v_l v_m$ describes components of the Reynolds tensor, according to (51). Especially important is its value near the wall (when $\delta \to 0$). This question can be easily investigated for the one-dimensional case (Eq. (7)):

$$\pi_{1111}|_{\delta=0} = -\rho\left[\alpha_1 v_1(\delta)\right]|_{\delta=0} = C\frac{v_1(\delta)}{\delta^{0.77}} = \frac{0}{0}. \tag{56}$$

According to the de l'Hospital theorem we can write:

$$\pi_{1111}|_{\delta=0} = C \lim_{\delta\to 0} \frac{\partial v_1/\partial n}{0.77\delta^{-0.23}} = 0 \tag{57}$$

(n – normal to the wall), because:

$$\frac{\partial v_1}{\partial n} = \frac{\tau_0}{\mu} \neq 0. \tag{58}$$

This result is consistent with experimental results (e.g. [2, 3]).

Symbol v_* in Eq. (54) denotes shear velocity, V – characteristic velocity, δ – distance from the nearest boundary, L – characteristic linear scale of the phenomenon, H_i – Lamé parameters. Indices (i, j) determine the axes of the coordinate system:

$i, j = x, y, z$ – Cartesian coordinates
$i, j = x, r, \varphi$ – cylindrical coordinates
$i, j = \Theta, r, \varphi$ – spherical system

It is an interesting thing that althoug this model has been derived in its original version [9] from the relation (57), it shows some similarities to the diffusive concept. The considerations presented above display that turbulent normal stress has the following structure in the convective model (a stationary boundary, i.e. $v=u$):

$$\Pi_{ii} = -\rho\,[\alpha_1 u_i^2 + \alpha_2 u_j^2 + \alpha_3 u_l^2]. \tag{59}$$

Turbulent shear stress in turn can be written as follows:

$$\Pi_{ij} = \rho\beta \frac{L}{V} u^2 S_{ij}. \tag{60}$$

Comparing Eqs. (59), (60) with contents of the Chapter 2 we can notice that convective model can be considered as a version of diffusive model, in which turbulent shear stress τ_t and normal stress τ_n has been separated. The first one has been described by means of the Boussinesq-Prandtl hypothesis with scalar coefficient of eddy viscosity given by the following formula:

$$\mu_{Tc} = \rho\beta \frac{L}{V} u^2. \tag{61}$$

The normal stress in turn is defined as a combination of the mean velocity components (related to the boundary of considered area).

Convective model of turbulence presents some advantages. It avoids all defects of classical models, which were listed in previous chapters (examples of its applications and results – see [11]). In particular the condition of traces equality is fulfilled by proper choice of the model parameters, in order that:

$$\operatorname{tr}\Pi = -2\rho k = -\rho(\alpha_1+\alpha_2+\alpha_3)v^2. \tag{62}$$

On the other hand we can enumerate the following flaws of the model:

– the model relates only to near-wall flows and does not describes free turbulence;

– constitutive equation (51) contains parameters V, L and v_*, which are not precisely determined;

– convective theory does not make possible to determine all parameters, which are identified empirically.

However it seems that these defects do not disqualify the proposal described in this chapter and it is purposeful to devote more efforts in order to obtain more adequate versions of convective model of turbulence.

5. Conclusions

The paper is devoted to the problem of modelling of spatial turbulence. Analysis of problem presented in the bibliography leads to the conclusion that two classical families of turbulence models (diffusive and differential) do not enable us to obtain full description of the averaged three-dimensional turbulent flow.

In both cases the main reason of this fault results from the form of turbulent normal stresses. The diffusive family can be corrected by combining turbulent normal stress with static pressure (Pascal assumption of isotropy). As an another example of Reynolds equations closure can serve a convective model of turbulence, presented in this paper.

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Analiza przestrzennych modeli turbulencji

Streszczenie

Praca poświęcona jest problemowi przestrzennego modelowania turbulencji. Przeanalizowano warianty równań konstytutywnych prezentowanych w literaturze – od najprostszej zależności jednorodnej ze skalarnym współczynnikiem lepkości burzliwej, do najogólniejszego równania (34) w ramach rodziny modeli „dyfuzyjnych". Następnie poddano badaniu ogólną koncepcję typu „różniczkowego". Pozwoliło to stwierdzić, że niezależnie od sposobu szczegółowego określania współczynnika lepkości burzliwej, propozycje literaturowe zawierają cały szereg mankamentów, które nie pozwalają stosować ich z pełnym przekonaniem do opisu trójwymiarowych przepływów turbulentnych.

Przedstawiono dwie propozycje innego rozwiązania problemu. Pierwsza z nich polega na dołączeniu turbulentnych naprężeń normalnych do ciśnienia statycznego, natomiast druga – nazwana „konwekcyjnym modelem turbulencji" – stanowi nową propozycję równania konstytutywnego, zgodnie z którą tensor Reynoldsa Π jest liniową funkcją diady utworzonej z wektora prędkości średniej względem ścianki.

Анализ пространственных модели турбулентности

Содержание

В статье представлено анализ классических модели турбулентных течений — „диффузивных" и „дифференциальных". Показано, что несмотря на метод выражения коэффициентов турбулентной вязкости, модели турбулентности существующие в литературе посвященной механике жидкости не дают возможности описать задачу о пространственным течении.

Представлено две возможности решения проблема. Первая из них заключается в добавлении нормальных турбулентных напряженый к статистическому давлению. Последующая состоит из конвективной модели турбулентности, согласно которой тензор Рейнольдса зависит от средней скорости движения.